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Linear Lusin-measurable functionals in case of a continuous cylinder measure

by

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RÉSUMÉ. — Soit μ une mesure cylindrique sur un e. v. t. E. On donne des résultats sur l'adhérence de E' pour la topologie de convergence en μ .

SUMMARY. — Let μ be a cylinder measure on a linear topological space E. Some results concerning the closure of E' in the topology of the convergence in μ are given.

1. INTRODUCTION

Let μ be a tight probability measure on a complete locally convex space E. A measurable linear functional is called Lusin-measurable if for every positive ε there exists a convex and compact set K such that $\mu(K) > 1 - \varepsilon$ and the functional restricted to K is continuous (Slowikowski [9]). Lusin-measurable functionals form the closure of E' in $L_0(E, \mu)$ [9]. In general not every linear measurable functional is Lusin-measurable (Kanter [6a]), see also Urbanik [15] and theorem 5.6 below). Using a notion of a pre-support introduced by Slowikowski [9] we define Lusin-measurable functionals in case when μ is a continuous cylinder measure. We obtain results similar to the case when μ is tight. We use

extensively a notion of a kernel introduced by Hoffmann-Jorgensen [5] and Borell [2].

The paper is nearly self-contained. In paragraph 2 we recall some definitions and facts concerning linear topological spaces and cylinder measures. In paragraphs 3 and 4 we prove some propositions about pre-supports and kernels of cylinder measures. Some of them are known; for a survey of results about kernels and pre-supports see [3] or [4]. Paragraph 5 contains main results of this paper.

2. PRELIMINARIES

By a locally convex space we will understand a linear space with a fixed locally convex topology. So, if we say for instance that a set is compact we mean compactness in this original fixed topology. The letter E will be reserved to denote a locally convex space. E' and E^a will denote its topological and algebraical duals respectively. If Z is a subset of E then Z^0 denotes the polar set of Z i. e.

$$Z^0 = \{ f \in E' : \forall e \in Z | \langle e, f \rangle | \leq 1 \}$$

2.1. DEFINITION. — Let U be a linear subspace of E , and let h be a linear functional defined on U . If h restricted to any compact and convex subset of U is continuous then we will say that it is almost uniformly continuous on U . A topology on E' given by polars of compact and convex subsets of U will be called the topology of almost uniform convergence on U and will be denoted by τ_U .

2.2. DEFINITION. — A linear subspace is called standard if it is a union of countably many compact convex sets. It is called quasi-standard if these sets are closed and convex only.

The following theorem is a version of Grothendieck's Completeness Theorem (cf. [6], p. 248).

2.3. THEOREM. — Let U be a dense standard subspace of a locally convex space E . Let U^* be the space of linear functionals almost uniformly continuous on U . Then U^* is the completion of E' in τ_U .

2.4. REMARK. — Since U is standard τ_U coincides with the Mackey topology $\tau(E', U)$.

A subset Z of E is called a cylinder set if it is of the form $Z = T^{-1}(B)$, where T is a continuous linear map from E into \mathbb{R}^n and B is a Borel subset of \mathbb{R}^n . A positive normed set function μ on the algebra of cylinder sets is a cylinder measure if for every T as above $\mu \circ T^{-1}$ is a σ -additive Borel measure on \mathbb{R}^n .

With every cylinder measure we can associate a linear map T_μ from E' into $L_0(\Omega, \mathcal{M}, P)$ (so called « adjoint linear stochastic process », cf. [1]). We say that a cylinder measure μ is continuous if T_μ is i. e. if, on E' , τ_E is stronger than the topology s_μ of the convergence in μ . A cylinder measure μ is full if for every non-zero element f of E' $\mu(f^{-1}(\{0\})) < 1$. If μ is full and U is a dense linear subspace of E then μ is also a full cylinder measure on U endowed with the induced topology.

3. PRE-SUPPORTS OF A CYLINDER MEASURE

Let μ be a full cylinder measure on a locally convex space E and let the dual space E' be endowed with the topology τ_E .

