ANNALES DE L'I. H. P., SECTION A

IAN MOSS

Quantum cosmology and the self observing universe

Annales de l'I. H. P., section A, tome 49, nº 3 (1988), p. 341-349
<http://www.numdam.org/item?id=AIHPA_1988__49_3_341_0>

Ann. Inst. Henri Poincaré,
Vol. 49, n° 3, 1988, p. 341-349. *Physique théorique*

Quantum Cosmology and the Self Observing Universe

by

Ian MOSS
Department of Physics, University of Newcastle upon Tyne,
Newcastle upon Tyne NE1 7RU

ABSTRACT. — We review some of the aspects of quantum cosmology with respect to quantum mechanics and general relativity.

RÉSUMÉ. — Nous présentons quelques aspects spécifiques de la cosmologie quantique par rapport à la mécanique quantique et la relativité générale.

1. INTRODUCTION

Quantum mechanics has inherited many basic features from its origin as a theory of microscopic phenomena. The structure of time in quantum mechanics is essentially Newtonian and it took several years to incorporate special relativity into quantum mechanical theory. Quantum mechanics has also inherited a commonly used interpretational framework which makes a complete separation between observers and the system, contrary to the spirit of the theory. This means that in quantum cosmology, where attempts are made to model the universe as a quantum system, the principles of quantum mechanics have had to be stretched. The result is arguably a more attractive theory. Some of the rigid framework has disappeared and many of us have to admit to being part of the quantum system.

Modern theories of cosmology make use of the principle of general relativity, that the laws of physics can be expressed in an equivalent way for any choice of coordinates. This means in particular that coordinate time should not be a physical observable. However, general relativity is not unique in having a time reparameterisation symmetry. The same symmetry can be written into any Hamiltonian theory and a theory formulated this way has many advantages. It is well known that symmetry can be used to reduce ambiguity in the form of the quantised theory. DeWitt [1], for example, demonstrated that covariance on configuration space could be used to fix the operator ordering in Schrodinger's equation. The time reparametrisation symmetry extends this still further, implying that Schrodinger's equation is covariant under conformal rescalings of the wave function.

There are occasions in quantum cosmology where it seems necessary to construct the quantum theory by a sum over random paths or a Feynman path integral, rather than by the action of operators on a Hilbert space [2]. The choice of paths is part of the quantisation procedure. A sum over paths which are closed and compact seems to suit applications both in cosmology and particle mechanics. This gives some indication that a general quantisation procedure exists which can be applied to all physical theories.

Finally, the tendency of quantum mechanics to mix levels of description is important for the interpretation of quantum cosmology. Somehow, the real world of experience has to be related to the quantum description in terms of wave functions and operators. This process is what is usually meant by « measurement ».

Our common experience is only directly about macroscopic phenomena. This restricts the class of physical observables to those which cannot discern microscopic changes of parameters. For these observables, the quantum state can be replaced by an averaged density matrix, summed over microstates. This averaging suppresses interference terms in the density matrix. In the words of J. A. Wheeler [3] microscopic phenomena are brought into our experience by « irreversible acts of amplification ». The expanding universe, continually increasing its own action, allows for irreversible acts of self amplification. This is part of what we mean by the self observing universe.

2. TIME REPARAMETERISATION SYMMETRY

In gravitational theory there is a freedom to choose the time coordinate, and the quantum theory of gravitational systems has to respect this freedom. The nearest that we come to a physical concept of time is when a clock of some kind is introduced as part of the system. Then time evolution can be defined in terms of correlations between events and configurations of the clock.

A useful prototype model with time reparameterisation symmetry [4], [5] consists of a relativistic particle moving on a curved manifold. The action is

$$S = \int (\dot{x}^a p_a - \mathrm{NH})dt$$

where x^a and p_a are the coordinates and momenta. The Hamiltonian

$$\mathrm{H} = \frac{1}{2}(g^{ab}(x)p_a p_b + m^2)$$

where $g_{ab}(x)$ is the spacetime metric. The lapse function $\mathrm{N}(t)$ measures the rate of change of coordinate time with respect to proper time t. From this action one obtains the classical equations from the variation with respect to N, p_a and x^a,

$$p^2 + m^2 = 0$$
$$p_a = \mathrm{N}^{-1} g_{ab}\dot{x}^b$$
$$\dot{p}_a = 0 .$$

These are the equations for geodesic motion in curved space.

Other models with time reparameterisation can be written in a very similar way. The non-relativistic particle has an action of the same form but with $\mathrm{H} = p^2/2m - \mathrm{E}$. Quantum gravity also has a similar action, though defined on superspace. The coordinates x^a are replaced by the three metric g_{ij}, and g_{ab} is replaced by the superspace metric G_{ijkl}. In this case, spatial reparameterisation symmetry leads also to three additional momentum constraints [*6*].

