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Trigonometric perturbation of the Gaussian generalized fields. Small coupling results

by

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RÉSUMÉ. — Nous étudions les équations fonctionnelles DLR correspondant aux perturbations trigonométriques de couplage faible des mesures Markoviennes Gaussiennes.

Sous des hypothèses techniques, nous démontrons que la propriété de Markov est préservée.

ABSTRACT. — We study functional DLR-equations corresponding to the trigonometric perturbations of the Gaussian, markovian measures in the weak coupling regime. We prove that under certain mild assumptions of the technical nature the global Markov property is preserved for such perturbations.

1. INTRODUCTION

The global Markov property with respect to the hyperplanes of the generalized random field indexed by the conventional nuclear spaces like the space $D(R^d)$ or $S(R^d)$ is one of the basic axiom of the Nelson axiomatic framework [*1*].

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After an euclideanisation of the constructive field theory mathematical technologies, the progress for the superrenormalizable interactions has been very fast and fruitfull in the decade of 70. (Almost) all superrenormalizable interactions have been controlled rigorously. For a background see [2] [3], ... for the construction of the three-dimensional Yang-Mills theories see [4] and references therein. However, already on the level of the simplest $P(\varphi)_2$ models the Nelson axioms have never been veryfield explicitly. There exists some eqzotic two-dimensional models like: $\cos \alpha\varphi_2$, $\exp \alpha\varphi_2$ where the most difficult Nelson axiom concerning the global Markov property, has recently been veryfied [5] [6] [7] [8]. The difficulties in its veryfication for the weakly coupled $P(\varphi)_2$ models seem to be technical but not conceptual. This was the reason that several papers appeared [9] [10] examining global Markov property for the lattice $P(\varphi)$ fields. These methods are based on a certain technics heavily depending on the ferromagneticity of the corresponding local specifications. However, it seems to be difficult to observe such a ferromagnetism for the continual local specifications. Our paper proposes a new possible strategy to attack the problem for at least: $\varphi^4 :_{2,3}$ like models without refering to ferromagnetism. It is well known that starting from the $\cos \alpha\varphi_2$-like theory, we can obtain φ_2^4 theory by a suitable limiting procedure [11] [12]. Our idea is based on this observation. We start with $\cos \alpha\varphi$ interactions for some Pauli-Villard regularization of the free field.

For such a class of theories we verify in a very economical way and quickly the Markov property. This is done in the present paper. The next two steps (in preparation) first translate the result for φ_2^4 theory with the Pauli-Villars regularisation fixed and then remove it.

The second novel aspect of our paper is its possible application to the classical statistical mechanics. It is well known that the grand canonical ensemble of the gas of classical particles interacting via a two-body potential of the positive type can be described by functional integrals of the type analysed here [13]. From the result proven here there follows the existence of the transfer matrix for such a class of systems.

This is an addendum to the earlier results of [14] [15] [16] [17].

2. PRELIMINARY DEFINITIONS AND RESULTS

a) Markov property.

Let $P : R^d \to R^1$ be a nonnegative polynomial and let $A(P)$ be a real differential operator with the constant coefficients the symbol of which is given by P i. e. $A(P)f = (P(k)\hat{f})^\vee$ where $\hat{}$ (resp. $\check{}$) denotes the Furier transformation (resp. inverse). We assume that $P^{-1}(k)$ is not too strongly singular at the origin. i. e. we will assume that

$$\int_{\{k\mid |k|<1\}} \frac{dk}{P(k)} < \infty . \tag{2.1}$$

With A(P) given, we define a map $A^{\frac{1}{2}}(P) : S(R^d) \to L^2(R^d)$ by the formula:

$$A^{\frac{1}{2}}(P)f = \left(\frac{1}{P(k)^{1/2}} \hat{f}\right)^{\vee} \tag{2.2}$$

which is nonnegatively defined and then we define on the space $S(R^d)$ an inner product by

$$(f, g)_A = (A^{\frac{1}{2}}f, A^{\frac{1}{2}}g)_{L_2(R^d)} . \tag{2.3}$$

The metric completion of the pair $\{ S(R^d), (.,.)_A \}$ is defined by H(P). It is obvious that the space H(P) can be identifield with a certain subset of the space of tempered distributions $S'(R^d)$ by

$$H(P) \approx \left\{ \varphi \in S'(R^d) \mid \varphi \in L^1_{loc}(R^d), \int \frac{|\hat{\varphi}(k)|^2}{P(k)} dk < \infty \right\}. \tag{2.4}$$

From our assumptions on $P(k)$ it follows that the space $C_0^\infty(R^d)$ is dense in H(P). For any open region $\Lambda \subset R^d$ we define $H_\Lambda(P)$ as a subspace (closed) of those $\varphi \in H(P)$ which have support in Λ. It is easy to check that then the space $C_c^\infty(\Lambda)$ is again dense in the space $H_\Lambda(P)$. For closed Λ we define: $H_\Lambda(P) = \bigcap_{\Lambda' \supset \Lambda, \Lambda'} \text{open } H_{\Lambda'}(P)$.

It follows easily that the bilinear from $V(f, g) = (f, g)_A$ defines a positive definite and continuous form on the space $S(R^d)$. From the Minlos theorem it follows that there exists a unique Gaussian centered measure μ_V^0 on $\{ S'(R^d), B(S'(R^d)) \}$ whose Fourier transform is given by:

$$\int_{S'(R^d)} \mu_V^0(d\varphi)e^{i\varphi(f)} = e^{-\frac{1}{2}V(f,f)} \tag{2.5}$$

Let us denote by $H(\mu_V^0)$ the metric completion (in the $L^2_{\mu_V^0}$ norm) of the linear hull of the random elements of the form $\varphi(f)$ where $f \in S(R^d)$. From the density of $S(R^d)$ in the space H(P) and from the fact that the map $f \to \varphi(f)$ is isometric from $H(P) \supset S(R^d) \to H(\mu_V^0)$ it easily follows that H(P) is isometric with $H(\mu_V^0)$. For $f \in H(P)$ we denote the corresponding elements in $H(\mu_V^0)$ again as $\varphi(f)$. Let Λ be an arbitrary nonempty subset of R^d. Then we define $\Sigma(\Lambda)$ (resp. $\hat{\Sigma}(\Lambda)$) the minimal (completed) σ-algebras generated by the family of random variables $\{ \varphi(f), f \in S(R^d) \text{ supp } f \subset \Lambda \}$ (resp. $\{ \varphi(f), f \in H_\Lambda(P) \}$). It is clear that $\Sigma(\Lambda) \subseteq \hat{\Sigma}(\Lambda)$. For $\Lambda \subset R^d$ being open, it follows from the density of $C_0^\infty(\Lambda)$ in $H_\Lambda(P)$ that $\Sigma(\Lambda) = \hat{\Sigma}(\Lambda)(\mu_V^0 - a.e.)$.

For $\Lambda \subset \mathrm{R}^d$ closed let $\mathscr{F}(\Lambda) = \bigcap_{\varepsilon>0} \Sigma(\Lambda^\varepsilon)$. From the above remark we obtain (for Λ closed)

$$\mathscr{F}(\Lambda) = \bigcap_{\varepsilon>0} \Sigma(\Lambda^\varepsilon) = \bigcap_{\varepsilon>0} \hat{\Sigma}(\Lambda^\varepsilon) \equiv \hat{\mathscr{F}}(\Lambda). \tag{2.6}$$

The following general result from the theory of Gaussian generalized random field is well known [*18*] [*19*] [*10*]:

LEMMA 2.1.

1. Let Λ be open in R^d and let Π_{Λ^c} be an orthogonal projector in the space H(P) onto the subspace $\mathrm{H}_{\Lambda^c}(\mathrm{P})$. Then, the conditional expectation value with respect to the measure μ_{V}^0 and the σ-algebra $\mathscr{F}(\Lambda^c)$ is given by

$$\mathrm{E}_{\mu_{\mathrm{V}}^0}\{ e^{i\varphi(f)} \mid \mathscr{F}(\Lambda^c) \}(\eta) = e^{i\eta(\Pi_{\Lambda^c}(f))}.\mu^0_{\mathrm{S}_{\Lambda^c}}(e^{i\varphi(f)}) = e^{i\eta(\Pi_{\Lambda^c}(f))}e^{-\frac{1}{2}\mathrm{S}_{\Lambda^c}(f,f)} \tag{2.7}$$

where S_{Λ^c} is the covariance given by the kernel:

$$\mathrm{S}_{\Lambda^c}(f, g) = ((1 - \Pi_{\Lambda^c})f, g)_{\mathrm{A}} = \mathrm{V}((1 - \Pi_{\Lambda^c})f, g). \tag{2.8}$$

Moreover formula (2.7) makes sense for μ_{V}^0-almost every $\eta \in \mathrm{S}'(\mathrm{R}^d)$.

2. The decomposition

$$\varphi = (1 - \Pi_{\Lambda^c})^*\varphi + \Pi^*_{\Lambda^c}(\varphi) \tag{2.9}$$

where $(1 - \Pi_{\Lambda^c})^*\varphi$ denotes symbolically the Gaussian centered random field with covariance equal to S_{Λ^c} and $\Pi^*_{\Lambda^c}(\varphi)$ is the Gaussian field with the covariance equal to $\mathrm{K}_{\Lambda^c} \equiv \mathrm{V} - \mathrm{S}_{\Lambda^c}$ is the orthogonal stochastic decomposition with respect to the σ-algebra $\mathscr{F}(\Lambda^c)$.

LEMMA 2.2. — For the covariance V as above and for Λ bounded and open, the corresponding process $\Pi^*_{\Lambda^c}(\varphi)$ is $\mathscr{F}(\partial\Lambda)$ measurable.

Lemme 2.2 expresses the notation of the higher-order markovianity introduced by McKean jr [*21*] and Pitt [*22*] and then studied by many peoples [*19*] [*20*] [*23*]. In the case when the operator A is of the second order it can be shown that $\mathscr{F}(\partial\Lambda) = \hat{\Sigma}(\partial\Lambda)$ for sufficiently regular $\partial\Lambda$ and then we have the standard markovianity of σ-order.

