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Section B :
Calcul des Probabilités et Statistique.

A queue with server of walking type (autonomous service) (*)

by

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ABSTRACT. — Queues with autonomous service (QAS) represent service systems in which the server becomes unavailable for a random time after each service epoch. Such systems have been used to model secondary memory devices in computer systems (e. g. paging disks or drums). The queue with « server of walking type » studied by Skinner [*1*] is a special instance of our model. This model has also been considered by Borovkov [*4*].

Assuming general independent interarrival times we obtain an operational formula relating the waiting time in stationary state of a QAS to the waiting time of the GI/G/1 queue. This result dispenses the need for analysis of the QAS in special cases and generalizes the result of Skinner [*1*], or that of Coffman [*2*] for a paging drum. Sufficient conditions for stability or instability of the system are also obtained.

RÉSUMÉ. — Les files d'attente avec « serveur autonome » représentent des files dans lesquelles la station de service s'absente après chaque service rendu aux clients. Elles sont utilisées en informatique pour modéliser les unités de mémoire secondaire (disques à têtes fixes ou à bras movible) qui comportent des temps d'accès ou de positionnement avant chaque transfert d'information.

Ce modèle a été étudié par Skinner [*1*] et par Borovkov [*4*] dans des cas particuliers.

Nous démontrons une formule générale permettant de calculer le temps d'attente à l'état stationnaire pour la file à serveur autonome à partir du temps d'attente à l'état stationnaire pour une file GI/G/1 qui lui correspond. Ce résultat permet d'éviter une analyse cas par cas de ce modèle, et généralise les résultats de Skinner [1]. Une application au modèle du tambour de pagination étudié par Coffman [2] est présentée.

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1. INTRODUCTION

We examine a single server, first-come-first-served service center to which customers arrive according to a renewal process. Let $A_1, A_2, \ldots, A_n, \ldots$ denote the interarrival times, and denote by $s_1, s_2, \ldots, s_n, \ldots$ the service times of the successive customers. After serving the n-th customer the server becomes idle for a time $T_n \geq 0$. We write $S_n \equiv s_n + T_n$, $n \geq 1$, and assume that $S_1, S_2, \ldots, S_n, \ldots$ is a sequence of i. i. d. (independent and identically distributed) random variables, independent also of the interarrival times.

Suppose that the queue is empty at time $s_k + T_k$; the server becomes once again available for service at times

$$s_k + T_k + \bar{S}_1^k + \bar{S}_2^k, \ldots, s_k + T_k + \bar{S}_1^k + \bar{S}_2^k + \ldots + \bar{S}_n^k, \ldots$$

That is, service will resume for the $(k+1)$-th customer which arrives at time $a_{k+1} \equiv \sum_{1}^{k+1} A_i$ at time $s_k + T_k + \sum_{1}^{l(a_{k+1})} \bar{S}_i^k$, where

$$l(a_{k+1}) = \inf \{ l : s_k + T_k + \Sigma_1^l \bar{S}_i^k \geq a_{k+1} \}$$

We assume that the $\{ \bar{S}_n^k \}_{n,k \geq 1}$ are i. i. d. and independent of the interarrival times and of the sequence $\{ S_n \}_{n \geq 1}$. In the sequel, we shall drop the index k associated with $\bar{S}_n^k$ in order to simplify the notation, though it will be understood that the variables associated with the end of different busy periods are distinct.

The model we consider arises in many applications. In computer systems [2, 3, 5] it serves as a model of a paging drum (in this case S and $\bar{S}$ are constant and equal). In data communication systems it can serve to represent a data transmission facility where transmission begins at predetermined instants of time.

Using the terminology of Skinner [1] who analyzed the model assuming Poisson arrivals, we shall call it a queue with server of walking type: after each service the server « takes a walk ». Borovkov [4] studies a related model which he calls a queue with « autonomous service ».

The purpose of this paper is to obtain a general formula relating the waiting time W_n of the n-th customer in our model to the waiting time of the n-th customer V_n in an equivalent GI/G/1 queue, $n \geqslant 1$. This equivalent GI/G/1 queue has the same arrival process, but the service times are $S_1, S_2, \ldots, S_n, \ldots$ and $V_{n+1} = [V_n + S_n - A_{n+1}]^+$. This result allows us to dispense with a special analysis of our queueing model in stationnary state since we can obtain the result directly from the known analysis of the corresponding GI/G/1 queue.

The formula (Theorem 4) is derived in Section 2 together with sufficient conditions for ergodicity. Section 3 contains an application to the paging drum model.