Returning to the relativistic particle, when this is quantised the operator version of the constraint $\mathrm{H} = 0$ becomes Schrodinger's equation $\mathrm{H}\psi = 0$ where ψ is a wave function. The form of H is potentially a source of ambiguity, and in general it could take a form

$$\mathrm{H} = \frac{1}{2}(\mathrm{F}_1(x)^{-1} g^{ab} p_a \mathrm{F}_1(x) p_b + m^2 + \hbar^2 \mathrm{F}_2(x))$$

with unknown functions $\mathrm{F}_1(x)$, $\mathrm{F}_2(x)$. In canonical quantisation, the first term is an obvious factor ordering ambiguity resulting from the commutation relations. The final term is needed to generate time evolution and is also ambiguous because of factor ordering problems. In Feynman's path integral quantisation, the first term reflects the skeletonisation ambiguity of the paths, and corresponds to the usual Ito-Stetanovich freedom to choose stochastic integrals. The final term depends upon how one interpolates the action between nearby points [*8*].

DeWitt pointed out in 1957 that the coordinate invariance of the action could be used to constrain some of this ambiguity and reduce H to

$$\mathrm{H} = \frac{1}{2}(-\nabla^2 + m^2 + \hbar^2 \mathrm{F}_2(\mathrm{R}))$$

where ∇ and R are the connection and curvature of the metric g. In fact, the relevant metric is not g_{ab} but rather

$$G_{ab} = N^{-1} g_{ab} .$$

Coordinate invariance does not fix the value of F_2 [7], though some results suggest that it should be only linear in R [8].

There is a way to fix the residual ambiguity by making use of the time reparameterisation. This is based on the observation that $N(t) \to N(t)\omega^2(x)$, then the action

$$S_\omega = \int (\dot{x}^a p_a - N\omega^2 H) dt$$

generates exactly the same classical equations as S. When we come to quantise the theory, there seems no reason to prefer any particular S over any other. Suppose, therefore, that we demand that when the quantisation procedure is applied to S_ω, it generates the same quantum theory as S. This assumption fixes the value of F_2.

Under the change of N,

$$G_{ab} = N^{-1}\omega^{-2} g_{ab} = N^{-1}\hat{g}_{ab}$$

the Hamiltonian becomes

$$H_\omega = \frac{1}{2}(-\hat{\nabla}^2 + \omega^2 m^2 + \hat{F})$$

where the derivative is now associated with the conformal metric $\hat{g}_{ab}$. We also have to transform ψ to ψ_ω to ensure that

$$H_\omega \psi_\omega = 0 .$$

This can only be achieved if H has the conformal coupling,

$$H = \frac{1}{2}\left(-\nabla^2 + m^2 + \frac{1}{4}\frac{n-2}{n-1}R\right)$$

and $\psi_\omega = \omega^{1-\frac{n}{2}}\psi$. However, the only physical appearance of ψ, which is in the probability itself, remains unaltered because the integration measure also rescales.

So far only the measure ambiguity of the path integral has been discussed, but there is also the question of what set of paths should be included in any given situation. A natural choice is to define the probability amplitude $\Delta(x, x')$ for a particle to be at position x at time x^0 and x' at time $x^{0\prime}$ to be

$$\Delta(x, x') = \int_{\mathscr{P}} e^{iS/\hbar}$$

where the path integral extends over all paths which are timelike with endpoints at x and x' (see fig. 1).

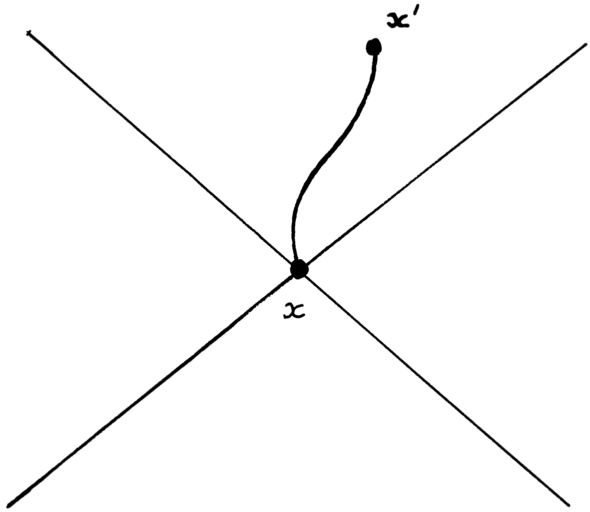

FIG. 1.

It also includes an integral over the lapse function N. The result is the same as the free field theory commutator function. If the spacetime manifold is Euclidean, then the result is the analytically continued Feynman Green function.