Whenever for any bounded open region $\Lambda \subset \mathrm{R}^d$ and for any two random functions G, F localized strictly in Λ (respectively in Λ^c) we have an equality

$$\mathrm{E}_{\mu_{\mathrm{V}}^0}\{ \mathrm{F}\cdot\mathrm{G} \mid \mathscr{F}(\partial\Lambda) \} = \mathrm{E}_{\mu_{\mathrm{V}}^0}\{ \mathrm{F} \mid \mathscr{F}(\partial\Lambda) \}\, \mathrm{E}_{\mu_{\mathrm{V}}^0}\{ \mathrm{G} \mid \mathscr{F}(\partial\Lambda) \} \tag{2.10}$$

we shall say that μ_{V}^0 is locally Markov. When the equality (2.10) holds for unbounded Λ, we shall also say that μ_{V}^0 is globally Markov.

In the following we choose A(P) to be of the form

$$\mathrm{A} = (-\Delta + \alpha_1)\cdot \ldots \cdot(-\Delta + \alpha_n), \qquad 0 < \alpha_1 \leq \alpha_2 \leq \ldots \leq \alpha_n.$$

With such a choice of A, there exists a constructive description of the projectors Π_{Λ^c} in the space H(P) at least for Λ sufficiently regular from the point of view of the theory of general hyperelliptic boundary Dirichlet problems.

For $g \in C_0^\infty(R^d)$ it can easily be proved that $\Pi_{\Lambda^c}(g)$ is given by the solution of the following Dirichlet problem:

$$(A(P)\cdot \Pi_{\Lambda^c}(g))(x) = 0 \quad \text{for} \quad x \in \operatorname{Int} \Lambda,$$
$$\partial_n^{k,\mathrm{int}}(\Pi_{\Lambda^c}(g))(x) = \partial_n^{k,\mathrm{int}}(g)(x) \quad \text{for} \quad x \in \partial\Lambda, \qquad k = 0, \ldots, n-1$$

where $\partial_n^{k,\mathrm{int}}$ denotes the k-th normal derivative at boundary point from the interior. (2.12)

In the paper [*24*] the description of the projectors $\Pi_{\Lambda^c}(f)$ for Λ with piecewise C^∞ smooth boundary and certain conical properties imposed on $\partial\Lambda$ has been given as a weak-solution of the problem (2.11) in the space $S'(R^d)$. More precisely, extending ideas of the basic work of Wiener [*25*] in the paper [*24*] it has been proved that for $\mu_V^0 - a$ every $\eta \in S'(R^d)$ the weak solution $\Pi_{\Lambda^c}^*(\eta)$ of the problem (2.12) exists if Λ fulfills the conditions as above. From the ellipticity it follows that then $\Pi_{\Lambda^c}^*(\eta)$ is a real analytic function inside Λ for $\mu_V^0 - a$. every $\eta \in S'(R^d)$ as a (weak) solution of hyperelliptic homogeneous equation.

It is an immediate consequence of the above remarks that the Gaussian Markov fields corresponding to the hyperelliptic operators as above have local Markov property. The question about the global Markov property is more delicate and has been resolved at least for hyperplanes by Molchan [*20*] see also [*19*].

b) Gibbsian perturbation.

In a strict analogy with the Euclidean field theory we shall introduce a concept of Gibbsian perturbation of the given Gaussian measure μ_V^0 as above. Let Λ be bounded open region in R^d. A multiplicative functional of the Gaussian field μ_V^0 is a random variable X_Λ which is positive, μ_V^0-integrable and for every open $\Lambda_1 \subset \Lambda$, X_Λ can be expressed as the product of two positive integrable random variables $X_\Lambda = X_{\Lambda_1} \cdot X_{\Lambda - \Lambda_1}$ where X_{Λ_1} is $\Sigma(\Lambda_1)$ measurable. In this paper we choose

$$X_1 = \exp\left(z \int_\Lambda \int :\cos \alpha\varphi :_V (x) dx dv(\alpha)\right),$$

v-bounded, real measure with compact support on R^1,

$$:\cos \alpha\varphi :_V(x) = \exp \frac{\alpha^2}{2} V(0) \cos \alpha\varphi(x) \qquad (2.14)$$

where $\alpha, z \in R^1$ and we assume $V(0) < \infty$ (see however remarks in section 5).

which corresponds to the assumption that $n > d/2$. With the help of X_Λ, we than define a new measure

$$\mu_\Lambda(d\varphi) \equiv (Z_\Lambda(z))^{-1} \cdot X_\Lambda(\varphi)\mu_V^0(d\varphi), \tag{2.15}$$

$$Z_\Lambda(z) = \mu_V^0(X_\Lambda(\varphi)) \tag{2.16}$$

which is called trigonometric perturbation of the measure μ_V^0 in the finite volume. Of particular interest is the thermodynamic limit $\Lambda \uparrow R^d$ of the measure $\mu_\Lambda(d\varphi)$. The simple Kirkwood-Salsburg [28] analysis gives.

LEMMA 2.3. — For $|z| < C(V)^{-1} \exp - \alpha_*^2 V(0) - 1$

where
$$C(V) = \sup_\alpha \int | e^{-\alpha\beta V(x)} - 1 | dx d\nu(\beta), \tag{2.17 a}$$

$$\alpha_* \equiv \sup \{ |\alpha| \mid \alpha \in \operatorname{supp} d\nu \} \tag{2.17 b}$$

the unique thermodynamic limit $\lim_\Lambda \mu_\Lambda(d\varphi) = \mu_\infty(z)$ as a weak limit exists. The measure $\mu_\infty(z)$ is translational invariant and is concentrated on the continuous functions. Moreover, the measure $\mu_\infty(z)$ is ergodic with respect to the translations.

Proof. — See [26] [27] where existence of $\mu_\infty(z)$ is proved. Algebraic formalism of [28] then gives the cluster property of $\mu_\infty(z)$. The support properties are the trace of the famous Kolmogorov criterion.

Our main interest concerns the Markov properties of the measure $\mu_\infty(z)$. Owing to the discussion from the earlier paper [15] we have:

PROPOSITION 2.4. — For $|z| < C^{-1}(V) \exp - \alpha_*^2 V(0) - 1$ the measure $\mu_\infty(z)$ is locally Markov.

The question about the global Markov property is much more delicate. Owing to the detailed discussion of [15] we have to check that certain $D-L-R$ equations have unique solutions. To explain the last we introduce several definitions.

DEFINITION 2.1. — Any probabilistic Borel measure μ on the space $S'(R^d)$ will be called V-regular iff there exists a positive constant $c \in R_+$ such that

$$\mu(\varphi^2(f)) \leq c V(f, f). \tag{2.19}$$

The class of V-regular measures is denoted by R(V).

DEFINITION 2.2. — Any probabilistic Borel measure μ on $S'(R^d)$ will be called the Gibbs measure corresponding to the trigonometric perturbation (2.14) of the Gaussian measure μ_V^0 iff for any bounded open region Λ the conditional expectation values with respect to the σ-algebras $\mathscr{F}(\Lambda^c)$ and the measure μ fulfill:

$$E_\mu\{ - \mid \mathscr{F}(\Lambda^c)\}(\cdot) = E_{\mu_\Lambda}\{ - \mid \mathscr{F}(\Lambda^c)\}(\cdot) \qquad (\mu - a.e.) \qquad (2.20)$$

and the measure μ is locally equivalent to μ_V^0.

The set of all Gibbs measures corresponding to the trigonometric perturbations is denoted by $\mathscr{G}(z)$ and its intersection with the set R(V) is denoted by $\mathscr{G}_r(z)$ and we call elements of $\mathscr{G}_r(z)$ V-regular Gibbs measures.

Remarks. — A more detailed definition of the set $\mathscr{G}(z)$ can be found in the paper (15). Some result concerning the structure of the set $\mathscr{G}_r(z)$ have been obtained in (42) using certain correlation inequalities as a basic tool.

From the results of [*15*] it follows easily.

LEMMA 2.5. — For any $|z| < C(V)^{-1} \exp - \alpha_*^2 V(0) - 1$ the measure $\mu_\infty(z)$ belongs to the $\mathscr{G}_r(z)$.

Let us take $\mu \in \mathscr{G}_r(z)$ and let $R_0 = \{ x \in R^d \mid x_1 = 0 \}$.

According to the discussion of Albeverio and Hoegh-Kröhn [*5*] the proof of the global Markov property for $\mu_\infty(z)$ amounts to the proof that for μ-almost every all $\eta \in S'(R^d)$ there exists a unique solution of the corresponding DLR (see eq. 5.1) to the conditioned interactions coming from the explicit form of $E_\mu\{ - \mid \mathscr{F}(R_0)\}(\eta)$, V-regular solution. Our verification of the global Markov property follows this strategy. Firstly, in section 3 we prove that the measure is the unique V-regular solution of the DLR-equation (2.20). In section 4 we extended the technique used in section 3, to cover the more general situation and check the global Markov property is some cases.

c) Some technical assumptions and results.