1.1. Relation to previous work

Let us briefly review previous work on the subject.

Borovkov ([4], Chapter 8) defines a system with arrivals according to a renewal process and in batches, and with service also in batches. According to the notations defined above, he assumes that the $T_n \equiv 0$ and that the $\bar{S}_n$ are distributed as the S_n, $n \geq 1$. Furthermore he considers various special cases for the distribution of the S_n and the A_n. His main result is that the queue length distribution (where the queue does not include the customers in service) of the above system is identical to the queue length distribution of a conventional queue (with batch arrivals and batch service) if the service times are exponentially distributed.

The model considered by Skinner [1] is a special case of the one we study since he assumes that the arrival process is Poisson; otherwise it is identical to ours. He obtains the generating function for the queue length distribution in stationary state.

2. PROPERTIES OF THE WAITING TIME PROCESS

Consider the sequence $W_1, W_2, \ldots, W_n, \ldots$ where W_n is the waiting time of the n-th customer arriving to the queue. We shall first prove that the W_n, $n \geqslant 1$, satisfy a simple recurrence relation. Let $\xi_n = S_n - A_{n+1}, n \geqslant 1$.

LEMMA 1. —

$$W_{n+1} = \eta(-W_n - \xi_n), \qquad n \geqslant 1 \tag{1}$$

where $\eta(.)$ is defined by

$$\eta(x) = \begin{cases} -x & \text{if } x \leqslant 0 \\ \Sigma_1^{l(x)} \bar{S}_j - x, & \text{if } x > 0 \end{cases}$$

where we define for $x > 0$:

$$(2) \qquad l(x) = \inf\{\, l : \Sigma_1^l \bar{S}_i \geqslant x,\ l > 0 \,\}$$

Proof. — The n-th customer arrives to the queue at time $\Sigma_1^n A_i$ and begins service at $\Sigma_1^n A_i + W_n$. The server will then be once again available (for the $(n+1)$-th customer) at time $\Sigma_1^n A_i + W_n + S_n$. Therefore

$$(3) \quad W_{n+1} = \begin{cases} \Sigma_1^n A_j + W_n + S_n - \Sigma_1^{n+1} A_j, & \text{if } W_n + S_n - A_{n+1} \geqslant 0 \\ \Sigma_1^{l(A_{n+1} - W_n - S_n)} \bar{S}_j - (A_{n+1} - W_n - S_n) & \text{if } W_n + S_n - A_{n+1} < 0 \end{cases}$$

where $l(x)$ is defined in (2).

This can be rewritten as

$$(4) \qquad W_{n+1} = \begin{cases} W_n + \xi_n & \text{if } W_n + \xi_n \geqslant 0 \\ \Sigma_1^{l(-W_n - \xi_n)} \bar{S}_j - (-W_n - \xi_n) & \text{if } W_n + \xi_n < 0 \end{cases}$$

which is the formula (1) given in the lemma.

As a consequence of Lemma 1 we have the following result.

LEMMA 2. — If $E\xi_n > 0$ for $n \geqslant 1$ then $W_n \to \infty$ with probability 1 as $n \to \infty$.

Proof. — Notice from (1) that $\eta(x) \geqslant -x$ for all x with probability 1: if $x \leqslant 0$ the statement is obvious; since $\eta(x) \geqslant 0$ with probability 1 it follows that $\eta(x) \geqslant -x$ if $x > 0$. Therefore, by lemma 1 we have

$$W_{n+1} \geqslant W_n + \xi_n, \qquad n \geqslant 1$$

Therefore $W_{n+1} \geqslant \Sigma_1^n \xi_n$, $n \geqslant 1$. If $E\xi_n > 0$, then the sum on the RHS converges with probability 1 to $+\infty$ as $n \to \infty$.

Henceforth we shall assume that $E\xi_n < 0$ for all $n \geqslant 1$.

Remark 3. — It is now clear that $W_1, W_2, \ldots, W_n, \ldots$ is a Markov chain since $\xi_1, \xi_2, \ldots, \xi_n, \ldots$ is a sequence of i. i. d. random variables and $\eta(.)$ is a random function which depends on $\bar{S}_1, \bar{S}_2, \ldots$ which are themselves independent of the $S_1, S_2, \ldots$, and of the $A_1, A_2, \ldots$.