Unitarity in the path integral formalism corresponds to the statement that

$$\Delta(x, x') = \int_{x''} \Delta(x, x'')\Delta(x'', x')$$

where the integral extends over all values of x which have a common time coordinate (see fig. 2).

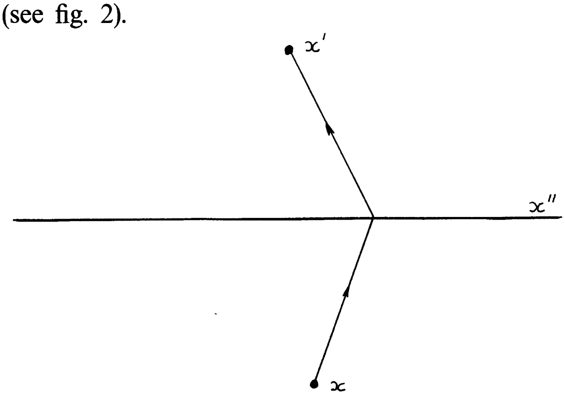

FIG. 2.

Recently, J. B. Hartle [2] has argued that this decomposition of the underlying space is too restrictive for quantum cosmology, and other possibilities such as the ones shown on fig. 3 have to be considered

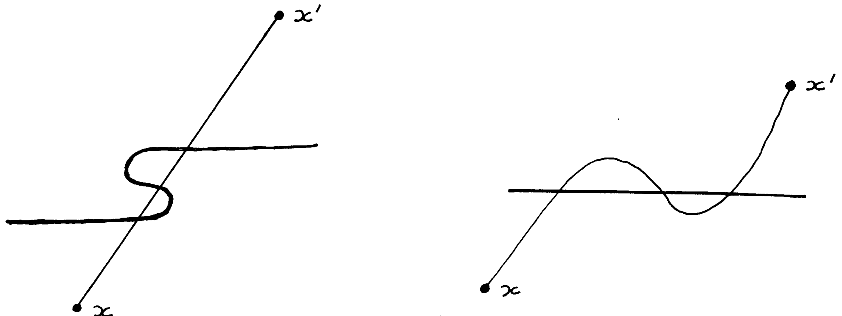

FIG. 3.

In either of these two cases the unitarity condition breaks down. As pointed out by Hartle this destroys any chance of constructing the usual Hilbert space structure. (In the diagram on the right of fig. 3, we can recover a Hilbert space structure at the expense of introducing many body theory or second quantization.) The path integral still makes sense. Suppose that the system preparation and observations which we make are expressed as a set of conditions $\mathscr{C}$ on the location of the particle. Then the amplitude for $\mathscr{C}$ is expressed by a sum over paths $\mathscr{P}$ which are consistent with $\mathscr{C}$.

This prescription still leaves unspecified what measure is to be associated with paths which go to infinity. A significant improvement can be made if the path integral is constructed not to give the amplitude but the probability directly. For example, the probability for a particle prepared at x to appear at x' is given by

$$P(x, x') = \int_{\mathscr{P}} e^{iS/\hbar}$$

where p is the closed path shown on fig. 4,

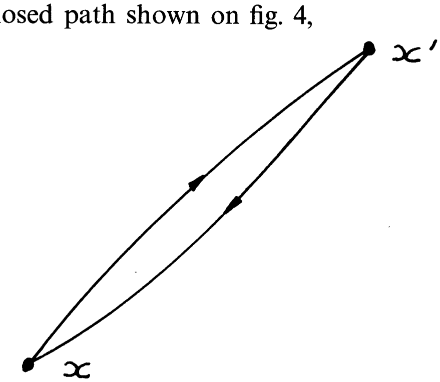

FIG. 4.

The intrinsic time runs in the direction shown by the arrows, with the result that

$$P(x, x') = \Delta(x, x')\Delta^*(x, x').$$

Quite generally, it would seem to be advantageous to define a probability for conditions $\mathscr{C}$ by

$$P(\mathscr{C}) = \int_{\mathscr{P}} e^{iS/\hbar}$$

where $\mathscr{P}$ is the set of paths which are consistent with $\mathscr{C}$ and also compact.