Let the open bounded region $\Lambda \subset R^d$ fulfills all the assumptions necessary to the existence and uniqueness of the stochastic boundary problem (2.12). Then, for μ_V^0-almost every $\eta \in S'(R^d)$ the solution of (2.12) is given by $\Pi_{\Lambda^c}^*(\eta)$. There exists a sequence of functions $(b_0(.,.), \ldots, b_{n-1}(.,.))$ each $\partial\Lambda \times R^d$ measurable and such that for η sufficiently smooth (say $\eta \in C_0^\infty(R^d)$) we can write:

$$\Pi_{\Lambda^c}^*(\eta)(x) = \sum_{i=0}^{n-1} \int_{\partial\Lambda} b_i(y, x)(\partial_n^{i,\mathrm{int}}\eta)(y)d\sigma(y) \qquad (2.21)$$

where $d\sigma$ is the surface measure on $\partial\Lambda$. From this it follows:

$$\mu_V^0(\Pi_{\Lambda^c}^*(\eta)^2(f)) = K_{\Lambda^c}(f, f)$$

$$= \sum_{i,i'=0}^{n-1} \int_{\partial\Lambda} d\sigma(y) \int_{\partial\Lambda} d\sigma(y') \int dx \int dy' b_i(y, x)$$

$$(\partial^{i,\mathrm{int}} f)(x) V(y, y')(\partial_n^{i',\mathrm{int}} f)(x') b_{i'}(y', x'). \quad (2.22)$$

For an operator A chosen by (2.11) the covariance $V(x)$ is real analytic with fast exponential decrease as $x \to \infty$. The same decrease property holds for its derivatives.

Therefore, assuming $\mu \in R(V)$ and that Λ is such that K_{Λ^c} is continuous in Int Λ we can define $\Pi_{\Lambda^c}^*(\eta)(x)$ for μ-*a.e.* $\eta \in S'(R^d)$ and all $x \in \mathrm{Int}\,\Lambda$ as a L_μ^2 random element with the covariance bounded as:

$$\mu(\Pi_{\Lambda^c}^*(\eta)^2(x)) \leq c K_{\Lambda^c}(x, x). \quad (2.23)$$

DEFINITION 2.3. — A open region $\Lambda \subset R^d$ is called V-regular iff the solution $\Pi_{\Lambda^c}^*(\eta)$ of (2.12) can be given by formula (2.20), and moreover, their exists a function $F : (0, \infty) \to [0, \infty)$ which is continuous and monotonously decreasing to zero as $r \uparrow \infty$ with an asymptotics at least like $r^{-d-\varepsilon}$ for some $\varepsilon > 0$ and such that

$$K_{\Lambda^c}(x, x) \leq F(\mathrm{dist}(x, \Lambda^c)). \quad (2.24)$$

Let $\Lambda \subset R^d$ be V-regular. Elementary calculation gives then

$$E_{\mu_\Lambda}\{ F(\varphi) \mid \mathscr{F}(\Lambda^c) \}(\eta) = \frac{E_{\mu_V^0}\{ F(\varphi) X_\Lambda(\varphi) \mid \mathscr{F}(\Lambda^c) \}(\eta)}{E_{\mu_V^0}\{ X_\Lambda(\varphi) \mid \mathscr{F}(\Lambda^c) \}(\eta)} \quad (2.25)$$

$$= \frac{\mu_{S_{\Lambda^c}}^0\left(F(\varphi + \Pi_{\Lambda^c}^*(\eta)) \exp\left(z \int_\Lambda d(\underline{x}) : e^{i\alpha\varphi} :_{S_{\Lambda^c}} : e^{i\alpha\Pi_{\Lambda^c}^*(\eta)} :_{K_{\Lambda^c}} \right)\right)}{\mu_{S_{\Lambda^c}}^0\left(\exp\left(z \int_\Lambda d(\underline{x}) : e^{i\alpha\varphi} :_{S_{\Lambda^c}}(x) : e^{i\alpha\Pi_{\Lambda^c}^*(\eta)} :_{K_{\Lambda^c}} \right)\right)}.$$

By the above remarks, local absolute continuity of $\mu \in \mathscr{G}_r(z)$ and μ_V^0 the above formula can be written for μ *a.e.* $\eta \in S^1(R^d)$.

DEFINITION 2.4. — A covariance V is regular iff there exists V-regular open bounded region Λ such that all its homothetic images Λ_λ are again V-regular.

We remark that in most of the interesting cases the regularity of the covariances corresponding to the operators A can be checked explicitly on spheres which gives also the exponential decay of the function F.

Let us define also the σ-algebra of events at infinity as

$$\mathcal{F}_\infty = \bigcap_{\Lambda \subset \mathbb{R}^d} \mathcal{F}(\Lambda^c) \tag{2.26}$$

Λ-bounded and open.

LEMMA 2.6. — A measure $\mu \in \mathcal{G}_r(z)$ is a pure Gibbs measure iff the algebra $\mathcal{F}_\infty$ is $0-1$ σ-algebra for it.

LEMMA 2.7. — A measure $\mu \in \mathcal{G}_r(z)$ is the unique Gibbs measure iff for μ-a.e. $\eta \in S^1(\mathbb{R}^d)$, any L^1_μ integrable random element $F(\varphi)$ and any countably generated filter of bounded subset of $\mathbb{R}^d$: $\{\Lambda\}$ tending to $\mathbb{R}^d$ we have.

$$\lim_{\{\Lambda\}} E_\mu \ \{ F(\varphi) \mid \mathcal{F}(\Lambda^c) \} = \mu(F(\varphi)). \tag{2.27}$$

The above results are well known [5] [15]. They provide as with the constructive tactic for veryfying the global Markov property for $\mu_\infty(z)$. Let $\{\Lambda\}$ be a countably generated filter of bounded subsets of $\mathbb{R}^d$ tending to $\mathbb{R}^d$. Assume that V is regular and let Y be a V-regular set. From the reverse martingale theorem it follows that it is enough to control the limit of the l.h.s. in formula (2.27) by controlling it on the monotonic sequence $(Y_n)_n$ for which formulas (2.25) can be used.

DEFINITION 2.5. — Any such sequence $(Y_n)_n$ will be called V-admissible sequence.

LEMMA 2.8. — Let $\mu \in R(V)$.

Let V be regular and let (Λ_n) and (Y_n) be two V-admissible sequences and such that dist $(Y_n, \Lambda_n^c) \to \infty$.

Then there exists a function $f : S^1(\mathbb{R}^d) \to [0, \infty]$ which is finite μ-a.e. and such that for any unit cube $\Delta \subset Y_n$ the following estimate is valid

$$\int_\Delta \Pi^*_{\Lambda_n^c}(\eta)^2(x)dx \leq f(\eta) F(\operatorname{dist}(\Delta, \Lambda_n^c))^\beta \tag{2.28}$$

valid for any β such that:

$$\sum_n F(\operatorname{dist}(Y_n, \Lambda_n^c))^{1-\beta} \mid Y_n \mid < \infty. \tag{2.29}$$

COROLLARY 2.9. — Let $\mu \in R(V)$ and let Λ be V-regular set for the regular covariance V. Then, for any unit cube $\Delta \subset \mathbb{R}^d$ we have:

$$\lim_{n\to\infty} \| \Pi^*_{\Lambda_n^c}(\eta) \|_{L^1(\Delta)} = \lim_{n\to\infty} \| \Pi^*_{\Lambda_n^c}(\eta) \|_{L^2(\Delta)} = 0. \tag{2.30}$$

for μ-a.e. $\eta \in S'(\mathbb{R}^d)$.

For the simple proof of Lemma 2.8 see our accompanying paper [42].

3. UNIQUENESS FOR SMALL $|z|$

Throughout this section we assume that V is regular. In this section we prove one of our main results of this paper. We prove that the measure $\mu_\infty(z)$ constructed in the previous section is pure Gibbs measure for small $|z| < C^{-1}(V)\exp - \alpha_*^2 V(0) - 1$. As follows from the discussion in section 2. We have to investigate the $\mu_\infty(z)$ content of the « σ-algebra at infinity ». To prove triviality of this algebra relative to the measure $\mu_\infty(z)$ it is sufficient to prove that:

$$\underset{f\in C_0^\infty(\mathbb{R}^d)}{\forall} \lim_{\Lambda\uparrow\mathbb{R}^d} E_{\mu_\infty(z)}\{ e^{i\varphi(f)} \mid \mathscr{F}(\Lambda^c)\} = \mu_\infty(e^{i\varphi(f)}) \tag{3.1}$$

Simple calculations give the following formulas by the conditioned expectation valus (for Λ V-regular)

$$\begin{aligned}
&E_{\mu_\infty(z)}\{ e^{i\varphi(f)} \mid \mathscr{F}(\Lambda^c)\}(\eta) \\
&= \frac{\mu^0_{S_{\Lambda^c}}(\exp(i\eta(\Pi_{\Lambda^c}(f)))\exp(i\varphi(f))X_\Lambda(\varphi + \Pi^*_{\Lambda^c}(\eta))}{\mu^0_{S_{\Lambda^c}}(X_\Lambda(\varphi+\Pi^*_{\Lambda^c}(\eta)))} \\
&\equiv \exp(i\eta(\Pi_{\Lambda^c}(f)))\exp\left(-\frac{1}{2}S_{\Lambda^c}(f,f)\right) \\
&\qquad \cdot \mu^\eta_\Lambda\left(\exp\left(z\int_\Lambda d(\underline{x}) : e^{i\alpha\varphi} :_{S_{\Lambda^c}}(x) : e^{i\alpha\Pi^*_{\Lambda^c}(\eta)} :_{K_{\Lambda^c}} \right.\right. \\
&\qquad \left.\left. \cdot (e^{-\alpha f * S_{\Lambda^c}.(x)} - 1)\right)\right) \\
&= \exp(i\eta(\Pi_{\Lambda^c}(f)))\exp\left(-\frac{1}{2}S_{\Lambda^c}(f,f)\right) \\
&\qquad \cdot \sum_{n=0}^\infty \frac{z^n}{n!}\int_\Lambda \cdots \int_\Lambda d(\underline{x})_n \prod_{i=1}^n [e^{-\alpha_i f * S_{\Lambda^c}(x_i)} - 1]\cdot \rho^\eta_\Lambda((\underline{x})_n)
\end{aligned} \tag{3.2}$$

where the conditioned correlation functions ρ^η_Λ are defined by the following formulas:

$$\begin{aligned}
\rho^\eta_\Lambda((\underline{x})_n) &= z^n \prod_{i=1}^n \chi_\Lambda(x_i) : e^{i\alpha_i \Pi^*_{\Lambda^c}(\eta)} :_{K_{\Lambda^c}}(x_i) \\
&\quad \cdot \mu^\eta_\Lambda\left(\prod_{i=1}^n : e^{i\alpha_i\varphi(x_i)} :_{S_{\Lambda^c}}\right)
\end{aligned} \tag{3.3}$$

and the measure μ^η_Λ by:

$$\mu^\eta_\Lambda(d\varphi) \equiv (Z^\eta_\Lambda(z))^{-1}\mu^0_{S_{\Lambda^c}}(d\varphi)\cdot\exp\left(z\int_\Lambda d(\underline{x}) : e^{i\alpha\varphi} :_{S_{\Lambda^c}}(x) : e^{i\alpha\Pi^*_{\Lambda^c}}(\eta) :_{K_{\Lambda^c}}(x)\right) \tag{3.4}$$

Formulas (3.2, 3.3) are valid for $\mu_\infty(z)$ almost every $\eta \in S^1(\mathbb{R}^d)$.

