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# Huygens' principle on Petrov type N space-times

by

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ABSTRACT. – By means of the moment equations it is proved that Maxwell's equations and Weyl's neutrino equation on a Petrov type $N$ space-time or on a $C$-space-time satisfy Huygens' principle if and only if the space-time is conformally equivalent to an exact plane wave space-time or conformally flat.

*Keywords :* Conformally invariant field equations, moments, Huygens' principle, Petrov type *N,* plane wave space-time.

RÉSUMÉ. – Grâce aux équations de moments on démontre que les équations de Maxwell et l'équation de Weyl sur un espace-temps de type $N$ de Petrov ou sur un espace-temps $C$ satisfont le principe d'Huygens si et seulement si l'espace-temps est conformément équivalent à un espace-temps des ondes planes.

## 1. INTRODUCTION

In an arbitrary four-dimensional pseudo-Riemannian manifold $(M,\, g)$ with a smooth metric of the signature $(+\,-\,-\,-)$ the following conformally invariant field equations are considered:

Scalar wave equation $\quad g^{ab}\,\nabla_a\,\nabla_b\,u - \dfrac{1}{6}\,Ru = O \quad$ (E1)

Maxwell's equations $\quad d\omega = O,\; \delta\omega = O \quad$ (E2)

Weyl's neutrino equation $\quad \nabla^A_{\dot X}\,\varphi_A = O \quad$ (E3) where $R$ denotes the scalar curvature, $\omega$ the Maxwell 2-form, $d$ the exterior derivative, $\delta$ the co-derivative, $\varphi$ a valence 1–spinor and $\nabla_{A\dot{X}}$ the covariant derivative on spinors. For one of the equations $E_1 - E_3$ Huygens' principle (in the sense of Hadamard's "minor premise") is valid if the solution of Cauchy's initial value problem in a sufficiently small neighbourhood of the initial space-like surface $F$ depends only on the Cauchy data in an arbitrarily small neighbourhood of the intersection of the past semi null cone with $F$ ([Ha]; [G2, 4]; [W4]). Only if Huygen's principle is valid is the wave propagation free of tails ([F]; [G4]; [McL2]; [W4]), that is the solution depends only on the source distribution on the past null cone of the field point and not on the sources inside the cone. Huygens' principle is valid if and only if the tail term with respect to $E_\sigma$, $\sigma = 1, 2, 3$ vanishes ([Ha], [F]; [G4]; [W4]). Because the functional relationship between the tail terms and the metric is very complicated the problem of determination of all metrics, for which any equation $E_\sigma$ satisfies Huygens principle, is not yet completely solved (*see* [G4]; [W2, 4, 8]; [McL3]; [CM]; [I]). The usual method for solving this problem is the derivation and the exploitation of the moment equations ([G4]; [W4])

$$I^\sigma_{i_1 \ldots i_r} = O, \qquad \sigma = 1, 2, 3, \quad r = 0, 1, 2, \ldots \qquad \text{(ME)}^\sigma_r$$

where the moments $I^\sigma_{i_1 \ldots i_r}$ are symmetric, trace-free, conformally invariant tensors. They are derived from the tail terms with respect to $E_\sigma$, $\sigma \in \{1, 2, 3\}$ by means of a certain conformal covariant differentiation process. If $g$ is analytic we have the following relationship between the moments and the validity of Huygen's principle: The equation $E_\sigma$, $\sigma \in \{1, 2, 3\}$, satisfies Huygens' principle if and only if all corresponding moments vanisch on $M$ ([G4]; [W8]). The moment equations $\text{(ME)}^\sigma_r$ are determined explicitly at present for $0 \leqq r \leqq 4$ (*see* [G4]; [McL2]; [W1, 2, 4]). Using some results on the theory of conformally invariant tensors [GW2, 3], in particular suitable linear independent systems of conformally invariant tensors, one obtains information about the general algebraic structure of the moments for $0 \leqq r \leqq 6$ ([RW]; [GeW1, 2]; [W8]).

If the equation $E_\sigma$, $\sigma \in \{1, 2, 3\}$ satisfies Huygens' principle, then, in particular the following moment equations must be satisfied (*see* [G4]; [W4, 8]; [McL2]; [RW]; [GeW1, 2])

$$I^\sigma_{i_1 i_2} \equiv \alpha^\sigma B_{i_1 i_2} = O \qquad \text{(ME)}^\sigma_2$$

$$I^\sigma_{i_1 \ldots i_4} \equiv \beta^\sigma [W^{(1)}_{i_1 \ldots i_4} - k_\sigma W^{(2)}_{i_1 \ldots i_4}] = O \qquad \text{(ME)}^\sigma_4$$

$$I^\sigma_{i_1 \ldots i_5} = O \qquad \text{(ME)}^\sigma_5$$

$$I^\sigma_{i_1 \ldots i_6} = O, \qquad \text{(ME)}^\sigma_6$$

where

$$B_{i_1 i_2} := \nabla_a \nabla_b C^{a}_{\cdot i_1 i_2 \cdot}{}^{b} - \frac{1}{2} C^{a}_{\cdot i_1 i_2 \cdot}{}^{b} L_{ab} \qquad \text{(Bach tensor)} \tag{1.1}$$

$$\begin{aligned} W^{(1)}_{i_1 \ldots i_4} := TS\, [&\nabla^a C^{b}_{\cdot i_1 i_2}{}^{c} \nabla_a C_{b i_3 i_4 c} + 16 \nabla_u C^{u}_{\cdot i_1 i_2 a} \nabla_k C^{k}_{\cdot i_3 i_4 \cdot}{}^{a} \\ &+ 4 C^{a}_{\cdot i_1 i_2 \cdot}{}^{b} \{2 \nabla_a \nabla_u C^{u}_{\cdot i_3 i_4 b} - C_{a i_3 i_4 \cdot}{}^{c} L_{bc}\}] \end{aligned} \tag{1.2}$$

$$\begin{aligned} W^{(2)}_{i_1 \ldots i_4} := TS\, [&2 \nabla_{i_1} C^{b}_{\cdot i_2 i_3}{}^{c} \nabla_u C^{u}_{\cdot a b i_4} + 2 \nabla_u C^{u}_{\cdot i_1 i_2 a} \nabla_k C^{k}_{\cdot i_3 i_4 \cdot}{}^{a} \\ &- C^{a}_{\cdot i_1 i_2 \cdot}{}^{b} \{2 \nabla_{i_3} \nabla_u C^{u}_{\cdot a b i_4} - C^{c}_{\cdot a b i_3} L_{c i_4}\}] \end{aligned} \tag{1.3}$$

$$L_{ab} := -R_{ab} + \frac{1}{6} R\, g_{ab}$$

$$\left. \begin{aligned} \alpha^\sigma \neq O, \quad \beta^1 = \frac{-1}{2^3 \cdot 5 \cdot 7}, \quad \beta^2 = \frac{-1}{2^3 \cdot 3 \cdot 7}, \quad \beta^3 = \frac{-1}{2^2 \cdot 3 \cdot 5 \cdot 7}, \\ k_1 = \frac{4}{3}, \quad k_2 = \frac{16}{5}, \quad k_3 = \frac{13}{8} \end{aligned} \right\} \tag{1.4}$$