We shall now study the characteristic function $Ee^{itW_{n+1}}$ for the waiting time process. Using (1) we have, for any real t

$$(5) \quad Ee^{itW_{n+1}} = Ee^{it(W_n+\xi_n)} I[W_n + \xi_n \geqslant 0]$$
$$+ \sum_{k=0}^{\infty} Ee^{it(W_n+\xi_n+\Sigma_1^{k+1}\bar{S}_i)} \cdot I[W_n + \xi_n + \Sigma_1^{k+1}\bar{S}_i \geqslant 0,\ 0 > W_n + \xi_n + \Sigma_1^k \bar{S}_i]$$

Let $f(t) = Ee^{it\bar{S}}$.

Then

$$(6) \quad \mathrm{E}e^{it\mathrm{W}_{n+1}} = \mathrm{E}e^{it(\mathrm{W}_n+\xi_n)} + [f(t)-1]\sum_{k=0}^{\infty} \mathrm{E}e^{it(\mathrm{W}_n+\xi_n+\Sigma_1^k\bar{\mathrm{S}}_i)} \cdot \mathrm{I}[\mathrm{W}_n+\xi_n+\Sigma_1^k\bar{\mathrm{S}}_i<0]$$

We are now ready to establish the main result of the paper.

THEOREM 4. — Suppose that

a) the random variable ξ is not arithmetic; that is $g(t) \equiv \mathrm{E}e^{it\xi}$ has a single real value t ($t = 0$) for which $g(t) = 1$,

b) $\mathrm{E}\xi < 0$, and $\mathrm{E}\bar{\mathrm{S}} < \infty$.

Then :

i) $\mathrm{W} \underset{p}{=} \lim_{n\to\infty} \mathrm{W}_n$ exists and is a proper random variable (1),

ii) $\mathrm{W} \underset{p}{=} \mathrm{V} + \gamma$, where $\mathrm{V} \underset{p}{=} \lim_{n\to\infty} \mathrm{V}_n$ (1),

$$\gamma \underset{p}{=} \lim_{x\to\infty} \sum_{1}^{l(x)} \bar{\mathrm{S}}_i - x\,,$$

and γ is independent of V.

That is, γ is the (limiting) forward recurrence time of the renewal process $\bar{\mathrm{S}}_1, \bar{\mathrm{S}}_1+\bar{\mathrm{S}}_2, \ldots, \bar{\mathrm{S}}_1+\ldots+\bar{\mathrm{S}}_n, \ldots$ It is well known that

$$\mathrm{P}[\gamma < x] = \int_0^x [1-\mathrm{F}_{\bar{\mathrm{S}}}(y)]dy/\mathrm{E}\bar{\mathrm{S}}$$

Proof. — Define

$$(7) \quad \phi_n(t) = \sum_{k=0}^{\infty} \mathrm{E}e^{it(\mathrm{W}_n+\xi_n+\Sigma_1^k\bar{\mathrm{S}}_i)} \cdot \mathrm{I}[\mathrm{W}_n+\xi_n+\Sigma_1^k\bar{\mathrm{S}}_i<0] = \int_{-\infty}^{0-} e^{itx} d\left\{\sum_{k=0}^{\infty} \mathrm{P}[\mathrm{W}_n+\xi_n+\Sigma_1^k\bar{\mathrm{S}}_i<x]\right.$$

Introduce the following notation :

$$\psi_n(t) = \mathrm{E}e^{it\mathrm{W}_n}$$

Then (9) becomes

$$(8) \qquad \psi_{n+1}(t) = \psi_n(t)g(t) + (f(t)-1)\phi_n(t)$$

Our proof will be complete if we can prove the existence and uniqueness

(1) lim W $\underset{p}{=}$ means limit in law.

of the characteristic function $\psi(t) \equiv \mathrm{E}e^{it\mathrm{W}}$ of a positive random variable W, solution of the stationary equation

$$\psi(t)(1 - g(t)) = (f(t) - 1)\phi(t) \tag{9}$$

obtained from (8), such that *i*) and *ii*) are satisfied.

Uniqueness. — We shall first show that if the solution $\psi(t)$ to (9) exists, then it is unique. If $\psi(t)$ exists, it must be continuous for real t and $\phi(t)$ must exist. Using (7) :

$$\phi(t) = \int_{-\infty}^{0^-} e^{ity} d\mathrm{G}(y)$$

where

$$\mathrm{G}(y) \equiv \sum_{k=0}^{\infty} \mathrm{P}[\mathrm{W} + \xi + \Sigma_1^k \bar{\mathrm{S}}_i < y]$$

Let us first show that $\phi(t)$ is a continuous function of t.