3.1. DEFINITION. — A linear subspace U of E is a pre-support of μ if $\forall \varepsilon > 0 \exists K_\varepsilon \subset U$, convex and compact such that

$$(*) \quad \forall f \in E' \quad f \in K_\varepsilon^0 \Rightarrow \mu(e \in E : |\langle e, f \rangle| \leq 1) \geq 1 - \varepsilon$$

A symmetric convex and compact set which fulfills (*) will be called a set of (up to) ε -concentration.

3.2. REMARK. — Let U be a dense subspace of E . The following conditions are equivalent:

- i) U is a pre-support of μ ;
- ii) μ is a continuous cylinder measure on U ;
- iii) U contains a standard subspace R such that on E' the topology τ_R is stronger than the topology s_μ of convergence in μ .

3.3. PROPOSITION. — The intersection of countably many pre-supports is a pre-support.

Proof. — Let $K = K_1 \cap K_2$, where K_1 and K_2 are sets of ε_1 - and ε_2 -concentration respectively. We have:

$$K^0 = \bigcup_{\lambda \in (0,1)} \lambda K_1^0 + (1 - \lambda) K_2^0$$

It results that $\mu(e \in E : |\langle e, f \rangle| \leq 1) \geq 1 - (\varepsilon_1 + \varepsilon_2)$.

Hence K is a set of $(\varepsilon_1 + \varepsilon_2)$ -concentration.

Now let (U_n) be a sequence of pre-supports and fix $\varepsilon > 0$.

For every n let K_n be a set of ε_n -concentration contained in U_n , where $\sum \varepsilon_n = \varepsilon/2$. We shall show that $K = \bigcap K_n$ is a set of ε -concentration. Suppose it is not so. Then for some $f \in K^0$ and for some $\delta > 0$ $\mu(e \in E : |\langle e, f \rangle| \geq 1 + \delta) > \varepsilon$.

Put $C_n = \bigcap_{i=1}^n K_i$. For each n C_n is a set of $\varepsilon/2$ concentration and the sequence (C_n) decreases to K .

By a standard topological argument there exists a number n_0 such that

$$C_{n_0} \subset \{e \in E : |\langle e, f \rangle| < 1 + \delta\}$$

But this is contradictory to the fact that C_{n_0} is a set of $\varepsilon/2$ -concentration. The proof is completed.

3.4. COROLLARY. — If μ is continuous then pre-supports and $\sigma(E, E')$ -pre-supports are the same.

Proof. — Obviously a pre-support in a stronger topology is a pre-support in a weaker one. Conversely, let U be a $\sigma(E, E')$ -pre-support. We may assume that U is $\sigma(E, E')$ -standard. Since μ is continuous there exist a standard pre-support \tilde{U} . By Proposition 3.3 $\tilde{U} \cap U$ is a $\sigma(E, E')$ -pre-support. But $\tilde{U} \cap U$ is a standard subspace. Thus U is a pre-support (in the original topology).

3.5. PROPOSITION. — If μ is σ -additive then every standard pre-support equals to the intersection of all measure one quasi-standard linear subspaces which contain it.

Proof. — Let U be a standard pre-support spanned by a decreasing (with the increase of epsilon) family of symmetric convex and compact sets $\{K_\varepsilon\}_{0 < \varepsilon < 1}$, where for each ε K_ε is a set of ε -concentration. Let $e_0 \in E \setminus U$. Take a decreasing sequence (ε_n) of positive numbers such that $\sum \varepsilon_n = 1$. Fix $n \geq 1$. For every positive integer k take a linear functional f_k^n such that $\langle e_0, f_k^n \rangle = k$ and $f_k^n \in K_{\delta(n,k)}^0$, where $\delta(n, k) = 2^{-k} \varepsilon_n$. Put

$$W_n = \bigcap_{k=1}^{\infty} \{e \in E : |\langle e, f_k^n \rangle| \leq 1\}.$$

Let $W = \bigcup_{m=1}^{\infty} \bigcap_{n=m}^{\infty} W_n$. It is easy to see that $\text{span}(W)$ is a measure one quasi-standard subspace not containing e_0 .