and where $TS\,(T)$ denotes the trace-free, symmetric part of the tensor $T$. The moments $I^{\sigma}_{i_1 \ldots i_r}$ were given by P. Günther [G2, 4] in the cases $r = 2$, $\sigma = 1$, 2, and by the author [W1, 2, 4] in the cases $r = 2$, $\sigma = 3$ and $r = 4$, $\sigma = 1$, 2, 3. The explicit form of $I^{1}_{i_1 \ldots i_6}$ one can find in ([GeW2]; [W8]). With respect to $I^{\sigma}_{i_1 \ldots i_r}$ for $r = 5$, 6 see also ([RW]; [GeW1, 2]). The moments are identically zero for $r \in \{0, 1, 3\}$ and $r = 5$, $\sigma = 1$ ([G4]; [W4]). Because of the conformal invariance of the equations $E_1 - E_3$ and of the corresponding tail terms ([G4]; [W4, 8]) a conformal transformation

$$\tilde{g}_{ab} = e^{2\phi} g_{ab}, \qquad \phi \in C^\infty (M), \tag{1.5}$$

preserve the Huygens' character of $E_\sigma$, $\sigma \in \{1, 2, 3\}$ ([G4]; [W4]; [McL3]). In particular, each of the equations $E_1 - E_3$ satisfies Huygens' principle for flat metrics ([G4]; [W4]), which implies that if $g$ is conformally flat for $E_1 - E_3$ Huygens' principle is (trivially) valid.

A step towards the determination of all Huygens' metrics is a program outlined by J. Carminati and R. G. McLenaghan, which is based on the conformally invariant Petrov classification [PR] of the Weyl conformal curvature tensor $C_{abcd}$ ([CM]; [W7, 8]). The procedure consists in considering separately space-times of the five possible Petrov types. To date the problem has been settled for $E_1$ on the type $N$ space-times [CM], for all equations $E_1 - E_3$ on type $D$ space-times ([W7]; [CM]) and on type III space-times [CM]. At present only partial results are available for type II space-times [CM]. Space-times of Petrov type $N$ are characterized by the existence of a null vector field $l$ such that the Weyl curvature tensor satisfies the equation [PR]

$$C_{abcd}\, l^d = O.$$

Special classes of type $N$ metrics are the generalized plane wave metrics, investigated by McLenaghan and Leroy ([McL, L]; [McL3])

$$\begin{aligned} ds^2 = {} & 2\, dx^1 \, [dx^2 + \{a\,(z+\bar{z})\, x^2 + Dz^2 + \bar{D}\bar{z}^2 + ez\bar{z} + Fz + \bar{F}\,\bar{z}\}\, dx^1 \\ & - 2\,[dz + az^2\, dx^1]\,[d\bar{z} + a\bar{z}^2\, dx^1], \end{aligned} \tag{1.6}$$

where $z = x^3 + ix^4$ and $a = \bar{a}$, $D$, $e$, $F$ are arbitrary functions of $x^1$ only. For the special case $a = 0$ we obtain the important subclass of plane wave metrics ([PR]; [McL, L]; [G4]; [S1, 2]).

If $g$ is a plane wave metric, then each of the equations $E_1 - E_3$ satisfies Huygens' principle ([G3]; [S1]; [W4]). Consequently, the moment equations $(\mathrm{ME})_r^\sigma$ hold for all $r$ and $\sigma$. If $g$ is an Einstein metric, a central symmetric metric, a (2, 2)-decomposable metric or a conformally recurrent metric and $\sigma \in \{1,\ 2,\ 3\}$, then from $\{(\mathrm{ME})_r^\sigma / r = 2,\ 4\}$ it follows, that $g$ is conformally flat or a plane wave metric [W4, 8]. Let $g$ be of Petrov type $D$ and $\sigma \in \{1,\ 2,\ 3\}$, then there are no metrics, for which the equations $\{(\mathrm{ME})_r^\sigma / r = 2,\ 4\}$ are valid ([CM]; [W7]; [McL, W]). In [W6] it was proved for $C$-spaces, *i. e.* space-times with $\nabla^a C_{abcd} = O$ :

PROPOSITION 1. – *Let $g$ be conformally equivalent to a $C$-space-metric and $\sigma \in \{1,\ 2,\ 3\}$. Then the equations $\{(\mathrm{ME})_r^\sigma / r = 2,\ 4\}$ imply, that $g$ is of Petrov type $N$.*

From $\{(\mathrm{ME})_r^1 / r = 2,\ 4,\ 6\}$ it follows [CM]

PROPOSITION 2. – *The equation $E_1$ for any Petrov type $N$ metric $g$ satisfies Huygens' principle if and only if $g$ is conformally equivalent to a plane wave metric.*

Let $g$ be of Petrov type $N$ and $\sigma \in \{\ 2,\ 3\}$. The moment equations $\{(\mathrm{ME})_r^\sigma / r = 2,\ 4,\ 5\}$ are satisfied if and only if $g$ is confomally equivalent to a generalized plane wave metric (*see* [AW]; [GM]). Consequently, one has [AW].

PROPOSITION 3. – *If the field equation $E_2$ or $E_3$ for any Petrov type $N$ metric $g$ satisfies Huygens' principle, then $g$ is conformally equivalent to a generalized plane wave metric* (1.6).

By means of $(\mathrm{ME})_6^\sigma$, $\sigma \in \{2, 3\}$ we prove in this paper the following extension of Proposition 2:

THEOREM 1. – *The field equation* $E_2$ or $E_3$ *satisfies Huygens' principle for any Petrov type N metric g if and only if g is conformally equivalent to a plane wave metric.*

As a consequence we get for $C$-spaces:

COROLLARY 1. – *Each of the equations* $E_1 - E_3$ *satisfies Huygens' principle for any metric g conformally equivalent to a C-space metric if and only if g is conformally equivalent to a plane wave metric or to a flat metric.*

A conjecture is ([W4, 6]; [CM]), that the moment equations $\{(\mathrm{ME})_r^\sigma / r \leqq 6\}$ are also sufficient for the validity of Huygens' principle for $E_\sigma$, $\sigma \in \{1, 2, 3\}$, and that these equations are satisfied if and only if $g$ is conformally equivalent to a plane wave metric or to a flat metric.