Set $\mathrm{W} = x$ in (6). Let us prove that the series on the right-hand-side of (7) is uniformly convergent on R^+ as function of x:

$$\left| \sum_{k=0}^{\infty} \mathrm{E}e^{it(x+\xi+\Sigma_1^{k+1}\bar{\mathrm{S}}_i)} \mathrm{I}[x + \xi + \Sigma_1^k \bar{\mathrm{S}}_i < 0] \right|$$
$$\leqslant \sum_{k=0}^{\infty} \mathrm{P}[x + \xi + \Sigma_1^k \bar{\mathrm{S}}_i < 0] < \sum_{k=0}^{\infty} \mathrm{P}[\xi + \Sigma_1^k \bar{\mathrm{S}}_i < 0] = \mathrm{EH}(-\xi)$$

where H(.) is the renewal function for the renewal process $\bar{\mathrm{S}}_1$, $\bar{\mathrm{S}}_1 + \bar{\mathrm{S}}_2$, $\bar{\mathrm{S}}_1 + \bar{\mathrm{S}}_2 + \bar{\mathrm{S}}_3, \ldots$ and $\mathrm{EH}(-\xi)$ is the expectation of $\mathrm{H}(-\xi)$ with respect to the random variable ξ. But $\mathrm{H}(y)$, which is the expected number of renewals in $[0, y]$ for $y > 0$ (and is zero for $y \leqslant 0$), is bounded by a function $\alpha + \beta y$ for α, $\beta \geqslant 0$. This completes the proof since a similar argument can be applied to the second series.

Therefore $\mathrm{G}(y)$ is a continuous function of y for almost all $y < 0$. It is obviously an increasing function of y and $\mathrm{G}(-\infty) = 0$ and $\mathrm{G}(0^-) < \infty$, since for $y < 0$

$$\mathrm{P}[\mathrm{W} + \xi + \Sigma_1^k \bar{\mathrm{S}}_i < y] \leq \mathrm{P}[\xi + \Sigma_1^k \bar{\mathrm{S}}_i < 0]$$

because $\mathrm{W} \geq 0$. Therefore $\mathrm{G}(y)$ is bounded for $y < 0$. Thus we have established that $\phi(t)$ is a continuous function of t.

Rewrite (9) as

$$\phi(t) = (1 - g(t)) \frac{\psi(t)}{f(t) - 1}$$

Since $\phi(t)$ is continuous and $g(t) \neq 1$ for $t \neq 0$ (by assumption (a)), it follows that every zero of $(f(t) - 1)$, if any, except $t = 1$, coincides with some zero of $\psi(t)$.

We now call upon a result of Borovkov [4]; if $\mathrm{E}\xi < 0$ (Chapter 4, p. 103, equation (1)) :

$$(1 - g(t)) = \frac{\mathrm{P}(\mathrm{V} = 0)}{\mathrm{E}e^{it\mathrm{V}}} [1 - \mathrm{E}e^{it\mathrm{X}}]$$

where X is a negative random variable. Therefore we may write,

$$\psi(t) \frac{\mathrm{P}(\mathrm{V} = 0)}{\mathrm{E}e^{it\mathrm{V}}} [1 - e^{it\mathrm{X}}] = (f(t) - 1)\phi(t)$$

Or

$$\frac{\psi(t)\mathrm{P}(\mathrm{V} = 0)it}{(f(t) - 1)\mathrm{E}e^{it\mathrm{V}}} = \frac{it\phi(t)}{1 - \mathrm{E}e^{it\mathrm{X}}} \tag{10}$$

Consider the LHS of (10). $\psi(t)$, $f(t)$ and $\mathrm{E}e^{it\mathrm{V}}$ are characteristic functions of positive random variables; they are therefore analytic in the upper half-plane ($\mathrm{Im}(t) > 0$) and continuous on the real line and bounded. Consider the RHS of (10). $\phi(t)$ is the characteristic function of a negative random variable and so is $\mathrm{E}e^{it\mathrm{X}}$; therefore the RHS of (10) is analytic on the lower half-plane ($\mathrm{Im}(t) < 0$) and continuous on the real line and bounded. Therefore by the theorem of Liouville the expression (10) is a constant, call it C. Let us write :

$$\psi(t) = \frac{\mathrm{C}(f(t) - 1)}{it\mathrm{P}(\mathrm{V} = 0)} \mathrm{E}e^{it\mathrm{V}}$$

Taking

$$1 = \psi(0) = \frac{\mathrm{CE}\bar{\mathrm{S}}}{\mathrm{P}(\mathrm{V} = 0)}$$

we have $\mathrm{C} = \mathrm{P}(\mathrm{V} = 0)/\mathrm{E}\bar{\mathrm{S}}$ and

$$\psi(t) = \frac{f(t) - 1}{it\mathrm{E}\bar{\mathrm{S}}} \mathrm{E}e^{it\mathrm{V}} \tag{11}$$

Therefore if $\psi(t)$ exists, then it is unique since it is given by (11). In fact, we have also shown that if it exists, it satisfies (ii) since (11) is simply the Fourier transform of the statement in (ii).