3.6. — PROPOSITION. — If μ is continuous and σ -additive then every measure one quasi-standard linear subspace is a pre-support.

Proof. — The proof can be done just like the proof of Proposition 3.3. So we omit it.

4. THE KERNEL OF A CONTINUOUS CYLINDER MEASURE

From now on we make an assumption that μ is continuous.

4.1. DEFINITION. — The intersection of all pre-supports is called the kernel of a continuous cylinder measure μ and will be denoted by J_μ . Let us denote by $\tilde{\mu}$ the probability measure on $E^{\prime a}$ (= the algebraic dual of E') which corresponds to μ in a natural way (cf. [1]). Since μ is continuous $\tilde{\mu}$ is continuous too.

4.2. PROPOSITION. — a) $J_\mu = J_{\tilde{\mu}}$.

b) $J_{\tilde{\mu}}$ equals to the intersection of all quasi-standard $\tilde{\mu}$ -measurable linear subspaces of $E^{\prime a}$ of measure $\tilde{\mu}$ one.

Proof. — If a linear subspace U of E is pre-support of μ it is also a pre-support of $\tilde{\mu}$. By the continuity of $\tilde{\mu}$ and proposition 3.3 we get that $J_\mu = J_{\tilde{\mu}}$. The second assertion follows directly from Propositions 3.5 and 3.6.

We give now two useful characterizations of the kernel J_μ .

4.3. PROPOSITION. — Let $F_\varepsilon = \{f \in E' : \mu(e \in E : |\langle e, f \rangle| \leq 1)\} \geq 1 - \varepsilon$. Let $B_\varepsilon = \bigcap_{f \in F_\varepsilon} \{e \in E : |\langle e, f \rangle| \leq 1\}$. Then $J_\mu = \text{span}(\{B_\varepsilon\}_{0 < \varepsilon < 1})$.

4.4. PROPOSITION. — $J_\mu = (E', s_\mu)'$.

Proof. — We will prove that $\text{span}(\{B_\varepsilon\}) \subset (E', s_\mu)' \subset J_\mu \subset \text{span}(\{B_\varepsilon\})$. Fix $0 < \varepsilon < 1$ and $\bar{e} \in B_\varepsilon$. Let $(f_n) \in E'$ converges to zero in s_μ . We have

$$\forall \delta > 0 \quad \exists n_0 \forall n \geq n_0 \quad \mu(e \in E : |\langle e, f_n \rangle| \leq \delta) \geq 1 - \varepsilon$$

Thus

$$\forall n \geq n_0 \quad \delta^{-1} f_n \in B_\varepsilon^0.$$

It follows that $(\langle \bar{e}, f_n \rangle)$ tends to zero. Since (f_n) was an arbitrary sequence converging to zero in s_μ this implies that $\bar{e} \in (E', s_\mu)'$. This proves the first inclusion.

Since μ is continuous $(E', s_\mu)'$ is contained in E . Take $\bar{e} \in E \setminus J_\mu$. By the

definition of J_μ there exists a pre-support U such that $\bar{e} \notin U$. For $0 < \varepsilon < 1$ let K_ε be a set of ε -concentration contained in U . There exists a sequence (f_n) of continuous linear functionals such that $nf_n \in K_{1/n}^0$ and $\langle \bar{e}, f_n \rangle = 1$. It follows that (f_n) converges to zero in s_μ . Hence $\bar{e} \notin (E', s_\mu)'$ and the second inclusion is proved. Let $\bar{e} \in E \setminus \text{span}(\{B_\varepsilon\})$. We want to show that \bar{e} is not an element of J_μ . By proposition 4.2 it is enough to show the existence of a quasi-standard linear subspace E_0 of E'^a such that $\tilde{\mu}(E_0) = 1$ and $\bar{e} \notin E_0$. Let \tilde{B}_ε denote the analogue of B_ε defined for $\tilde{\mu}$. It is easy to see that $\tilde{B}_\varepsilon \cap E = B_\varepsilon$. Thus $\bar{e} \in E'^a \setminus \text{span}(\{\tilde{B}_\varepsilon\})$. By the definition of $\{\tilde{B}_\varepsilon\}$ for every $\varepsilon > 0$ there exists a sequence $(f_n) \in E'$ such that $\langle \bar{e}, f_n \rangle = n$ and $\tilde{\mu}(e \in E'^a : |\langle e, f_n \rangle| \leq 1) \geq 1 - \varepsilon/2^n$. Now it is enough to use an argument from the end of the proof of Proposition 3.5.