## 2. PRELIMINARIES

Let $(M, g)$ be a space-time, *i.e.* a 4-manifold together with a smooth metric of Lorentzian signature, and $g_{ab}$, $g^{ab}$, $\nabla_a$, $R_{abcd}$, $R_{ab}$, $R$, $C_{abcd}$ the local coordinates of the covariant and contravariant metric tensor, the Levi-Civita connection, the curvature tensor, the Ricci tensor, the scalar curvature and the Weyl curvature tensor, respectively. $\mathcal{J}$ and $\Lambda^p$ denote the space of the $C^\infty$ scalar fields and the $p$-forms of class $C^\infty$, respectively. On $\Lambda^p$ the exterior derivative $d$, the coderivative $\delta$ and $\Delta := -(d\delta + \delta d)$ are defined. Assuming that $(M, g)$ can be equipped with a spin structure we denote the complex spinor bundles of covariant and contravariant 1-spinors and their conjugates by $S$, $S^*$, $\bar{S}$, $\bar{S}^*$, the set of all cross sections of $S$, $S^*$, $\bar{S}$, $\bar{S}^*$, by $\mathcal{S}$, $\mathcal{S}^*$, $\bar{\mathcal{S}}$, $\bar{\mathcal{S}}^*$, respectively, the coordinates of $\varphi \in \mathcal{S}$, $\psi \in \bar{\mathcal{S}}$, the connection quantities (generalized Pauli-matrices), the Levi-Civita spinor, the spinor covariant derivative and the connection coefficients by [PR]

$$\left.\begin{array}{c}\varphi_A,\ \psi_{\dot X},\ \sigma^a_{\cdot A\dot X},\ \varepsilon_{AB},\ \nabla_{A\dot X} := \sigma^a_{\cdot A\dot X}\,\nabla_a;\\ A\in\{1,\,2\},\qquad \dot X\in\{\dot 1,\,\dot 2\}\end{array}\right\}\tag{2.1}$$

$$\Gamma_{aB}^{\;A} = \frac{1}{2}\,\sigma_d^{\;A\dot Y}\,(\sigma^b_{\cdot B\dot Y}\,\Gamma^d_{ab} + \partial_a\,\sigma^d_{\cdot B\dot Y}).\tag{2.2}$$

If we define for $\varphi \in \mathcal{S}$, $\psi \in \bar{\mathcal{S}}$

$$(\mathcal{M}\,\varphi)_{\dot X} := \nabla^A_{\dot X}\,\varphi_A, \qquad (\mathcal{N}\,\psi)_A := \nabla^{\dot X}_A\,\psi_{\dot X},\tag{2.3}$$

we have ([W4]; [PR])

$$-2\,(\mathcal{N}\mathcal{M}\,\varphi)_A = g^{ab}\,\nabla_a\,\nabla_b\,\varphi - \frac{1}{4}\,R\,\varphi_A =: (\mathcal{L}^{(1/2)}\,\varphi)_A. \tag{2.4}$$

In the following we consider the conformally invariant wave equation

$$\mathcal{L}^{(0)}\,u \equiv g^{ab}\,\nabla_a\,\nabla_b\,u - \frac{1}{6}\,R\,u = O, \qquad u \in \mathcal{J}, \tag{E$_1$}$$

the (source-free) Maxwell equations

$$du = O, \qquad \delta u = O, \qquad u \in \Lambda^2 \tag{E$_2$}$$

and Weyl's neutrino equation

$$\mathcal{M}\,u = O, \qquad u \in \mathcal{S}. \tag{E$_3$}$$

Let $M$ be a causal domain ([F]; [G4]) and $\Gamma\,(x,\,y)$ the square of the geodesic distance of $x,\,y \in M$. For any fixed $y \in M$ the set $\{x \in M/\Gamma\,(x,\,y) > 0\}$ decomposes naturally into two open subsets of $M$; one of them is called the future $D_+\,(y)$ and the other one the past $D_-\,(y)$ of $y$. The characteristic semi null cones $C_\pm\,(y)$ are defined as the boundary sets of $D_\pm\,(y)$, respectively.

Let $G_\pm^{(0)}\,(y)$, $G_\pm^{(1)}\,(y)$ and $G_\pm^{(1/2)}\,(y)$ be the fundamental solutions of the linear operators $\mathcal{L}^{(0)}$, $\Delta =: \mathcal{L}^{(1)}$, $\mathcal{L}^{(1/2)}$ and $T^{(\alpha)}\,(\cdot,\,y)$, $\alpha = 0, 1,\, 1/2$ the tail terms of $G_\pm^{(\alpha)}\,(y)$ with respect to $y$, respectively. The tail term is just the factor of the regular part of the corresponding fundamental solution, which is a distribution supported inside the future of $y$ ([F]; [G4]).

For $T^{(\alpha)}$ there is an asymptotic expansion in $\Gamma$

$$T^{(\alpha)} \sim \sum_{k=0}^{\infty} \frac{1}{2^k.\,k\,!}\,\mathcal{U}_{(k+1)}^{(\alpha)}\,\Gamma^k, \tag{2.5}$$

where the Hadamard coefficients $\mathcal{U}_{(k)}^{(\alpha)}$ are determined recursively by the transport equations ([F], [G4]; W4])

$$\begin{aligned} &\nabla^a\,\Gamma\,\nabla_a\mathcal{U}_{(k)}^{(\alpha)} + \frac{1}{2}\,(g^{ab}\,\nabla_a\,\nabla_b\,\Gamma - 8 + 2\,k)\,\mathcal{U}_{(k)}^{(\alpha)} \\ &\quad = -\mathcal{L}^{(\alpha)}\,\mathcal{U}_{(k-1)}^{(\alpha)}, \qquad \left(k \geqq O;\ \alpha = 0,\ 1,\ \frac{1}{2}\right) \end{aligned} \tag{2.6}$$

with the initial conditions

$$\mathcal{U}_{(-1)}^{(\alpha)} \equiv O, \qquad \mathcal{U}_{(0)}^{(\alpha)}\,(y,\,y) = I_y, \tag{2.7}$$

where $I_y$ denotes the identity.

In ([Ha]; [F]; [G2, 4]; [W4]) it was proved:

PROPOSITION 2.1. – *The equation $E_\sigma$, $\sigma \in \{1, 2, 3\}$ satisfies Huygens'principle iff*

$$\forall x,\, y \in M : T^{(0)}(x,\, y) = O \ \textit{in the case} \ \sigma = 1, \tag{2.8}$$

$$\forall x,\, y \in M : K(x,\, y) := d^{(1)}\, d^{(2)}\, T^{(1)}(x,\, y) = O \quad \textit{in the case} \ \sigma = 2, \tag{2.9}$$

$$\forall x,\, y \in M : N(x,\, y) := \mathcal{M}^{(1)}\, T^{(1/2)}(x,\, y) = O \quad \textit{in the case} \ \sigma = 3. \tag{2.10}$$

Here the superscripts $^{(1)}$, $^{(2)}$ indicate whether the derivative is meant with respect to $x$ or $y$.