LEMMA 3.1. — Let $(\Lambda_n)_n$ be V-regular sequence.
For $\mu_\infty(z)$ *a.e.* η we have

$$\lim_{n\to\infty} e^{i\eta(\Pi_{\Lambda_n}(f))} = 1 . \tag{3.5}$$

wherenever $f \in C_0^\infty(\mathbb{R}^d)$. The whole essential volume dependence in formula (3.2) comes from that of ρ_Λ^η. Therefore, we concentrate on them.
Making out the shift transformation

$$\varphi \to \varphi - if * S_{\Lambda^c} \tag{3.6}$$

we obtain the following identities between the conditioned correlation functions:

$$\begin{aligned}\rho_\Lambda^\eta((\underline{x})_n) = z^n \cdot \prod_{i=1}^{n} :e^{i\alpha_i \Pi^*_{\Lambda^c}(\eta)}:_{K_{\Lambda^c}}(x_i)\\ \cdot \exp\left[-\alpha_1 \cdot \sum_{i=2}^{n} \alpha_i S_{\Lambda^c}(x_1 - x_i)\right]\\ \cdot \mu_\Lambda^\eta\left(\prod_{i=2}^{n} :e^{i\alpha_i\varphi}:_{S_{\Lambda^c}}(x_i) \exp\left(z\int_\Lambda d(\underline{x}) : e^{i\alpha\varphi}:_V(x) : e^{i\alpha\Pi^*_{\Lambda^c}(\eta)}:(x)\right)\right)\\ \cdot (e^{-\alpha_1\alpha S\ (x_1-x)} - 1)).\end{aligned} \tag{3.7}$$

which are nothing more then the familiar Kirkwood-Salsburg identities. In the following we concentrate on the analysis of the identities (3.7). Let B_ξ be a Banach space which consists of all sequences of the functions $\{\phi_n(x,\alpha)_n\}_{n=1}^\infty = \phi$ where $\phi_n(x,\alpha)_n$ are complex measurable functions defined on $(\mathbb{R}^d \otimes \mathbb{R}^1)^{\otimes n}$ so that the norm

$$\| \phi \|_\xi = \sup_n \xi^{-n} \operatorname*{ess\,sup}_{(x,\alpha)_n} | \phi^n(x,\alpha)_n | \tag{3.8}$$

is finite. The positive real number ξ is fixed and will be chosen later. In the space B_ξ we define the operators $\hat{\mathbb{K}}_{\Lambda^c}$ by the following formulas:

$$(\hat{\mathbb{K}}_{\Lambda^c}\phi)_1(x,\alpha) = \sum_{m=1}^{\infty}\int d(\underline{y})_m \prod_{i=1}^{m} [e^{-\alpha\beta_j\cdot S\ (x-y_j)} - 1]\phi_m(\underline{y}_m), \tag{3.9}$$

$$\begin{aligned}(\hat{\mathbb{K}}_{\Lambda^c}\phi)_n(x,\alpha)_n = \exp\left(-\sum_{j=2}^{n}\alpha_1\alpha_j S_{\Lambda^c}(x_1 - x_j)\right)\\ \cdot\left\{ \phi_{n-1}(x,\alpha)'_{n-1} + \sum_{m=1}^{\infty}\frac{1}{m!}\int d(y)_m \prod_{j=1}^{m} [e^{-\alpha_1\beta_j S_{\Lambda^c}(x_1-y_j)} - 1]\right.\\ \left.\cdot\, \phi^{n+m-1}((x,\alpha)'_{n-1},(y,\beta)_m)\right\}\end{aligned} \tag{3.10}$$

Let us define also the following operator:

$$(\hat{\chi}_\Lambda \phi)_n(x, \alpha)_n = \prod_{i=1}^{n} \chi_\Lambda(x_i)\phi_n(x, \alpha)_n \equiv \chi_\Lambda(x)_n \phi_n(x, \alpha)_n \tag{3.11}$$

and the vector $\underset{\sim}{\alpha} : \underset{\sim}{\alpha}_n = \delta_{n,1}$.

With these definitions we can rewrite the identities (3.7) as:

$$\begin{aligned}\rho^\eta_\Lambda(\underline{x})_n = z\chi_\Lambda(x)_n : e^{i\alpha_1 \Pi^*_{\Lambda^c}(\eta)} :_{K_{\Lambda^c}}(x_1) \cdot \underset{\sim}{\alpha} \\ + z\chi_\Lambda(x)_n : e^{i\alpha_1 \Pi^*_{\Lambda^c}(\eta)} :_{K_{\Lambda^c}}(x_1)(\hat{\mathbb{K}}_{\Lambda^c}\rho^\eta_\Lambda)(\underline{x})_n .\end{aligned} \tag{3.12}$$

A crucial step in the proof of the uniqueness is done by the following result.

PROPOSITION 3.1. — Let $\{\Lambda_n\}$ be arbitrary V-admissible sequence of bounded regions in R^d. Then for $\mu_\infty(z)$ almost every $\eta \in S^1(R^d)$, any compact $\Xi \subset R^{dm}$ we have:

$$\lim_{n\to\infty} \sup_{(\underline{x})_m \in \Xi} |\rho^\eta_{\Lambda_n}(\underline{x})_m - \rho^{\eta=0}_\infty(\underline{x})_m| = 0 \tag{3.12}$$

for $$|z| < C^{-1}(V) \exp - \alpha_*^2 V(0) - 1 .$$

We start the proof of this proposition by a series of simple remarks. Let $\hat{\Pi}$ denote the permutation operator (see [28], then because of the symmetry of ρ^n_Λ we can rewrite equations (3.11) with $\hat{\Pi}\hat{\mathbb{K}}_{\Lambda^c}$ incread of $\Pi\hat{\mathbb{K}}_{\Lambda^c}$. In the following we choose those permutations $\Pi^n(\Lambda_m)$ for which we have:

$$\sum_{j=2}^{m} \alpha_1\alpha_j S_{\Lambda^c_n}(x_1 - x_j) \geq - \alpha_*^2 \cdot S_{\Lambda^c_n}(0) . \tag{3.13}$$

LEMMA 3.2.

1. For $\xi = C(V)^{-1}, |z| < C(V)^{-1} \exp - \alpha_*^2 V(0) - 1$ we have

$$\| z e^{i\alpha\Pi^*_{\Lambda^c_n}(\eta)(x)} \hat{\Pi}\hat{\mathbb{K}}_{\Lambda^c} \|_\xi < 1 \tag{3.14}$$

uniformly in η and Λ.

2. For $|z| < C(V)^{-1} \exp - \alpha_*^2 V(0) - 1$ the following estimate

$$\| \{ \rho^\eta_\Lambda(\underline{x})_n \} \|_\xi \leq \frac{zC(V)}{1 - zC(V)e^{\alpha_*^2}V(0) + 1} e^{\alpha_*^2 V(0)} \tag{3.15}$$

is uniformly valid in η and Λ.

Proof. — Let us take $\phi \in \mathbb{B}_\xi$ and observe:

$$\begin{aligned} &\xi^{-n} \mid z^n e^{i\alpha\Pi^*_{\Lambda^c}(\eta)(x)} (\hat{\Pi}\mathbb{K}_{\Lambda^c})(\phi)_n(\underline{x})_n \mid \\ &\qquad \leq \xi^{-1} \mid z \mid \exp(\xi C(S_{\Lambda^c})) \exp(\alpha_*^2 K_{\Lambda^c}(0)) \exp \alpha_*^2 S_{\Lambda^c}(0) \parallel \phi \parallel_\xi \\ &\qquad \leq \xi^{-1} \mid z \mid \exp \alpha_*^2 V(0) \exp \xi C(V) \parallel \phi \parallel_\xi . \end{aligned}$$

Tp prove the second part we use the equality (3.11) and we estimate

$$\parallel \rho^\eta_\Lambda \parallel_\xi \leq \xi^{-1} \exp\left(\frac{\alpha_*^2}{2} K_{\Lambda^c}(0)\right) + \xi^{-1} \mid z \mid \exp\left(\frac{\alpha_*^2}{2} V(0)\right) \exp(\xi C(V)) \parallel \rho^\eta_\Lambda \parallel_\xi$$

from which the estimate follows.

The space $\mathbb{B}_\xi$ is the dual space of the space ${}^*\mathbb{B}_\xi$ consisting of sequences of functions $\mathbb{f} = \{ f_n(x, \alpha)_n \}_{n=1,2,\dots}$ with measurable components equipped in the norm:

$$^*\parallel \mathbb{f} \parallel_\xi = \sum_{n=0}^{\infty} \xi^n \int d(\underline{x})_n \mid f_n(\underline{x})_n \mid . \tag{3.16}$$

From Lemma 3.2 (2) it follows that for $\mu_\infty(z)$-almost every $\eta \in S^1(R^d)$ the vectors $\{ \rho^\eta_{\Lambda_n} \}_{\Lambda_n}$ form the $*$-weakly precompact set in the space $\mathbb{B}_\xi$. From the Banach Alaglou theorem it follows that there exists a convergent (in the $*$-weak topology) subsequence $(n') \subset (n)$ for any V-admissible sequence (Λ_n).