4.5. REMARK. — It is clear that sets B_ε which appeared in Proposition 4.3 are closed, absolutely convex and they increase when epsilon decreases. They are also compact because each B_ε is contained in every set of ε -concentration. Conversely, the intersection of all sets of ε -concentration is contained in $B_{\varepsilon/2}$.

4.6. COROLLARY. — The kernel is a standard subspace.

The kernel is defined as the intersection of all pre-supports. By Proposition 3.3 an intersection of countably many pre-supports is a pre-support. The following theorem gives a necessary and sufficient condition to ensure that the kernel is a pre-support.

4.7. THEOREM. — Let μ be a full and continuous cylinder measure on a locally convex space E . The following conditions are equivalent:

- i) the kernel J_μ of μ is a pre-support of μ ;
- ii) E' is locally convex in the topology s_μ of the convergence in μ .

Proof. — If J_μ is a pre-support then τ_{J_μ} is stronger than s_μ . On the other hand by Proposition 4.4 s_μ is stronger than $\sigma(E', J_\mu)$. It follows (bornology argument) that s_μ is stronger than $\tau(E', J_\mu)$. Since J_μ is standard this ends the proof that (i) implies (ii).

Conversely, if (E', s_μ) is locally convex then since it is metrisable and since $J_\mu = (E', s_\mu)'$ it follows that $(E', s_\mu) = (E', \tau(E', J_\mu))$. By Corollary 3.4 the last equality implies that J_μ is a pre-support.

4.8. REMARK. — It can be shown (cf. [8]) that (E', s_μ) is nuclear if and only if μ is σ -additive and $\mu(J_\mu) = 1$.

5. LINEAR LUSIN-MEASURABLE FUNCTIONALS

Let μ be a full and continuous cylinder measure on a locally convex space E . Let D be a standard pre-support of μ and let h be a linear functional almost uniformly continuous on D . We will denote by X the space of such pairs (h, D) factored by the following equivalence relation:

$$(h_1, D_1) \sim (h_2, D_2) \quad \text{if there exists} \quad (h_3, D_3)$$

such that

$$D_3 \subset D_1 \cap D_2 \quad \text{and} \quad h_1|_{D_3} = h_2|_{D_3} = h_3.$$

By Proposition 3.3 X is a linear space.

5.1. DEFINITION. — Elements of X will be called linear Lusin-measurable functionals. They will be denoted by x or by (h, D) .

5.2. THEOREM. — The above constructed space X is the completion of E' in the topology s_μ of convergence in $\mu : X = \overline{(E', s_\mu)}$. More precisely:

- a) for every linear Lusin-measurable functional (h, D) there exists a Cauchy sequence in (E', s_μ) converging to h almost uniformly on D ;
- b) every Cauchy sequence in (E', s_μ) contains a subsequence which converges almost uniformly on some pre-support D ;
- c) the following conditions are equivalent:

- i) $(h_1, D_1) \sim (h_2, D_2)$
- ii) if for $i = 1, 2$ (f_n^i) is a sequence of elements of E' converging to h_i almost uniformly on D_i then $(f_n^1 - f_n^2)$ converges to zero in s_μ .