DEFINITION 2.1. – *The terms*

$$T^{(0)}(x,\, y),\ K(x,\, y),\ N(x,\, y) \tag{2.11}$$

*are called tail terms of the equation $E_1$, $E_2$, $E_3$, respectively.*

The tail term $K(x,\, y)$ is a double differential form of degree 2:

$$K(x,\, y) = K_{ij,\,\alpha\beta}(x,\, y)\, dx^i \underset{(1)}{\wedge} dx^j\, dy^\alpha \underset{(2)}{\wedge} dy^\beta.$$

One defines for every $y \in M$ and $r \geqq 2$ the coincidence values ([G4; W8])

$$M_{i_1 \ldots i_r}(y) := g^{j\beta}(y)\, \overset{c}{\nabla}{}^{(1)}_{i_1} \ldots \overset{c}{\nabla}{}^{(1)}_{i_{r-2}} K_{i_{r-1} j,\, i_r \beta}(y,\, y), \tag{2.12}$$

where $\overset{c}{\nabla}_i$ denotes the conformal covariant derivative (*see* [GW1, 2]; [W8]). Furthermore, if $N_{\dot{X}\underline{A}}(x,\, y)$(1) are the coordinates of the tail term $N(x,\, y)$, then we define for every $y \in M$ and $r \geqq 1$ the (complex) coincidence values (*see* [W8]; [W4])

$$N_{i_1 \ldots i_r}(y) := \sigma^{A\dot{X}}_{(i_1}(y)\, \overset{c}{\nabla}{}^{(1)}_{i_2} \ldots \overset{c}{\nabla}{}^{(1)}_{i_r)} N_{\dot{X}\underline{A}}(y,\, y). \tag{2.13}$$

The usual method for solving the problem of determination of all metrics, for which any equation $E_\sigma$ satisfies Huygens' principle, is the derivation and the exploitation of the moment equations (*see* [G4]; [W2, 4, 8]; [McL1, 2]

$$I^\sigma_{i_1 \ldots i_r} = O; \qquad \sigma = 1,\, 2,\, 3; \qquad r =,\, 1,\, 2,\, \ldots, \qquad (\mathrm{ME})^\sigma_r$$

(1) The underlined indices refer to $y$.

where the moments $I^{\sigma}_{i_1 \dots i_r}$ are symmetric, trace-free, conformally invariant tensors of weight $(-1)$ ([G4]; [W8]). They are derived from the tail terms with respect to $E_\sigma$, $\sigma \in \{1, 2, 3\}$, by means of a conformal covariant differentiation process. If $g$ is analytic it holds ([G4]; [W8]):

PROPOSITION 2.2. – *The equation* $E_\sigma$, $\sigma \in \{1, 2, 3\}$, *satisfies Huygens' principle if and only if all corresponding moments vanish on* $M$.

In particular, in the case of $B_{i_1 i_2} = O$ [*see* $(\mathrm{ME})^{\sigma}_{2}$] we have (*see* [G4]; [W8])

$$I^{2}_{i_1 \dots i_6} = TS \left[ M_{i_1 \dots i_6} - \frac{18}{11} \overset{c}{\nabla}_{i_1} \overset{c}{\nabla}_{i_2} I^{2}_{i_3 \dots i_6} \right] \tag{2.14}$$

$$I^{3}_{i_1 \dots i_6} = TS \left[ \mathfrak{Re}\, N_{i_1 \dots i_6} - \frac{30}{11} \overset{c}{\nabla}_{i_1} \overset{c}{\nabla}_{i_2} I^{3}_{i_3 \dots i_6} \right], \tag{2.15}$$

where $\mathfrak{Re}$ denotes the real part of the tensor.

On the other hand, using the linear independent, generating systems of conformally invariant tensors of rank 6 and weight $(-1)$ (*see* [GeW2]; [W8])

$$\{ S^{(2,\, m)}_{i_1 \dots i_6},\ S^{(3,\, p)}_{i_1 \dots i_6};\ m = 1,\ 2;\ p = 1,\ \dots,\ 6 \}^{2)} \tag{2.16}$$

one has the following information about the general algebraic structure of $I^{\sigma}_{i_1 \dots i_6}$ ([GeW2]; [W8]).

$$I^{\sigma}_{i_1 \dots i_6} = \sum_{m=1}^{2} \delta^{(2,\, \sigma)}_{m}\, S^{(2,\, m)}_{i_1 \dots i_6} + \sum_{p=1}^{6} \delta^{(3,\, \sigma)}_{p}\, S^{(3,\, p)}_{i_1 \dots i_6} \qquad (\sigma = 2,\ 3), \tag{2.17}$$

where $\delta^{(2,\, \sigma)}_{m}$, $\delta^{(3,\, \sigma)}_{p} \in \mathbb{R}$.

If the equation $E_\sigma$, $\sigma \in \{1, 2, 3\}$, satisfies Huygens' principle, then, in particular the moment equations $\{(\mathrm{ME})^{\sigma}_{r} / r \leqq 6\}$ must hold. In [CM] it was proved that the general solution of $\{(\mathrm{ME})^{\sigma}_{r} / r = 2,\ 4\}$ for Petrov type $N$ space-times are conformally equivalent to special cases of the complex recurrent space-times of McLenaghan and Leroy [McL, L]. If in addition $(\mathrm{ME})^{\sigma}_{5}$, $\sigma \in \{2, 3\}$, is satisfied, then the metric is conformally equivalent to a generalized plane wave metric (1.6) (*see* [AW]; [W8]) and Proposition 3). Conversely, in [AW], [GeW2] it was shown:

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(2) For the definition of the tensors $S^{(2,\, m)}$, $S^{(3,\, p)}$, which contain many monomials see [GeW2].

LEMMA 2.1. – *For a generalized plane wave metric (1.6) one has*

$$I^{\sigma}_{i_1 \dots i_r} \equiv O \qquad for \quad 0 \leqq r \leqq 5, \quad \sigma \in \{1,\, 2,\, 3\},$$

$$S^{(3,\,p)}_{i_1 \dots i_6} \equiv O \qquad for \quad 1 \leqq p \leqq 6$$

*and*

$$S^{(2,\,1)}_{i_1 \dots i_6} = 11\, S^{(2,\,2)}_{i_1 \dots i_6} = 363\, Z_{i_1 \dots i_6},$$

*where*

$$Z_{i_1 \dots i_6} := TS\,[C^{a}_{\,.\,i_1 i_2\,.}{}^{b}\, C_{a i_3 i_4 b}\, R_{i_5\,.}^{\;\;c}\, R_{i_6 c}].$$

LEMMA 2.2. – *A generalized plane wave metric is a plane wave metric iff* $Z_{i_1 \dots i_6} = O$.

Consequently, in virtue of (2.17) it holds for a generalized plane wave metric

$$I^{\sigma}_{i_1 \dots i_6} = 33\,(11\,\delta_1^{(2,\,\sigma)} + \delta_2^{(2,\,\sigma)})\, Z_{i_1 \dots i_6}, \tag{2.18}$$

and under the assumption

$$11\,\delta_1^{(2,\,\sigma)} + \delta_2^{(2,\,\sigma)} \neq O, \qquad \sigma \in \{2,\, 3\} \tag{2.19}$$

from $I^{\sigma}_{i_1 \dots i_6} = O$, $\sigma \in \{2,\, 3\}$ it follows, that $g$ is a plane wave metric. $I^{1}_{i_1 \dots i_6}$ is explicitly known ([RW]; [GeW2]), the condition (2.19) is satisfied for $\sigma = 1$ and our problem is completely solved for the equation $E_1$ (*see* Proposition 2 [CM]). Because of Proposition 3, $I^{\sigma}_{i_1 \dots i_6} = O$ and (2.18) for the proof of Theorem 1 it remains to show the property (2.19). For this the following method is used.