Existence. — We must now prove the existence of the solution $\psi(t)$ given by (11), of the equation (9).

Using (7), we shall show that $\psi(t)$ of (11) is a solution to (9). We write, from (7) :

$$\phi(t) = \int_{-\infty}^{0^-} d\left[\sum_{k=0}^{\infty} \mathrm{P}(\mathrm{W} + \xi + \Sigma_1^k \bar{\mathrm{S}}_n < x)\right] e^{itx} \tag{12}$$

It is the Fourier transform of the restriction to R^- of a mesure μ. μ is the convolution of two measures.

* μ_1, corresponding to the random variable ξ, on R
* μ_2, defined on R^+ with

$$\mu_2[0, x[= \sum_{k=0}^{\infty} P\left(W + \sum_{n=1}^{k} \bar{S}_n < x\right).$$

The Fourier transform of μ_2 is given by

$$\frac{\psi(t)}{1 - f(t)} (\,|f(t)| < 1 \text{ when } \mathrm{Im}(t) > 0)$$

But, from (11),

$$\frac{\psi(t)}{1 - f(t)} = \frac{-\,\mathrm{E}e^{it\mathrm{V}}}{it\mathrm{E}(\bar{S})}$$

Using the fact that $-\dfrac{1}{it\mathrm{E}(\bar{S})}$ is the Fourier transform of the Lebesgue measure on R^+, with density $\dfrac{1}{\mathrm{E}(\bar{S})}$, μ_2 itself is obtained as the convolution of this Lebesgue measure and of the measure of V.

Hence, μ is σ-finite and its Fourier transform is

$$\int_R e^{itx}\mu(dx) = -\,g(t)\frac{\mathrm{E}e^{it\mathrm{V}}}{it\mathrm{E}\bar{S}} = [1 - g(t)]\frac{\mathrm{E}e^{it\mathrm{V}}}{it\mathrm{E}\bar{S}} + \left(-\frac{\mathrm{E}e^{it\mathrm{V}}}{it\mathrm{E}\bar{S}}\right)$$
$$= \frac{p(\mathrm{V} = 0).(1 - \mathrm{E}e^{it\mathrm{X}})}{it\mathrm{E}\bar{S}} + \int_{R^+} e^{itx}\mu_2(dx)$$

We deduce $\mu = \mu^* + \mu_2$, where

. μ_2 is μ restricted to R^+.

. μ^* is the restriction of μ to R^- and therefore has the Fourier transform $\phi(t)$ which is

$$\phi(t) = p(\mathrm{V} = 0)\frac{1 - \mathrm{E}(e^{it\mathrm{X}})}{it\mathrm{E}(\bar{S})}$$

Hence replacing $\phi(t)$ above and (11) in (9) we see that the equality (9) is satisfied completing the existence proof.

We have established the existence and uniqueness of the stationary solution $\psi(t)$ of equation (8). We now have to prove that

$$\lim_{n\to\infty} \mathrm{W}_n \underset{p}{=} \mathrm{W}$$

i. e. that this stationary solution is the limit in the above equation. For this we shall call upon general results on the ergodicity of Markov chains as presented by Revuz [*6*]. In particular:

① We first show that W_n is irreducible.

② We use the Theorem (Revuz [*6*], Theorem 2.7, Chapter 3) that states that if a chain is irreducible and if a finite invariant measure exists, then it is *recurrent in the sense of Harris* (i. e. a *Harris chain*). Thus we show that W_n is a Harris chain.

③ Finally we use Orey's theorem (Revuz [*6*], Theorem 2.8, Chapter 6) which states that if a finite invariant measure m exists for an aperiodic Harris chain W_n, then $W_n \underset{p}{\rightarrow} W$; if the measure m is a probability measure then it is the measure of W.

Let us proceed with this proof.