Proof. — a) Follows immediatly from Theorem 2.3 and from the definition of pre-support.

b) Let (f_n) be a Cauchy sequence in (E', s_μ) . Thanks to Egoroff's theorem there exists a subsequence (f_{n_k}) of (f_n) and an increasing sequence (F_m) of closed subsets of E^a such that for every m (f_{n_k}) converges uniformly on F_m and $\tilde{\mu}(F_m) > 1 - \frac{1}{m}$. Evidently, F_m can be replaced by its closed absolutely convex hull F'_m . Let U be a standard pre-support of μ . $U = \bigcup_{m=1}^{\infty} K_m$, where K_m is a set of $\frac{1}{m}$ -concentration and $K_m \subset K_{m+1}$. Let us put

$$D = U \cap \text{span} (\{ F'_m \}_{m=1}^{\infty}).$$

By Propositions 3.3 and 3.6 D is a pre-support of μ . On the other hand for every m (f_{n_m}) converges uniformly on $C_m = m(K_m \cap F'_m)$ and $D = \bigcup_{m=1}^{\infty} C_m$. This proves (b).

c) (i) implies (ii) by the definition of pre-support. The reversed implication can be proved in the same way as (b).

5.3. REMARK. — Every linear Lusin-measurable functional is almost uniformly continuous on J_μ . However, it is not true in general that every linear functional defined and almost uniformly continuous on J_μ can be extended to a Lusin-measurable one. (For instance if $E = \mathbb{R}^\infty$ and μ is an infinite product of p -stable laws, $0 < p < 1$, then $J_\mu = l^\infty$, but $X = l^p$ not l^1). On the other hand it can happen that $J_\mu = \{0\}$ (cf. [12]).

The following theorem is a completion of Theorem 4.7.

5.4. THEOREM. — The following conditions are equivalent:

- i) every linear functional almost uniformly continuous on J_μ has a unique extension to a Lusin-measurable one;
- ii) $(X, s_\mu) = \overline{(E', \tau(E', J_\mu))}$
- iii) (X, s_μ) is locally convex
- iv) J_μ is a pre-support of μ .

Proof. — Obviously we only have to prove that (i) implies (ii). Let J_μ^* denote the space of functionals almost uniformly continuous and linear on J_μ . By (i) J_μ is dense in E , so by Theorem 2.3 and Remark 2.4

$$(J_\mu^*, \tau_{J_\mu}) = \overline{(E', \tau(E', J_\mu))}$$

Let I denote the map from X into J_μ^* that associates with every Lusin-measurable functional its restriction to J_μ . By (i), Theorem 5.2.b and Corollary 4.6. I is a continuous linear bijection from (X, s_μ) onto (J_μ^*, τ_{J_μ}) . Thus, by the Open Mapping Theorem of Banach, I^{-1} is also continuous. This finishes the proof.

Let us call a pre-support hilbertien if it is of the form $\text{span } K$, where K is a compact absolutely convex set and the Minkowski functional of K can be induced by a scalar product. There are, of course, continuous cylinder measures which have no hilbertien pre-support. However, every tight probability measure on a Frechet space has « sufficiently rich » family of hilbertien pre-supports.

5.5. THEOREM. — Let μ be a tight probability measure on a Frechet

space E . Then there exists a family $(U_\sigma)_{\sigma \in \Sigma}$ of pre-supports of μ with the following properties:

- i) U_σ is hilbertien for every $\sigma \in \Sigma$;
- ii) for every Lusin-measurable functional x there exists $\sigma \in \Sigma$ such that x admits a representation (h, U_σ) ;

iii) $\bigcap_{\sigma \in \Sigma} U_\sigma = J_\mu$.

Proof. — By a result of Kuelbs (cf. [7]) there exists a Banach space E_0 continuously embeded in E such that μ is a tight measure on E_0 . Thus without loss of generality we can assume that E is a separable Banach space. Let Σ be the set of bounded Borel functions σ on E , μ -almost everywhere positive and such that $\int_E \|e\|^2 \sigma(e) d\mu(e)$ is finite. For every $\sigma \in \Sigma$ let T_σ be the identity operator from E' into $L_2(\sigma d\mu)$. T_σ is compact (cf. [14], Proposition 3a).