Let $g$ be an arbitrary anlytic metric, $\mathfrak{R}$ the tensor algebra which is generated by the fundamental tensor, the Riemannian curvature tensor and its covariant derivatives by means of the usual tensor operations and $\mathfrak{R}_k$ the subset of those tensor of $\mathfrak{R}$ with the covariant rank $k$. Each such polynomial tensor $T \in \mathfrak{R}_k$ can be represented by a linear combination of monomials of $\mathfrak{R}_k$ [GW2, 3]. For example, the tensors $M_{i_1 \dots i_6}$, $\mathfrak{Re}\, N_{i_1 \dots i_6}$, $I^{\sigma}_{i_1 \dots i_6}$, $S^{(2,\,k)}_{i_1 \dots i_6}$ ($\sigma = 1,\, 2,\, 3;\ k = 1,\, 2\}$ are elements of $\mathfrak{R}_6$ ([G4]; [W4, 8]). Under their monomials there are only two, which are quadratic in the zeroth and fourth derivatives of the Weyl tensor, namely

$$\left.\begin{aligned} T_{i_1 \dots i_6} &:= TS\,[C^{a}_{\,.\,i_1 i_2\,.}{}^{b}\, \nabla_{i_3} \nabla_{i_4} \nabla_{i_5} \nabla_u\, C^{u}_{\,.\,a b i_6}], \\ T'_{i_1 \dots i_6} &:= TS\,[C^{a}_{\,.\,i_1 i_2\,.}{}^{b}\, \nabla_{i_3} \nabla_{i_4} \nabla_a \nabla_u\, C^{u}_{\,.\,i_5 i_6 b}]. \end{aligned}\right\} \tag{2.20}$$

We are interested on the coefficients only of the monomials (2.20) and write for $\overset{1}{T}$, $\overset{2}{T} \in \mathfrak{R}_6$

$$\overset{1}{T}_{i_1 \ldots i_6} \equiv \overset{2}{T}_{i_1 \ldots i_6} \tag{2.21}$$

iff the tensor $TS\,[\overset{1}{T}_{i_1 \ldots i_6} - \overset{2}{T}_{i_1 \ldots i_6}]$ does not contain the monomials (2.20). Then we have by $(\mathrm{ME})_4^\sigma$, (1.2), (1.3)

$$\left.\begin{array}{c} \overset{c}{\nabla}_{(i_1} \overset{c}{\nabla}_{i_2} I^\sigma_{i_3 \ldots i_6)} \equiv \nabla_{(i_1} \nabla_{i_2} I^\sigma_{i_3 \ldots i_6)} \\ \equiv \beta^\sigma \,[8\, T'_{i_1 \ldots i_6} + 2\, k_\sigma\, T_{i_1 \ldots i_6}], \\ (\sigma \in \{2,\, 3\}) \end{array}\right\} \tag{2.22}$$

and (*see* [GeW2])

$$S^{(2,\,1)}_{i_1 \ldots i_6} \equiv -228\, T_{i_1 \ldots i_6} - 180\, T'_{i_1 \ldots i_6}, \quad S^{(2,\,2)}_{i_1 \ldots i_6} \equiv 48\, T_{i_1 \ldots i_6}, \quad S^{(3,\,p)}_{i_1 \ldots i_6} \equiv O,$$

hence by (2.17)

$$\left.\begin{array}{c} I^\sigma_{i_1 \ldots i_6} \equiv (-228\, \delta_1^{(2,\,\sigma)} + 48\, \delta_2^{(2,\,\sigma)})\, T_{i_1 \ldots i_6} - 180 \delta_1^{(2,\,\sigma)}\, T'_{i_1 \ldots i_6}, \\ (\sigma = 2,\, 3). \end{array}\right\} \tag{2.23}$$

On the other hand the ansatz

$$M_{i_1 \ldots i_6} \equiv \rho_2\, T_{i_1 \ldots i_6} + \rho'_2\, T'_{i_1 \ldots i_6} \tag{2.24}$$

$$\mathfrak{Re}\, N_{i_1 \ldots i_6} \equiv \rho_3\, T_{i_1 \ldots i_6} + \rho'_3\, T'_{i_1 \ldots i_6} \qquad (\rho_2,\, \rho'_2,\, \rho_3,\, \rho'_3 \in \mathbb{R}) \tag{2.25}$$

and (2.14), (2.15), (2.22) imply

$$I^2_{i_1 \ldots i_6} \equiv \left(\rho_2 - \frac{24}{385}\right) T_{i_1 \ldots i_6} + \left(\rho'_2 - \frac{6}{77}\right) T'_{i_1 \ldots i_6}, \tag{2.26}$$

$$I^3_{i_1 \ldots i_6} \equiv \left(\rho_3 + \frac{13}{616}\right) T_{i_1 \ldots i_6} + \left(\rho'_3 - \frac{4}{77}\right) T'_{i_1 \ldots i_6}. \tag{2.27}$$

We continue the proof of (2.19) assuming the opposite of (2.19) *i. e.*

$$\delta_2^{(2,\,\sigma)} = -11\, \delta_1^{(2,\,\sigma)}, \qquad \sigma = 2,\, 3, \tag{2.28}$$

then we obtain from (2.23)-(2.27)

$$I^\sigma_{i_1 \ldots i_6} \equiv -36\, \delta_1^{(2,\,\sigma)}\, [21\, T_{i_1 \ldots i_6} + 5\, T'_{i_1 \ldots i_6}], \qquad (\sigma = 2,\, 3)$$

and

$$5\, \rho_\sigma - 21\, \rho'_\sigma = \frac{\nu_\sigma}{11}, \qquad (\sigma = 2, 3), \tag{2.29}$$

where

$$\nu_2 = -102/7, \qquad \nu_3 = 607/56.$$

DEFINITION 2.2. – *Let a rational number $r$ said to have the Property $F$, if $r$ is representable in the form $r = \frac{n}{m}$ where $n$, $m$ are integer numbers with $n \not\equiv 0 \,(\mathrm{mod}\, 11)$, $m \equiv 0 \,(\mathrm{mod}\, 11)$.*

From (2.29) it follows, that under the assumption (2.28) at least one of the rational coefficients $\rho_\sigma$ or $\rho'_\sigma$ $(\sigma = 2, 3)$ must have the Property $F$. In the following Section we show, that this cannot be satisfied. Consequently, we obtain contradictions to the assumption (2.28).