LEMMA 3.4. — For $\mu_\infty(z)$ almost every $\eta \in S'(R^d)$ and any V-admissible sequence $\{ \Lambda_n \}$ we have:

$$*\text{-}\omega \lim_{n\to\infty} ; e^{i\alpha\Pi^*_{\Lambda_n^c}(\eta)} :_{K_{\Lambda_n^c}}(x) = 1 . \tag{3.17}$$

Proof. — This follows immediately from Lemma 2.8 by taking into account the assumed decay of K_{Λ^c} and the local L_2-convergence of $\Pi^*_{\Lambda_n^c}(\eta)$ to zero.

LEMMA 3.5. — For $\mu_\infty(z)$ almost every $\eta \in S'(R^d)$ and any V-admissible sequence $\{ \Lambda_n \}$ we have

i) $$\hat{\mathbb{K}}_{\Lambda_n^c} \xrightarrow[n\to\infty]{*\text{-}w} \hat{\mathbb{K}}_\infty \tag{3.18}$$

ii) $$\hat{\Pi}\hat{\mathbb{K}}_{\Lambda_n^c} \xrightarrow[n\to\infty]{*\text{-}w} \widehat{\Pi\mathbb{K}}_\infty \tag{3.19}$$

Proof. — The limits in *i)* and *ii)* are taken in the $*$-weak topology of the space $\mathbb{B}_\xi$ i.e. (3.18) and (3.19) are equivalent to the statements that:

$$\forall_{\mathbb{f} \in {}^*\mathbb{B}_\xi} \; *\text{-}w \lim_{n\to\infty} (\hat{\mathbb{K}}_{\Lambda_n^c} - \hat{\mathbb{K}}_\infty) \mathbb{f} = 0 \tag{3.20}$$

and

$$\underset{\mathfrak{f}\in{}^*\mathbb{B}_\xi}{\forall} \ *\text{-}w \lim_{n\to\infty} (\hat{\Pi}\hat{\mathbb{K}}_{\Lambda_n^c} - \hat{\Pi}\hat{\mathbb{K}}_\infty)\,\mathfrak{f} = 0. \tag{3.21}$$

It is well known that the strong convergence of the duals $*\hat{\mathbb{K}}_{\Lambda^c}$ (resp. $*(\hat{\Pi}\hat{\mathbb{K}}_{\Lambda^c})$) in the dual pair $({}^*\mathbb{B}_\xi, \mathbb{B}_\xi)$ to the corresponding $*\hat{\mathbb{K}}_\infty$ (resp. $*(\hat{\Pi}\hat{\mathbb{K}}_\infty)$) yield the assumed convergence in (3.20) and (3.21). The dual operators $*\hat{\mathbb{K}}_{\Lambda^c}$ and $*(\hat{\Pi}\hat{\mathbb{K}}_{\Lambda^c})$ can easily be calculated (see [*30*]) for the similar calculation with the results

$$(*\hat{\mathbb{K}}_{\Lambda^c})(\mathfrak{f})_n(x,\alpha) = \left[*\hat{k} \exp - \sum_{i=2}^{n} \alpha_1\alpha_i S_{\Lambda^c}(x_1 - x_i)\hat{\chi}_\Lambda(\mathfrak{f}) \right]_n (x,\alpha)_n, \tag{3.22}$$

$$(*\hat{\mathbb{K}}_\infty)(\mathfrak{f})(x,\alpha)_n = \left[*\hat{k} \exp - \sum_{i=2}^{n} \alpha_1\alpha_i V(x_1 - x_i)\mathfrak{f} \right]_n (x,\alpha)_n, \tag{3.23}$$

$$*\hat{\Pi}(\Lambda) = \sum_{j=1}^{\infty} \hat{\Pi}_j \hat{V}_j, \qquad *(\hat{\Pi}(\Lambda)\hat{\mathbb{K}}_{\Lambda^c}) = (*\hat{\mathbb{K}}_{\Lambda^c})(*\hat{\Pi}(\Lambda)) \tag{3.24}$$

where the operator $*\hat{k}$ is defined by

$$(*\hat{k}(\Lambda^c)\mathfrak{f})(x,\alpha)_n = \sum_{m=0}^{\infty} \frac{1}{m!} \int d(\underline{y}) \prod_{j=1}^{m} [e^{-\beta S_{\Lambda^c}(y - x_j)} - 1]$$
$$f_{m+1+n}((\underline{y})_m \vee ((\underline{x})_n - (\underline{x})_m)) \cdot \Theta(n - 1 + m) \tag{3.25}$$

(with an obvious notation) and the operators $\hat{\Pi}_j$, $\hat{V}_j$ are defined in ([*30*], formula (3.4) and (3.5). On the basis of these formulas and the assumed fast convergence of S_{Λ^c} to V it is not difficult to finish the proof of the convergence in (3.20) and (3.21). From Lemmas 3.4 and 3.5 we extract now the proof of the following corollary.

COROLLARY 3.6. — Let $|z| < C(V)^{-1} \exp - \alpha_*^2 V(0) - 1$ and let (Λ_n) be an arbitrary V-admissible sequence of bounded regions in R^d. Then for $\mu_\infty(z)$ almost every $\eta \in S'(R^d)$ we have:

$$*\text{-}w \lim_{n\to\infty} (\rho^\eta_{\Lambda_n}(\underline{x})_m) = *\text{-}w \lim_{n\to\infty} (\rho^{\eta=0}_{\Lambda_n}(\underline{x})_m) = (\rho_\infty(\underline{x})_m) \tag{3.26}$$

Proof. — From the Philips theorem [*31*] it follows that we have equalities for the resolvent:

$$R_z(*(\hat{\Pi}\hat{\mathbb{K}}_{\Lambda^c})) = *(R_z(\hat{\Pi}\hat{\mathbb{K}}_{\Lambda^c})), \tag{3.27}$$

$$R_z(*(\hat{\Pi}\hat{\mathbb{K}}_\infty)) = *(R_z(\hat{\Pi}\hat{\mathbb{K}}_\infty)). \tag{3.28}$$

and the corresponding convergence of them. Taking into account Lemma 3.2 (1), Lemma 3.5 *i*), Lemma (3.4) and the fact that in any dual pair $({}^*\mathbb{B}_\xi, \mathbb{B}_\xi)$, from the convergence $\mathfrak{f}_n \xrightarrow{*\|\ \|} \mathfrak{f}_\infty$ and $\phi_n \xrightarrow{*-w} \phi_\infty$ it follows that $(\mathfrak{f}_n, \phi_n) \to (\mathfrak{f}_\infty, \phi_\infty)$ as $n \to \infty$, we easily conclude that for any $\mathfrak{f} \in {}^*\mathbb{B}_\xi$ and $|z| < C(V)^{-1} \exp - \alpha_*^2 V(0) - 1$ we have the equality:

$$\lim_{n\to\infty} \langle \mathfrak{f}, \rho^\eta_{\Lambda_n}(z) - \rho^\eta_\infty(z) \rangle$$

$$= \lim_{n\to\infty} \langle \mathfrak{f}, R_z(: e^{i\alpha\Pi^*_{\Lambda^c_n}(\eta)} :_{K_{\Lambda^c_n}} (\hat{\Pi}\hat{\mathbb{K}}_{\Lambda^c_n})(\hat{\chi}_\Lambda : e^{i\alpha\Pi^*_{\Lambda^c_n}(\eta)} :_{K_{\Lambda^c_n}} \underset{\sim}{\alpha}) - R_z(\hat{\Pi}\hat{\mathbb{K}}_\infty)\underset{\sim}{\alpha} \rangle$$

$$= \lim_{n\to\infty} \{ \langle ({}^*R_z(: e^{+i\alpha\Pi^*_{\Lambda^c_n}(\eta)} :_{K_{\Lambda^c_n}} \hat{\Pi}\hat{\mathbb{K}}_{\Lambda^c_n}) - {}^*R_z(\hat{\Pi}\hat{\mathbb{K}}_\infty))\mathfrak{f},$$

$$(\hat{\chi}_\Lambda : e^{i\alpha\Pi^*_{\Lambda^c_n}(\eta)} :_{K_{\Lambda^c_n}} - 1)\underset{\sim}{\alpha} \rangle$$

$$\langle \mathfrak{f}, {}^*(R_z(: e^{+i\alpha\Pi^*_{\Lambda^c_n}(\eta)} :_{K_{\Lambda^c_n}} \hat{\Pi}\hat{\mathbb{K}}_{\Lambda^c_n}) - {}^*R_z(\hat{\Pi}\hat{\mathbb{K}}_\infty))\tilde{\alpha} \rangle$$

Finally, we scetch a proof of Proposition 3.1.

Let $|z| < C(V)^{-1} \exp - \alpha_*^2 V(0) - 1$ and (Λ_n) be an arbitrary V-admissible sequence of bounded regions in R^d. Both $\rho^\eta_{\Lambda_n}$ and ρ_∞ fulfill the well known Mayer-Montroll identities which in the case of $\rho^\eta_{\Lambda_n}$ can be obtained by successively integrating all the exponentials in formula (3.3) with the result:

$$\rho^\eta_\Lambda(\underline{x})_n = z^n \cdot \chi_\Lambda(\underline{x})_n \cdot \prod_{i=1}^{n} : e^{i\alpha_i\Pi^*_{\Lambda^c}(\eta)} :_{K_{\Lambda^c}}(x_i) \cdot \exp\left(- \sum_{i<j}^{n} \alpha_i\alpha_j S_{\Lambda^c}(x_i - x_j) \right)$$

$$\cdot \mu^\eta_\Lambda\left(\exp\left(z \int_\Lambda d(\underline{x}) : e^{i\alpha\varphi} :_V (x) e^{i\alpha\Pi^*_{\Lambda^c}(\eta)}(x) \left(\prod_{i=1}^{n} e^{-\alpha\alpha_i S_{\Lambda^c}(x_i - x)} - 1 \right) \right) \right) \quad (3.30)$$

Now the comparison with the corresponding Mayer-Montroll equations and the assumed fast convergence $S_{\Lambda^c} \to V$ yields the assured results by standard considerations (see [*30*] [*32*]). Hawing proven Proposition 3.1 and taking into account formula (3.2) (or using the fact that the moments $\rho_\infty(z)$ determine in a unique way the measure $\mu_\infty(z)$, we can conclude the following:

THEOREM 3.1. — Assume that $|z| \langle C(V)^{-1} \exp - \alpha_*^2 V(0) - 1$. Then the measure $\mu_\infty(z)$ is the unique Gibbs measure in the set $\mathcal{G}_r(z)$ corresponding to the interaction (2.13).