The adjoint operator T' is given by the Bochner integral:

$$L_2(\sigma d\mu) \ni g \xrightarrow{T'_\sigma} \int_E g(e)\sigma(e)ed\mu(e) \in E.$$

We put $U_\sigma = T'_\sigma(L_2(\sigma d\mu))$. From Chebyshev Inequality and from the compactness of T'_σ it follows that U_σ is a pre-support. Obviously U_σ is hilbertien. Let x be a Lusin-measurable functional and let (f_n) be a sequence of elements of E' converging μ -almost surely to x . Then for

$$\sigma(e) = \min (\|e\|^{-2}, (1 + \sup_n \langle e, f_n \rangle^2)^{-1})$$

(f_n) converges almost uniformly on U_σ . This proves (ii).

Finally let $e \in \bigcap_{\sigma \in \Sigma} U_\sigma$ and let $(f_n) \in E'$ converge to zero in s_μ . To finish the proof we have to show that $(\langle e, f_n \rangle)$ converges to zero. Suppose it is not so. Taking, if necessary, a subsequence we can assume that (f_n) converges to zero μ -almost surely but $\langle e, f_n \rangle > \varepsilon > 0$. This contradicts the fact that $e \in U_\sigma$, where σ is constructed as above. This finishes the proof.

At the end of this paragraph we give an example of an infinite dimensional probability measure with interesting properties. A construction of this example is based on the following theorem of S. Mazur:

THEOREM. — (S. Mazur) ⁽¹⁾. Let (p_n) be an increasing sequence of integers

⁽¹⁾ To appear.

with $p_0 = 0$ such that $p_{n+1}(p_n)^{-1} > 1 + \varepsilon$, $\varepsilon > 0$, $\varepsilon(1 + \varepsilon)^{1 + \varepsilon^{-1}} > 2^{-1}$, $n = 1, 2, \dots$. Let (f_k) , $k = 1, 2, \dots$ be a sequence of functions on the interval

$[0, 1)$ of the form $f_k(t) = \sum_{n=0}^{\infty} c_{k,n} t^{p_n}$. Suppose that (f_k) converges in

Lebesgue measure to some function f . Then

1) for every n $(c_{k,n})$ converges to c_n ;

2) $f(t) = \sum_{n=0}^{\infty} c_n t^{p_n}$;

3) (f_k) converges to f uniformly on every subinterval $[0, r]$, $r < 1$.

5.6. THEOREM. — Let E be an infinite dimensional Frechet space. Then there exists a tight probability measure μ on E with the following properties:

1) every μ -measurable functional is a μ -measurable linear functional;
 2) there are μ -measurable linear functionals which are not Lusin-measurable;

3) $\mu(J_\mu) = 1$.

Proof. — Assume first that $E = \mathbb{R}^\infty$. Let T be a map from the unit interval $[0, 1]$ into \mathbb{R}^∞ given by $T(t) = (t^{p_n})$, where (p_n) is as above.

We put $\tilde{\mu} = \lambda \circ T^{-1}$, where λ is the Lebesgue measure. Since $\tilde{\mu}$ is supported by a linearly independent subset $\{(t^{p_n})_{n=1}^\infty\}_{0 < t < 1}$ of \mathbb{R} property 1 is clearly fulfilled. Properties 2 and 3 follow directly from Mazur's theorem.

For general E let K be an infinite dimensional symmetric convex compact subset of E (such K exists in every complete infinite dimensional linear metric space by Mazur's argument (cf. [1a], p. 268)). There exists an affine homeomorphism G from the subset $[-1, 1]^\infty$ of \mathbb{R}^∞ on to a subset of K such that $G(0) = 0$ (cf. [16], p. 321). It is easy to see that $\mu = \tilde{\mu} \circ G^{-1}$ has properties 1, 2, 3.

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