## 3. THE PROOF FOR MAXWELL'S EQUATION $(\sigma = 2)$

We prove, that the coefficients $\rho_2, \rho'_2$ defined by (2.24) have not the Property $F$. From (2.5) and (2.9) we imply ([G2, 4]; [GW1]; [W2])

$$K(x, y) = \sum_{k=0}^{\infty} \frac{\overset{k}{K}(x, y)}{2^k \cdot k!}\, \Gamma^k, \tag{3.1}$$

where

$$\begin{aligned}\overset{k}{K}(x, y) := \Big\{ & d^{(1)} d^{(2)} \mathcal{U}^{(1)}_{(k+1)} + \frac{1}{2}\, d^{(1)} \Gamma \underset{(1)}{\wedge} d^{(2)} \mathcal{U}^{(1)}_{(k+2)} \\ & + \frac{1}{2}\, d^{(2)} \Gamma \underset{(2)}{\wedge} d^{(1)} \mathcal{U}^{(1)}_{(k+2)} + \frac{1}{2}\, d^{(1)} d^{(2)} \Gamma \underset{(1)}{\wedge} \underset{(2)}{\wedge} \mathcal{U}^{(1)}_{(k+2)} \\ & + \frac{1}{4}\, d^{(1)} \Gamma\, d^{(2)} \Gamma \underset{(1)}{\wedge} \underset{(2)}{\wedge} \mathcal{U}^{(1)}_{(k+3)} \Big\} (x, y) \\ =: & \overset{k}{K}_{ij,\alpha\beta}(x, y)\, dx^i \underset{(1)}{\wedge} dx^j\, dy^\alpha \underset{(2)}{\wedge} dy^\beta.\end{aligned}$$

Because of (3.1) and $\Delta K = 0$ (*see* [G4]) in (2.12) one can replace $K_{ij,\alpha\beta}$ by $\overset{o}{K}_{ij,\alpha\beta}$. By $B_{ab} = O$ we have

$$K_{ij,\alpha\beta}(y, y) = O, \qquad \partial^{(1)}_{i_1} K_{ij,\alpha\beta}(y, y) = O. \tag{3.2}$$

Further, under consideration of the equivalence relation (2.21), the properties of $\overset{c}{\nabla}_i$ (*see* [GW2, 3]; [W8]) and (3.2) it is easy to see that

$$M_{i_1 \ldots i_6}(y) \equiv g^{\alpha\beta}(y)\, \partial^{(1)}_{i_1} \ldots \partial^{(1)}_{i_4}\, \overset{o}{K}_{i_5\alpha, i_6\beta}(y, y). \tag{3.3}$$

Let $V(y)$ be a normal neighbourhood of $y$ and $\{x^i\}$ normal coordinates in $y$. Then the following equations hold ([G2, 4]; [W4]):

$$\Gamma(x, y) = g_{ab}(y)\, x^a\, x^b, \qquad d^{(1)}\,\Gamma(x, y) = 2\, g_{ab}(y)\, x^b\, dx^\alpha$$
$$d^{(2)}\Gamma(x, y) = -2\, g_{a\,\beta}(y)\, x^a\, dy^\beta,$$
$$d^{(1)}\, d^{(2)}\,\Gamma(x, y) = -2\, g_{i\alpha}(y)\, dx^i\, dy^\alpha.$$

Let be

$$\mathcal{U}^{(1)}_{(k)}(x, y) = \mathcal{U}^k_{i,\,\alpha}(x, y)\, dx^i\, dy^\alpha$$

and

$$\mathcal{U}^k_{i,\,\alpha}(x, y) = \sum_{r=0}^{\infty} \mathcal{U}^k_{i,\,\alpha|i_1\ldots i_r}(y)\, x^{i_1}\ldots x^{i_r},$$

where

$$\mathcal{U}^k_{i,\,\alpha|i_1\ldots i_r}(y) := \frac{1}{r!}\,\partial^{(1)}_{i_1}\ldots\partial^{(1)}_{i_r}\,\mathcal{U}^k_{i,\,\alpha}(y, y).$$

Then one obtains after a straightforward calculation ([GW1]; [W2]; [G4])

$$\begin{aligned} M_{i_1\ldots i_6}(y) \equiv 4!\, g^{i\alpha}(y)\,[-30\,\mathcal{U}^1_{(i,\,\alpha)|i_1\ldots i_6}(y) + 5\,\partial_{i_1}\,\mathcal{U}^1_{i,\,\alpha|i_2\ldots i_6}(y)]_{[ii_1]\,[\alpha i_2]} \\ - 12\,\mathcal{U}^2_{i_1,\,i_2|i_3\ldots i_6}(y) \end{aligned} \tag{3.4}$$

From the transport equations (2.6) it follows for $\alpha = 1$ and $k \geqq 0$ in normal coordinates [G2, 4].

$$\begin{aligned} 2\,(x^a\,\partial_a + k)\,\mathcal{U}^k_{i,\,\alpha} + x^a\left(\frac{1}{2}\,\delta^l_i\, g^{mn}\,\partial_a\, g_{mn} - g^{lm}\,\partial_a\, g_{mi}\right)\mathcal{U}^k_{l,\,\alpha} \\ = -\Delta\mathcal{U}^{k-1}_{i,\,\alpha} \end{aligned} \tag{3.5}$$

with the initial conditions $\mathcal{U}^{(-1)}_{i,\,\alpha} = O$, $\mathcal{U}^0_{i,\,\alpha}(y, y) = g_{i\alpha}(y)$.

In order to determine $M_{i_1\ldots i_6}(y)$ we need the derivatives of $\mathcal{U}^2_{i,\,\alpha}$ up to fourth order and the derivatives of $\mathcal{U}^1_{i,\,\alpha}$ up to sixth order, which we can express by means of (3.5) by the derivatives of $g_{ab}$, $g^{ab}$ up to eighth order [G1]. The coefficients of these representation formulas (*see* [G1]) up to eighth order have not the Property $F$. Consequently, if we express in this way the summands of (3.4) by the covariant derivates of $R_{abcd}$ the coefficients have not the Property $F$. Finally, symmetrizations and alternations up to eight incides, the Ricci identity, the Bianchi identity and the elimination of monomials, which are linearly with respect to the covariant derivatives of $R_{abcd}$ by means of $B_{ab} = 0$ don't imply the Property $F$. Hence, the coefficients $\rho_2$, $\rho'_2$ have not the Property $F$.