Remark. — We note that using the method of comparing the corresponding Lioville-Neumann expansions, it is possible to extend Theorem 3.1 to the following sharpened form:

THEOREM 3.1'. — Let (Λ_n) and (Y_n) be two V-admissible sequences of the bounded regions in Λ such that $\lim_{n\to\infty} \operatorname{dist}(Y_n, \Lambda^c_n) = \infty$ and for every

$n : Y_n \subset \Lambda_n$. Then for $|z| < C(V)^{-1} \exp - \alpha_*^2 V(0) - 1$ and for $\mu_\infty(z)$ almost every $\eta \in S'(R^d)$ we have

$$\lim_{n\to\infty} \| \hat{\chi}_{Y_n}(\dot{\rho}^\eta_{\Lambda_n} - \rho_\infty) \|_\xi = 0$$

if we choose $\xi = C^{-1}(V)$.

4. FROM THE LOCAL TO THE GLOBAL MARKOV PROPERTY

Let V be a markovian covariance with $V(0) < \infty$. In this section we prove that the unique Gibbs regular measure $\mu_\infty(z)$ has the global Markov property assuming that μ_V^0 has this property and $|z| < C(V)^{-1} \exp - \alpha_*^2 V(0) - 1$.

In the following we restrict ourselves to verifying the global Markov property in the hyperplanes. Due to the euclidean invariance it is enough to consider the case $\Sigma_0 = \{ x \in R^d, x_1 = 0 \}$.

However, our techniques applies to the more general hypersurfaces as well.

According to the general strategy originated by Albeverio and Hoegh-Krohn in order to verify the global Markov property for the measure $\mu_\infty(z)$ we have to prove the uniqueness of the (regular) solution of DLR equations corresponding to the measure $\mu^0_{S\Sigma_0}$ and the interactions

$$z \int_\Lambda : e^{i\alpha\varphi} :_V (x) e^{i\alpha \Pi^*_{\Sigma_0}(\eta)}(x) d(x)$$

for $\mu_\infty(z)$-almost every $\eta \in S'(R^d)$. See the basic paper [*5*] and papers [*6*] [*7*] [*8*] [*15*] for some application to the quantum field theory.

Let (Λ_n) be V-admissible sequence of bounded regions in R^d and let us denote $\Lambda_n^\pm = \Lambda_n \cap R^d_\pm$ where $R^d_\pm = \{ x \in R^d \,|\, x_1 \rangle (\langle,) 0 \}$ and we assume that $\partial\Lambda_n \cap \Sigma_0$ are of dimensions $d - 2$. Furthermore, let us introduce $\overline{\partial\Lambda_n} = \partial\Lambda_n - \Sigma_0$. From the DLR equations and the local Markov property we obtain the following expressions for the conditioned expectation values with respect to the σ-algebra $\mathscr{F}(\Sigma_0 \cup \overline{\partial\Lambda_n})$:

$$\begin{aligned}
&E_{\mu_\infty(z)} \{ F \mid \mathscr{F}(\Sigma_0 \cup \overline{\partial\Lambda_n}) \} (\eta) \\
&= \frac{E_{\mu_V^0} \{ F \mid X_\Lambda \mid \mathscr{F}(\Sigma_0 \cup \overline{\partial\Lambda_n}) \} (\eta)}{E_{\mu_V^0} \{ X_\Lambda \mid \mathscr{F}(\Sigma_0 \cup \overline{\partial\Lambda_n}) \} (\eta)} \\
&= \frac{\mu^0_{S\Sigma_0 \cup \overline{\partial\Lambda_n}} \left(F(\varphi + \Pi^*_{\Sigma_0 \cup \overline{\partial\Lambda_n}}(\eta)) \exp \left(z \int_{\Lambda_n} d(\underline{x}) : e^{i\alpha\varphi} :_V (x) e^{i\alpha \Pi^*_{\Sigma_0 \cup \partial\Lambda_n}(\eta)}(x) \right) \right)}{\mu^0_{S\Sigma_0 \cup \overline{\partial\Lambda_n}} \left(\exp \left(z \int_{\Lambda_n} d(\underline{x}) : e^{i\alpha\varphi} :_V (x) e^{i\alpha \Pi^*_{\Sigma_0 \cup \partial\Lambda_n}(\eta)}(x) \right) \right)}
\end{aligned} \tag{4.1}$$

valid for any $\Lambda_n - \Sigma_0$ measurable and $L^1_{\mu_\infty(z)}$ integrable F and $\mu_\infty(z)$ almost every η. The objects $\Pi^*_{\Sigma_0\cup\overline{\partial\Lambda_n}}$ are defined in terms of the corresponding projections $\Pi_{\Sigma_0\cup\overline{\partial\Lambda_n}}$ in the space H(P). In particular for $F(\varphi) = \exp i\varphi(f)$ with $\operatorname{supp} f \subset \Lambda_n - \Sigma_0$ we obtain

$$E_{\mu_\infty(z)}\{e^{i\varphi(f)} \mid \mathscr{F}(\Sigma_0 \cup \overline{\partial\Lambda_n})\}(\eta) = \exp\left(-\frac{1}{2} S_{\Sigma_0\cup\overline{\partial\Lambda_n}}(f,f)\right)\exp\left(i\Pi^*_{\Sigma_0\cup\overline{\partial\Lambda_n}}(\eta)(f)\right)$$

$$\cdot\,\mu^\eta_{\Sigma_0\cup\overline{\partial\Lambda_n}}\left(\exp\left(z\int_{\Lambda_n} d(\underline{x}) : e^{i\alpha\varphi} :_V(x) e^{i\alpha\Pi^*_{\Sigma_0\cup\overline{\partial\Lambda_n}}(\eta)(x)}\right.\right.$$

$$\left.\left.\cdot\,(e^{-\alpha(S_{\Sigma_0\cup\overline{\partial\Lambda_n}} * f)(x)} - 1)\right)\right).$$

According to the outlined before A-HK general strategy we have to prove that

$$\lim_{n\to\infty} E_{\mu_\infty(z)}\{e^{i\varphi(f)} \mid \mathscr{F}(\Sigma_0 \cup \overline{\partial\Lambda_n})\}(\eta) = E_{\mu_\infty(z)}\{e^{i\varphi(f)} \mid \mathscr{F}(\Sigma_0)\}(\eta) \quad (4.3)$$

for $\mu_\infty(z)$ almost every $\eta \in S'(R^d)$. Comparing with (4.2) we conclude that to prove (4.3) we have to show:

A)

$$\lim_{n\to\infty} \langle \eta, \Pi_{\Sigma_0\cup\overline{\partial\Lambda_n}}(f)\rangle = \langle \eta, \Pi_{\Sigma_0}(f)\rangle\ (\mu_\infty(z)\text{-}a.e.) \quad (4.4)$$

and

B) Defining the following conditioned correlation functions:

$$\rho^{\eta,\Sigma_0}_{\Lambda_n}(\underline{x})_m \equiv \mu^\eta_{\Sigma_0,\Lambda_n}\left(\prod_{i=1}^m : e^{i\alpha_i\varphi} :_{S_{\Sigma_0\cup\overline{\partial\Lambda_n}}}(x_i)\right)\cdot z^m \cdot \prod_{i=1}^m : e^{i\alpha_i\Pi^*(\eta)} :_{K_{\Sigma_0\cup\partial\Lambda_n}}, (x_i)$$

where

$$\mu^\eta_{\Sigma_0,\Lambda_n}(-) = (Z^\eta_{\Sigma_0,\Lambda_n})^{-1}\mu^0_{S_{\Sigma_0\cup\overline{\partial\Lambda_n}}}\left(-\exp\left(z\int_{\Lambda_n} d(\underline{x}) : e^{i\alpha\varphi} :_V(x)\cdot e^{i\alpha\Pi^*_{\Sigma_0\cup\partial\Lambda_n}(\eta)}(x)\right)\right), \quad (4.5)$$

$$Z^\eta_{\Sigma_0,\Lambda_n} = \mu^0_{S_{\Sigma_0\cup\overline{\partial\Lambda_n}}}\left(\exp\left(z\int d(\underline{x}) : e^{i\alpha\varphi} :_V(x)\cdot e^{i\alpha\Pi^*_{\Sigma\cup\partial\Lambda_n}(\eta)(x)}\right)\right). \quad (4.6)$$

we have to show that $\rho^{\eta,\Sigma_0}_{\Lambda_n}$ tend in the limit $n = \infty$ to the following moments:

$$\rho^{\eta,\Sigma_0}_\infty(\underline{x})_m = z^m \prod_{i=1}^m : e^{i\alpha_i\Pi^*_{\Sigma_0}(\eta)} :_{K_{\Sigma_0}}(x_i)\cdot \mu^\eta_{\Sigma_0,\infty}\left(\prod_{i=1}^m : e^{i\alpha_i\varphi} :_{S_{\Sigma_0}}(x_i)\right) \quad (4.7)$$

where

$$\mu^\eta_{\Sigma_0,\infty} = \underset{n\to\infty}{\text{w-lim}}\, \bar\mu^\eta_{\Sigma_0,Y_n}. \quad (4.8)$$

Here (Y_n) is an arbitrary V-admissible sequence and $\bar\mu^\eta_{\Sigma_0,Y_n}$ is defined as

$$\bar\mu^\eta_{\Sigma_0,Y_n}(d\varphi) = (\bar Z^\eta_{\Sigma_0,Y_n})^{-1}\exp\left(z\int_{Y_n} d(\underline{x}) : e^{i\alpha\varphi} :_V(x) : e^{i\alpha\Pi^*_{\Sigma_0}(\eta)}(x)\right)\mu^0_{S_{\Sigma_0}}(d\varphi), \quad (4.9)$$

$$\bar Z^\eta_{\Sigma_0,Y_n} = \mu^0_{S_{\Sigma_0}}\left(\exp\left(z\int_{Y_n} d(\underline{x}) : e^{i\alpha\varphi} :_V(x)\cdot e^{i\alpha\Pi^*_{\Sigma_0}(\eta)(x)}\right)\right) \quad (4.10)$$

The standard Kirkwood-Salburg analysis gives the existence of the limits (4.8) for $|z| < C(V)^{-1} \exp\left(-\frac{\alpha_*^2}{2} K_{\Sigma_0}(0)\right) \exp(-\alpha_*^2 S_{\Sigma_0}(0) - 1)$ and for $\mu_\infty(z)$ almost every $\eta \in S'(R^d)$.