It is instructive to see how this prescription works in the case that $\mathscr{C}$ consists of a single point x. Suppose, first of all, that the particle is a simple harmonic oscillator with $H = p^2/2m + mw^2x^2/2$. The saddle point paths with imaginary N start from x and return to x after bouncing of the potential as shown on fig. 5

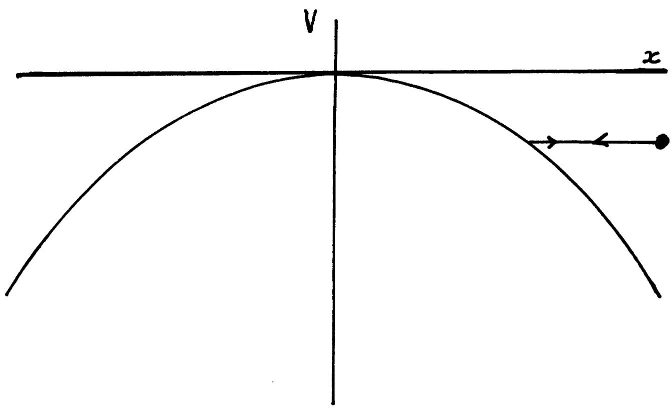

FIG. 5.

The time coordinate is also reversed at the bounce point. The main contribution to the path integral comes from the limit where the path asymptotically reaches $x = 0$, and gives

$$P(x) = \frac{w}{\sqrt{\pi}} e^{-w^2x^2}$$

which is the ground state probability distribution. In this case the prescription is equivalent to the one given by Landau for the purposes of calculating the ground state wave function [*9*].

Now consider a particle in a gas with temperature T, which is related to a particle in a spacetime which has the imaginary time coordinate periodically identified with period $\beta = T^{-1}$. The path integral prescription for finding a particle at position x now contains a contribution from closed curves (see fig. 6).

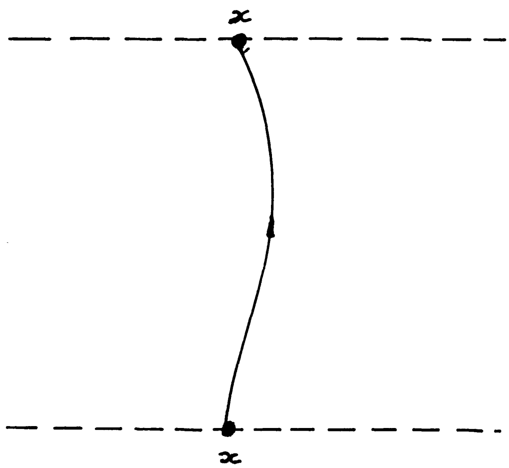

FIG. 6.

which move a distance β in the imaginary time direction. This is identical to Feynman's path integral prescription for the density matrix [10], that is

$$P(x) = \rho(x, x).$$

Finally, in quantum cosmology the point x represents a three dimensional manifold with metric g_{ij}. The probability of measuring g_{ij} is given by a path integral over compact four geometries (fig. 7).

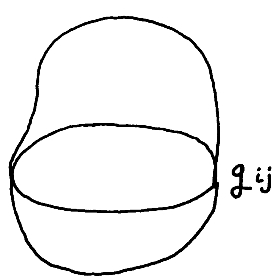

FIG. 7.

Those who are familiar with the Hartle-Hawking formula [11] for the quantum state of the universe ψ_H will recognise the result that, for simply connected four geometries,

$$P(g) = \psi_H^*(g)\psi_H(g).$$

This probability only applies in the absence of any further information about the state of the universe. However, in practice we are interested most of all by what is accessible to our observations, which of necessity is a universe in which we are able to exist. This fact implies a set of conditions which allows at the very least for the existence of information processing systems. These conditions would include such things as a minimum size for the universe and restrictions on the phase of its matter content. The application of such conditions is also a part of what we mean by the self observing universe.

REFERENCES

[1] B. S. DeWitt, *Rev. Mod. Phys.*, t. **29**, 1957, p. 377.
[2] J. B. Hartle in *Gravitation in astrophysics* (NATO ASI, 1986).
[3] J. A. Wheeler and W. Zurek, *Quantum Theory and Measurement* (Princeton University Press, 1983).
[4] J. A. Hartle and K. V. Kuchar, *Phys. Rev.*, t. **D 34**, 1986, p. 2323.
[5] C. Teitelboim, *Phys. Rev.*, t. **D 25**, 1982, p. 3159.
[6] R. Arnowitt, S. Deser and C. W. Misner in *Gravitation: An introduction to current research*, ed. L. Witten (Wiley, New York, 1962).
[7] J. S. Dowker, *J. Phys.*, t. **A 7**, 1974, p. 1256.
[8] L. Parker, *Phys. Rev.*, t. **D 19**, 1979, p. 438.
[9] L. D. Landau and E. M. Lifshitz, *Quantum mechanics* (Pergamon Press, Oxford).
[10] R. P. Feynman, *Statistical physics*.
[11] J. B. Hartle and S. W. Hawking, *Phys. Rev.*, t. **D 28**, 1983, p. 2960.

(Manuscrit reçu le 10 juillet 1988)