## 4. PROOF FOR WEYL'S EQUATION ($\sigma = 3$)

Now we show in analogues manner, that the coefficients $\rho_3$, $\rho_3'$ defined by (2.25) haven't the Property $F$. As in Section 3 we use normal coordinates $\{x^i\}$ in $y$ and additionally an adapted basis system in the spinor space. Then we have ([G1]; [W4])

$$\left.\begin{array}{c} x^a\,\Gamma^B_{aA}(x) = O, \qquad x^a\,(\sigma_a^{A\dot X}(x) - \sigma_a^{A\dot X}(y)) = O, \\ \nabla_{A\dot X}\,\Gamma(x,\,y) = 2\,x^a\,\sigma_{aA\dot X}(y). \end{array}\right\} \tag{4.1}$$

From (2.5), (2.10) if follows

$$N_{\dot X\underline{A}}(x,\,y) = O \Leftrightarrow \left\{\nabla^K_{\dot X}\,(\mathcal{U}_1^{(1/2)})_{K\underline{A}} + \frac{1}{2}\,(\mathcal{U}_2^{(1/2)})_{K\underline{A}}\,\nabla^K_{\dot X}\,\Gamma\right\}(x,\,y) = O$$
$$\text{for}\quad \Gamma(x,\,y) = O. \tag{4.2}$$

Putting

$$(\mathcal{U}_k^{(1/2)})_{K\underline{A}}(x,\,y) =: \mathcal{U}^k_{K\underline{A}}(x,\,y) = \sum_{r=0}^{\infty} \mathcal{U}^k_{K\underline{A}|i_1\ldots i_r}(y)\,x^{i_1}\ldots x^{i_r},$$

where

$$\mathcal{U}^k_{K\underline{A}|i_1\ldots i_r}(y) := \frac{1}{r!}\,\partial^{(1)}_{i_1}\ldots\partial^{(1)}_{i_r}\,\mathcal{U}^k_{K\underline{A}}(y,\,y)$$

we obtain because of (4.2), (2.13) and

$$\sigma^{A\dot X}_{i_1}\,\sigma_{i_2A\dot X} \equiv O, \qquad \varphi_{[AB]} = \frac{1}{2}\,\varepsilon_{AB}\,\varphi_{C\cdot}^{\ C}, \qquad B_{ab} = O$$

as in the case of Maxwells equations

$$\begin{aligned}
&\mathfrak{Re}\,N_{i_1\ldots i_6}(y)\\
&\equiv \mathfrak{Re}\,[\sigma^{A\dot X}_{i_1}(y)\,\nabla^{(1)}_{i_2}\ldots\nabla^{(1)}_{i_6}\,\nabla^{(1)\,K}_{\dot X}\,\mathcal{U}^1_{K\underline{A}}(y,\,y)]\\
&\equiv \mathfrak{Re}\,[\sigma^{A\dot X}_{i_1}(y)\,\partial^{(1)}_{i_2}\ldots\partial^{(1)}_{i_6}\,\{\sigma^{rK}_{\dot X}(x)\,(\partial^{(1)}_r\,\mathcal{U}^1_{K\underline{A}}(x,\,y)\\
&\quad - \Gamma^L_{rK}(x)\,\mathcal{U}^1_{L\underline{A}}(x,\,y))\}]_{x=y}\\
&\equiv \mathfrak{Re}\,[3\cdot 5!\,\varepsilon^{K\underline{A}}\,\mathcal{U}^1_{K\underline{A}|i_1\ldots i_6}(y)\\
&\quad + 6!\,\sigma^{A\dot X}_{i_1}(y)\,\sigma^{rK}_{\dot X}(y)\,\mathcal{U}^1_{(K\underline{A})|ri_2\ldots i_6}(y)\\
&\quad + 2\cdot 5!\,\sigma^{A\dot X}_{i_1}(y)\,(\partial_{i_2}\,\partial_{i_3}\,\sigma^{rK}_{\dot X})(y)\,\mathcal{U}^1_{(K\underline{A})|ri_4i_5i_6}(y)\\
&\quad - 5!\,\sigma^{A\dot X}_{i_1}(y)\,\sigma^{rK}_{\dot X}(y)\,(\partial_{i_2}\,\Gamma^L_{rK})(y)\,\mathcal{U}^1_{L\underline{A}|i_3\ldots i_6}]
\end{aligned} \tag{4.3}$$

Using normal coordinates and an adapted basis system from the transport equations (2.6) it follows for $\alpha = 1/2$ and $k = 0,\ 1$ [W4]

$$2\,(x^a\,\partial_a + k)\,\mathcal{U}^k_{X\underline{A}} + \frac{1}{2}\,x^a\,(g^{ij}\,\partial_a\,g_{ij})\,\mathcal{U}^k_{X\underline{A}} = -(g^{ab}\,\nabla_a\,\nabla_b - R/4)\;\mathcal{U}^{k-1}_{X\underline{A}} \tag{4.4}$$

with the initial conditions

$$\mathcal{U}^{(-1)}_{X\underline{A}} = O, \qquad \mathcal{U}^0_{X\underline{A}}\,(y,\,y) = \varepsilon_{X\underline{A}}\,.$$

For the determination of $\mathfrak{Re}\,N_{i_1\ldots i_6}\,(y)$ we need the derivatives of $\mathcal{U}^1_{K\underline{A}}$ up to sixth order, which we can express by means of (4.4) by the derivatives of $g_{ab}$, $g^{ab}$ and $\sigma_a^{A\dot{X}}$ up to eighth order [W4]. Using normal coordinates and an adapted basis system one can represent these derivatives by $\sigma_a^{A\dot{X}}\,(y)$ and by the covariant derivatives of $R_{abcd}$ up to sixth order (*see* [G1]). The coefficients of these represention formulas up to eighth order have not the Property $F$. Consequently, if we express in this way the summands of (4.3) by the covariant derivatives of $R_{abcd}$ the coefficients have not the Property $F$. Finally, symmetrizations and alternations up to eight indices, the Ricci- and Bianchi identities and the elimination of the linear monomials by means of $B_{ab} = 0$ don't imply the Property $F$. Hence, the coefficients $\rho_3$, $\rho'_3$ have not the Property $F$, *i. e.*, the condition (2.19) is correct for $\sigma \in \{2,\ 3\}$ and the Theorem 1 is proved.

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## REFERENCES

[A] M. ALVAREZ, Zum Huygensschen Prinzip bei einigen Klassen spinorieller Feldgleichungen in gekrümmten Raum-Zeit-Mannigfaltrigkeiten, *Dissertation A, Pädgogische Hochschule.*

[AW] M. ALVAREZ and V. WÜNSCH, Zur Gültigkeit des Huygensschen Prinzips bei der Weyl-Gleichung und den homogenen Maxwellschen Gleichunge für Metriken vom Petrow-Typ N, *Wiss. Zeitschr. d. Päd. Hochschule Erfurt/Mühlhausen, Math.-naturwissensch.* Reihe **27**, 1991, H. 2, pp. 77-91.

[B] Y. BRUHAT, Sur la théorie des propagateurs, *Ann. Mat. Pura Appl.*, Vol. **64**, 1964, pp. 191-228.

[CM] J. CARMINATI and R. G. MCLENAGHAN, An Explicit Determination of the Space-times on Which the Conformally Invariant Scalar Wave Equation Satisfies Huygens' Principle, *Ann. Inst. Henri Poincaré, Phys. théor.*, Vol. **44**, 1986, pp. 115-153; Part. II, Vol. **47**, 1987, pp. 337-354; Part. III, Vol. **48**, 1988, pp. 77-96; Vol. **54**, 1991, p. 9.