Some arguments from the theory of martingales give the existence of $\mu^\eta_{\Sigma_0,\infty}$ and the equality (4.8) for $|z| < C(V)^{-1} \exp - \alpha_*^2 V(0) - 1$. Now we impose a certain technical regularity condition on the Markovian covariance V under which we shall be able to verify A) and B).

DEFINITION 4.1. — Let $\partial B(\rho_n)$ be the sequence of spheres of the form: $\partial B(\rho_n) = \{ x \in R^d \mid |x| = \rho_n \}$ where we assume that $\rho_n \to \infty$ as $n \to \infty$ and let Δ be any unit cube contained in $B(\rho_n) - \Sigma_0$.

We will say that a given covariance V is strongly regular iff V is regular and fulfill:

SM1) for $\mu_\infty(z)$-*a.e.*

$$\lim_{n\to\infty} \| \Pi^*_{\Sigma_0 \cup \partial B(\rho_n)}(\eta) - \Pi^*_{\Sigma_0}(\eta) \|_{L_1(\Delta)} = \lim_{n\to\infty} \| \Pi^*_{\Sigma_0 \cup \partial B(\rho_n)}(\eta) - \Pi^*_{\Sigma_0}(\eta) \|_{L_2(\Delta)} = 0$$

SM2)
$$K_{\Sigma_0 \cup \partial B(\rho_n)}(x, x) \to K_{\Sigma_0}(x, x)$$

uniformly on compacts in $R^d - \Sigma_0$.

The class of the strongly regular covariance we shall denote by SM. For $V \in SM$ the verification of A) is trivial. In a strict analogy with the analysis of section 3 we shall verify point B) of our strategy by a comparison of the corresponding Kirkwood-Salsburg equations. The correlation function $\rho^{\eta,\Sigma_0}_\infty$ fulfill the following Kirkwood-Salsburg identities:

$$\rho^{\eta,\Sigma_0}_\infty = z : e^{i\alpha\Pi^*_{\Sigma_0}(\eta)} :_{K_{\Sigma_0}}(x) \cdot \underset{\sim}{\alpha} + z : e^{i\alpha\Pi^*_{\Sigma_0}(\eta)} :_{K_{\Sigma_0}}(x) \hat{\Pi} \hat{\mathbb{K}}(\Sigma_0) \rho^{\eta,\Sigma_0}_\infty \quad (4.11)$$

at least for small z (uniformly in η), where the operator $\hat{\mathbb{K}}(\Sigma_0)$ is defined (in the corresponding space $\mathbb{B}_\xi$) by the following formulas:

$$(\hat{\mathbb{K}}(\Sigma_0)\phi)_1(x, \alpha) = \sum_{m=1}^{\infty} \int d(\underline{y}_m) \prod_{i=1}^{m} [e^{-\alpha\beta_j S_{\Sigma_0}(x-y_j)} - 1] \phi_m(\underline{y})_m ; \quad (4.12)$$

$$(\hat{\mathbb{K}}(\Sigma_0)\phi)_m(x, \alpha)_m = \exp\left(-\sum_{j=2}^{m} \alpha_1 \alpha_j S_{\Sigma_0}(x_1 - x_j)\right)$$
$$\cdot \left\{ \phi_{n-1}(x, \alpha)'_{n-1} + \sum_{m=1}^{\infty} \frac{1}{m!} \int d(\underline{y})_m \prod_{j=1}^{m} [e^{-\alpha_1 \beta_j S_{\Sigma_0}(x_1 - y_j)} - 1] \right.$$
$$\left. \cdot \phi_{n+m-1}((x, \alpha)'_{n-1}, (y, \beta)_m) \right. . \quad (4.13)$$

The corresponding Kirkwood-Salsburg identities for the conditioned correlation functions are

$$\rho^{\eta,\Sigma_0}_{\mathrm{B}(\rho_n)} = z : e^{i\alpha\Pi^*_{\Sigma_0\cup\partial\overline{\mathrm{B}}(\rho_n)}(\eta)} :_{\mathrm{K}_{\Sigma_0\cup\partial\overline{\mathrm{B}}(\rho_n)}}(x)\cdot \underset{\sim}{\alpha}$$
$$+ z : e^{i\alpha\Pi^*_{\Sigma_0\cup\partial\overline{\mathrm{B}}(\rho_n)}(\eta)} :_{\mathrm{K}_{\Sigma_0\cup\partial\overline{\mathrm{B}}(\rho_n)}}(x)\cdot \hat{\Pi}\hat{\mathbb{K}}(\Sigma_0\cup\partial\mathrm{B}(\varphi_n))\cdot\rho^{\eta,\Sigma_0}_{6(\rho_n)}$$

where the operator $\hat{\mathbb{K}}(\Sigma_0\cup\partial\mathrm{B}(\rho_n))$ is defined by formulas like (4.12) and (4.13) but with $\mathrm{S}_{\Sigma_0\cup\partial 6(\rho_n)}$ and $\mathrm{K}_{\Sigma_0\cup\partial\mathrm{B}(\rho_n)}$ instead of S_{Σ_0} and K_{Σ_0}.

LEMMA 4.1. — Let $\mathrm{V}\in\mathrm{SM}$.

1) For $\mu_\infty(z)$ almost every $\eta\in\mathrm{S}'(\mathrm{R}^d)$ we have the convergence

$$\hat{\Pi}\hat{\mathbb{K}}(\Sigma_0\cup\overline{\partial\mathrm{B}}(\rho_n)) \xrightarrow{n\to\infty} \hat{\Pi}\hat{\mathbb{K}}(\Sigma_0)$$

in the $*$-weak topology.

2) For $\mu_\infty(z)$ almost every $\eta\in\mathrm{S}'(\mathrm{R}^d)$ we have the convergence :

$$*\text{-}w\lim_{n\to\infty} : e^{i\alpha\Pi^*_{\Sigma_0\cup\partial\overline{\mathrm{B}}(\rho_n)}(\eta)(x)} :_{\mathrm{K}_{\Sigma_0\cup\partial\overline{\mathrm{B}}(\rho_n)}} = : e^{i\alpha\Pi^*_{\Sigma_0}(\eta)} :_{\mathrm{K}_{\Sigma_0}}(x).$$

The proof follows exactly the arguments used in the proof of the corresponding proof of section 3 using additionally the regularity assumption made on V and will be not written here. Applying further the arguments of the section 3 we derive the equality

$$*\text{-}w\lim_{n\to\infty}\rho^{\eta,\Sigma_0}_{\mathrm{B}(\rho_n)} = \rho^{\eta}_{\Sigma_0}$$

valid at least for

$$|z| < \exp\left(-\frac{\alpha_*^2}{2}\mathrm{K}_{\Sigma_0}(0)\right)\mathrm{C}(\mathrm{S}_{\Sigma_0}(0))^{-1}\exp\left(-\alpha_*^2\mathrm{S}_{\Sigma_0}(0)-1\right)$$

and $\mu_\infty(z)$ almost every η.

Thus we have essentially the following theorem:

THEOREM 4.1. — Let $\mathrm{V}\in\mathrm{SM}$ and μ^0_V have the global Markov property with respect to the hyperplane Σ_0. Then, for $|z|$ sufficiently small the measure $\mu_\infty(z)$ has the global Markov property with respect to the hyperplane Σ_0.

Finally, let us say a few words about the regularity condition imposed in Definition 4.1. Let A(V) be an elliptic operator corresponding to the Markovian covariance V. Then, the function $\Pi^*_{\Sigma_0,\partial\mathrm{B}(n)}(\eta)(x)$ is a solution of the equation:

$$\mathrm{A}(\mathrm{V})\Pi^*_{\Sigma_0,\partial\mathrm{B}(n)}(\eta)(x) = 0$$

for $x\in\mathrm{R}^d - (\Sigma_0\cup\partial\mathrm{B}(n))$ such that for $x\in\Sigma_0$ we have

$$\Pi^*_{\Sigma_0,\partial\mathrm{B}(n)}(\eta)(x) = 0, \ldots, \partial_n^{\mathrm{int},i}(\Pi^*_{\Sigma_0,\partial\mathrm{B}(n)}(\eta))(x) = 0, \ldots$$

up to $\imath = n-1$ or $d(\mathrm{V})$ and for $x\in\overline{\partial\mathrm{B}(n)}$ we have

$$\Pi^*_{\Sigma_0,\partial\mathrm{B}(n)}(\eta)(x) = \eta(x) - \Pi^*_{\Sigma_0}(\eta)(x), \ldots, \partial_n^{\mathrm{int},n-1}(\Pi^*_{\Sigma_0\partial\mathrm{B}(n)}(\eta))$$
$$= \partial_n^{\mathrm{int},n-1}(\eta(x) - \Pi^*_{\Sigma_0}(\eta))(x).$$

According to the general theory and assuming moreover that the operator A is such that for $\Sigma_0 \cup \partial B(n)$, Σ_0 there exists potentials $\{ b_i^{\Sigma_0}(y, x) \}_{i=0,\dots,n-1}$, $\{ b_i^{\Sigma_0 \cup \partial \overline{B}(n)}(y, x) \}_{i=0,\dots,k-1}$ giving $\Pi^*_{\dots}(\eta)$ by the formulas

$$\Pi^*_{\Sigma_0}(\eta)(x) = \sum_{i=0}^{n-1} \int_{\Sigma_0} d\sigma(y) b_i^{\Sigma_0}(y, x)(\partial_n^{\mathrm{int}, i}\eta)(y),$$

$$\Pi^*_{\Sigma_0 \cup \partial B(n)}(\eta)(x) = \sum_{i=0}^{n-1} \int_{\Sigma_0 \cup \partial B(n)} d\sigma(y) b_i^{\Sigma_0 \cup \partial B(n)}(\underline{y}, x)(\partial_n^{\mathrm{int}, i}\eta)(y)$$

valid for $x \notin \Sigma_0$.