① To show irreducibility, consider the measure m whose Fourier transform is $\psi(t)$. By (11) we can write

$$m = v * s$$

where $*$ denotes the convolution, v is the measure whose transform is Ee^{itV} and s is the measure whose transform is $[f(t) - 1]/itE\bar{S}$. Clearly,

$$v_c(A) > 0 \Rightarrow m(A) > 0 \quad \text{where } v_c \text{ is the continuous component of } v$$

for a subset A of the non-negative real line. We shall show that, for each initial state $x \in [0, \infty[$, there exists a positive integer n such that

$$P(W_n \in A \mid W_0 = x) > 0 .$$

For this, notice that V_n is ergodic (Borovkov [*4*], Theorem 7) if *a*) and *b*) are satisfied. Thus

$$v_c(A) > 0 \Leftrightarrow P(V \in A) > 0 \Rightarrow [\exists m \ni P(V_m \in A \mid V_0 = x) > 0]$$

But since $P(V_m = W_m \in A) > 0$ for each finite m (the case where the queue with automous server does not empty up to, and including, the m-th customer), then

$$v_c(A) > 0 \Rightarrow [\exists m \ni P(W_m \in A \mid W_0 = x) > 0]$$

Therefore (by Revuz [*6*], Definition 2.1 of Chapter 3) W_n is v_c-irreducible.

② Theorem 2.7, Chapter 3 of Revuz [*6*] states that W_n is a Harris chain if it is v-irreducible and if there exists finite invariant measure m such that $v(A) > 0 \Rightarrow m(A) > 0$ for all A($m \gg v$, in Revuz's notation). This has already been proved. Therefore W_n is indeed recurrent in the sense of Harris.

③ We now have to show, in order to use Orey's theorem, that W_n is aperiodic. We call again upon the classical result that V_n is ergodic if $E\xi < 0$ and ξ is not arithmetic (both of which we have assumed). Therefore V_n is aperiodic, and so is W_n since for each finite m

$$P(V_m = W_m \in A) > 0$$

Thus, by Orey's theorem W_n is ergodic and

$$\lim_{n \to \infty} W_n \underset{p}{=} W$$

This complete the proof of Theorem 4.

3. APPLICATION TO A PAGING DRUM MODEL

In this section we shall apply the theoretical results obtained in the previous sections to a model arising in the analysis of computer system behaviour [*2*, *3*, *5*]. Here the customers are requests for the transfer of *pages* (blocks of information of fixed size) from a paging drum (a secondary memory device used in computer systems). For the purpose of efficiency, described in [*2*, *3*, *5*], page requests are addressed to one of N sector queues; each paging drum sector when traversed permits to deliver one page. Since the paging drum rotates at constant speed, if T is the time for one complete rotation then one page will be transfered in time T/N and service at this particular sector queue will not be available for a time $T(N - 1)/N$ until the paging drum can be once again positioned at the beginning of the same sector.

Let W be the stationary waiting time at a sector queue with general independent interarrivals, and V be the stationary waiting time of the corresponding GI/D/1 queue with constant service time T. We then have:

THEOREM 5. — If $EA < T$ and ξ is not arithmetic with $E\xi < 0$ then W and V are proper random variables related by the formula

$$W \underset{p}{=} V + Y$$

where V and Y are independent and Y is uniformly distributed in $[0, T]$.

In particular we can obtain the average waiting time for the case of Poisson arrivals derived by Coffman [*2*]:

$$EW = \frac{T}{2} + \frac{\lambda T^2}{2(1 - \lambda T)}$$

where λ is the arrival rate of transfer requests.

Clearly, the response time R (waiting time plus service time) is simply

$$R = V + Y + \frac{T}{N}$$

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REFERENCES

[1] C. E. SKINNER, A priority queueing model with server walking type, *Opérations Research*, vol. **15**, 1967, p. 278-285.
[2] E. G. COFFMAN, Analysis of a drum input-output queue under scheduled operation, *J. A. C. M.*, vol. **16**, n° 1, 1969, p. 73-90.
[3] E. GELENBE, J. LENFANT, D. POTIER, Response time of a fixed-head disk to transfer of variable length, *S. I. A. M. J. on Computing*, vol. **4**, n° 4, 1975, p. 461-473.
[4] A. A. BOROKOV, *Stochastic Processes in Queueing Theory*, Springer Verlag, New York, 1976.
[5] S. H. FULLER, F. BASKETT, *An analysis of drum storage units*, Tech. Rep. 29, Digital Syst. Lab. Stanford University, Stanford, Cal., 1972.
[6] D. REVUZ, *Markov chains*, North Holland, Amsterdam, 1975.

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