[DC] L. DEFRISE-CARTER, Conformal Groups and Conformally Equivalent Isometry Groups, *Comm. Math. Phys.*, Vol. **40**, 1975, pp. 273-282.

[F] F. G. FRIEDLANDER, *The Wave Equation on a Curved Space-time,* Cambridge University Press; Cambridge, London, New York, Melbourne, 1975.

[GeW1] R. GERLACH and V. WÜNSCH, Über konforminvariante Tensoren ungerader Stufe in gekrümmten Raum-Zeiten, *Wiss. Zeitschr. d. Päd. Hochschule Erfurt-Mühlhausen, Math.-naturwissensch.* Reihe **26**, 1990, H. 1, pp. 20-32.

[GeW2] R. GERLACH and V. WÜNSCH, Über konforminvariante Tensoren der Stufe sechs in gekrümmten Raum-Zeiten, *Wiss. Zeitschr. d. Päd. Hochschule Erfurt-Mühlhausen, Math.-naturwissensch.* Reihe **27**, 1991, H. 2, pp. 117-143.

[G1] P. GÜNTHER, Spinorkalkül und Normakoordinaten, *ZAMM*, Vol. **55**, 1975, pp. 205-210

[G2] P. GÜNTHER, Einige Sätze über Huygenssche Differentialgleichungen. *Wiss. Zs. Karl-Marx-Univ. Leipzig,* Vol. **14**, 1965, pp. 497-507.

[G3] P. GÜNTHER, Ein Beispiel einer nichttrivialen Huygensschen Differentialgleichung mit vier unabhängigen Veränderlichen. *Archive Rat. Mech. and Analysis,* Vol. **18**, 1965, pp. 103-106.

[G4] P. GÜNTHER, *Huygens' Principle and Hyperbolic Equations,* Boston, Academic Press, 1988.

[GW1] P. GÜNTHER and V. WÜNSCH, Maxwellsche Gleichungen und Huygenssches Prinzip I, *Math. Nachr.*, Vol. **63**, 1974, pp. 97-121.

[GW2] P. GÜNTHER and V. WÜNSCH, Contributions to a theory of polynomial conformal tensors, *Math. Nachr.*, Vol. **126**, 1986, pp. 83-100.

[GW3] P. GÜNTHER and V. WÜNSCH, On Some Polynomial Conformal Tensors, *Math. Nachr*, Vol. **124**, 1985, pp. 217-238.

[Ha] J. HADAMARD, *Lectures on Cauchy's Problem in Linear Partial Differential Equations*, Yale University Press, New Haven, 1923.

[He] S. HELGASON, Huygens' Principle for Wave Equations on Symmetric Spaces, *J. Funkt. Anal.*, Vol. **107**, 1992, pp. 279-288.

[I] R. ILLGE, Zur Gültigkeit des Huygensschen Prinzips bei hyperbolischen Differentialgleichungssystemen in statischen Raum-Zeiten, *Zs. für Anal. und Anwendungen,* Vol. **6**, 1987, (5) pp. 385-407.

[L] A. LICHNEROWICZ, Champs spinoriels et propagateurs en relativité générale, *Bull. Soc. math. Fr.*, Vol. **92**, 1964, pp. 11-100.

[McL1] R. G. MCLENAGHAN, An Explicit Determination of the Empty Space-times on which the Wave Equation Satisfies Huygens' Principle, *Proc. Cambridge Phil. Soc.*, Vol. **65**, 1969, pp. 139-155.

[McL2] R. G. MCLENAGHAN, On the Validity of Huygens' Principle for Second Order Partial Differential Equations With Four Independent Variables. Part. I. Derivation of Necessary Conditions, *Ann. Inst. H. Poincaré,* Vol. A **20**, 1974, pp. 153-188.

[McL3] R. G. MCLENAGHAN, Huygens' principle, *Ann. Inst. H. Poincaré,* Vol. A **37**, 1982, pp. 211-236.

[McL, L] R. G. MCLENAGHAN and J. LEROY, Complex Recurrent Space-Times. *Proc. Roy. Soc. London*, Vol. A **327**, 1972, pp. 229-249.

[McL, W] R. G. MCLENAGHAN and G. C. WILLIAMS, An explicit Determination of the Petrov Type D Space-times on which Weyl's Neutrino Equation and Maxwell's Equations Satisfy Huygens' Principle, *Ann. Inst. Henri Poincaré,* Vol. A **53**, 1990, p. 217.

[Ø] B. ØRSTED, The Conformal Invariance of Huygens' Principle, *Diff. Geom.*, Vol. **16**, 1981, pp. 1-9.

[PR] R. PENROSE and W. RINDLER, *Spinors and Space-time,* Vol. **1**, 1984, Vol. **2**, 1986, Cambridge Univ. Press.

[RW] B. RINKE and V. WÜNSCH, Zum Huygensschen Prinzip bei der skalaren Wellengleichung, *Beiträge zur Analysis,* Vol. **18**, 1981, pp. 43-75.

[S1] R. SCHIMMING, Zur Gültigkeit des Huygensschen Prinzips bei einer speziellen Metrik, *ZAMM,* Vol. **51**, 1971, pp. 201-208.

[S2] R. SCHIMMING, Riemannsche Räume mit ebenfrontiger und ebener Symmetrie, *Math. Nachr,* Vol. **59**, 1974, pp. 129-162.

[W1] V. WÜNSCH, Über selbstadjungierte huygenssche Differentialgleichungen, *Math. Nachr.,* Vol. **47**, 1970, pp. 131-154.

[W2] V. WÜNSCH, Maxwellsche Gleichungen und Huygenssches Prinzip II, *Math. Nachr.,* Vol. **73**, 1976, pp. 19-36.

[W3] V. WÜNSCH, Über eine Klasse konforminvarianter Tensoren, *Math. Nachr.,* Vol. **73**, 1976, pp. 37-58.

[W4] V. WÜNSCH, Cauchy-Problem und Huygenssches Prinzip bei einigen Klassen spinorieller Feldgleichungen I, II, *Beiträge zur Analysis,* Vol. **12**, 1978, pp. 47-76; Vol. **13**, 1979, pp. 147-177.

[W5] V. WÜNSCH, Konforminvariante Variationsprobleme und Huygenssches Prinzip, *Math. Nachr.,* Vol.**120**, 1985, pp. 175-193.

[W6] V. WÜNSCH, C-Räume und Huygenssches Prinzip, *Wiss. Zeitscher. d. Päd. Hochschule Erfurt-Mühlhausen, Math.-naturwissensch,* Reihe **23**, 1987, H. 1, pp. 103-111.

[W7] V. WÜNSCH, Huygens' Principle on Petrov Type D Space-Times, *Ann. d. Physik,* 7. Folge, Bd. **46**, H. 8, 1989, pp. 593-597.

[W8] V. WÜNSCH, Moments and Huygens' Principle for Conformally Invariant Field Equations in Curved Space Times, *Ann. Inst. Henri Poincaré,* Vol. **A60**, 4, 1994.

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