From the proceding remarks we obtain

$$\begin{aligned}\Pi^*_{\Sigma_0, \partial B(n)}(\eta)(x) &= \int_{\partial B(n)} \sum_{i=0}^{n-1} b_i^{\Sigma_0 \cup \partial \overline{B}(n)}(y, x)(\partial_n^{\mathrm{int}, i}(\eta(y) - \Pi^*_{\Sigma_0}(\eta)(y))d\sigma(y) \\ &= \int_{\partial B(n)} \sum_{i=0}^{n-1} b_i^{\Sigma_0 \cup \partial \overline{B}(n)}(y, x)(\partial_n^{\mathrm{int}, i}\eta)(y)d\sigma(y) \\ &\quad - \int_{\partial B(n)} \sum_{i=0}^{n-1} b_i^{\Sigma_0 \cup \partial \overline{B}(n)}(y, x)\partial_n{}^{\mathrm{int}, i}(\Pi^*_{\Sigma_0}(\eta))(y)d\sigma(y).\end{aligned}$$

valid for $x \notin \Sigma_0 \cup \partial \overline{B}(n)$ and μ_V^0-almost every $\eta \in S'(R^d)$. The above formulas seem to be basic for the constructive description of the set SM of covariances observing additionally the bound: (which follows from the V-regularity of $\mu_\infty(z)$)

$$\begin{aligned}&\mu_\infty(\Pi^*_{\Sigma_0, \partial\overline{B}(n)}(\eta)^2(x)) \\ &\leq 2c' \int_{\partial\overline{B}(n)} d\sigma(y) \int_{\partial\overline{B}(n)} d\sigma(y') \cdot \sum_{i,i'=0}^{n-1} b_i^{\Sigma_0 \cup \partial \overline{B}(n)}(y, x) b_{i'}^{\overline{\Sigma}_0 \cup \partial \overline{B}(n)}(y', x)\mu_\infty((\partial_n^{\mathrm{int}, i}\eta)(y) \\ &\qquad\qquad (\partial_n^{\mathrm{int}, i'}\eta)(y')) \\ &+ 2c \int_{\partial\overline{B}(n)} d\sigma(y) \int_{\partial\overline{B}(n)} d\sigma(y') \cdot \sum_{i,i'=0}^{n-1} b_i^{\Sigma_0 \cup \partial \overline{B}(n)}(y, x) b_{i'}^{\Sigma_0 \cup \partial \overline{B}(n)}(y', x)\mu_\infty \\ &\qquad\qquad (\partial_L{}^{\mathrm{int}, i}(\Pi^*_{\Sigma_0}(\eta)(y)) \cdot \partial_n^{\mathrm{int}, i'}(\Pi^*_{\Sigma_0}(\eta)(y'))) \\ &2c \int_{\partial\overline{B}(n)} d\sigma(y) \int_{\partial\overline{B}(n)} d\sigma(y') \sum_{i,i'=0}^{n=1} b_i^{\Sigma_0 \cup \partial^{-}(n)}(\underline{y}, x) b_{i'}^{\Sigma_0 \cup \partial\ (n)}(\underline{y}', x) \partial_y^{\mathrm{int}, i} \partial_{y'}^{\mathrm{int}, i'} V(\underline{y}, \underline{y}') \\ &+ 2c \int_{\partial\overline{B}(n)} d\sigma(y) \int_{\partial\overline{B}(n)} d\sigma(y') \sum_{i,i'=0}^{n-1} b_i^{\Sigma_0 \cup \partial \overline{B}(n)}(y, x) b_{i'}^{\Sigma_0 \cup \partial B(n)}(y', x) \partial_{\underline{y}}^{\mathrm{int}, i} \partial_{y'}^{\mathrm{int}, i'}(K_{\Sigma_0}(\underline{y}, \underline{y}')).\end{aligned}$$

The last estimate should play a basic role in the verification of the conditions imposed in Def. 4.2 on the covariance V. The detailed description of the set of Markovian covariances fulfilling those conditions will published elsewhere.

5. ADDITIONAL REMARKS AND COMMENTS

1. It is well known that the notion of the Markovianity depends heavely on an indexing space chosen. For example, if we take the space $C_0^\infty(\mathbb{R}^d)$ as index space then the Markov property should be defined in different fashion. This notion enlarges the class of admissible covariances [33]. In particular for the case $d = 1$ the covariances with spectral functions equal to entire functions of the infraexponential type are in this class. Generalized random fields indexed by other function spaces have also been discussed in relation with the theory of hyperdistributions [34]. Our point of view corresponds to the indexation by space of measures (in our cases they are the classical Sobolev spaces) in the full analogy with [5] [19] [24]. For recent developments in the theory of the Gaussian measures corresponding to the strongly elliptic differential operators of second order, see [36] [38] and for a review of such questions see [39].

2. In the paper [40] there has been developed a theory of generalized Ornstein-Uhlenbeck processes corresponding to the markovian covariances of the type used in this paper and taking values in the space $S'(\mathbb{R}^d)$. It would be of certain interest to develop the theory of differential stochastic equations for such processes in the spirit of [41].

3. We would like to stress that up to the present authors knowledge there does not exist any satisfactory version of the general Dobrushin-like theory to treat the functional DLR equations. We hope to discuss this problem elsewhere.

4. In the case when $V(0) = \infty$ we can introduce $U - V$ regularization of the perturbation in the following way. Let $\chi_\varepsilon \in C_0^\infty(\mathbb{R}^d)$ be positive and with support contained in the ball $B^\varepsilon = \{ |x| < \varepsilon \}$. Then, we can consider the perturbation of the Gaussian measure μ_V^0 of the form:

$$\mu_\Lambda^\varepsilon(d\varphi) = (Z_\Lambda^\varepsilon(z))^{-1} \exp\left(z \int_\Lambda : \cos \alpha\varphi_\varepsilon :_{V_\varepsilon}(x) dx \right) \mu_V^0(d\varphi)$$

where

$$\varphi_\varepsilon = \varphi * \chi_\varepsilon, \qquad V_\varepsilon(0) = ((\chi_\varepsilon \otimes \chi_\varepsilon) * V)(0).$$

Assuming that V is the Markovian covariance we note the following formulas for the conditional exceptation values with respect to the measure μ_Λ^ε:

$$E_{\mu_\Lambda^\varepsilon}\{F(\varphi_\varepsilon|\mathscr{F}(\Lambda^c)\}(\eta) = \frac{\mu^0_{V_\varepsilon}\left(F(\varphi+\Pi^{*,\varepsilon}_{\Lambda^c}(\eta))\exp\left(z\int_\Lambda d(\underline{x}) : e^{i\alpha\varphi} x_{V_\varepsilon} e^{i\alpha\Pi^*(\eta)}\right)\right)}{\mu^0_{V_\varepsilon}\left(\exp\left(z\int_\Lambda d(\underline{x}) : e^{i\alpha\varphi} :_{V_\varepsilon} e^{i\alpha\Pi^*_{\Lambda^c},\varepsilon(\eta)(x)}\right)\right)}$$

where

$$V_\varepsilon(\Lambda^\varepsilon) = \begin{cases} S_{\partial(\Lambda-(\partial\Lambda)_\varepsilon)}(x,y) : x, y\in\Lambda-\partial(\Lambda)_\varepsilon \\ 0 \quad : x\in(\partial\Lambda)_\varepsilon,\ \underline{y}\notin(\partial\Lambda)_\varepsilon \quad \text{or} \quad x\notin(\partial\Lambda)_\varepsilon,\ \underline{y}\in(\partial\Lambda)_\varepsilon \\ S_{\partial(\Lambda^c-(\partial\Lambda)_\varepsilon)}(x,y) : x, y\in\Lambda^\varepsilon \end{cases}$$

and

$$\Pi^{*,\varepsilon}_{\Lambda^c}(\eta) = \begin{cases} \Pi^*_{\partial(\Lambda^c-(\partial\Lambda_\varepsilon)}(\eta)(x), & x\in\Lambda-(\partial\Lambda)_\varepsilon \\ \eta(x) \quad, & x\in(\partial\Lambda)_\varepsilon, \end{cases}$$

$$(\partial\Lambda)_\varepsilon = \{x \mid \operatorname{dist}(x,\partial\Lambda)\le\varepsilon\}.$$

Now let us consider the stochastic decomposition of the free field with respect to σ-algebra $\mathscr{F}((\Sigma_0\cup\partial B(n))_\varepsilon)$. For this let us define the following functionals of the field φ

$$\Pi^*_{(\Sigma_0,\partial B(n))_\varepsilon}(\varphi)(x) = \begin{cases} \Pi^*_{\partial(B(n)\cap R^d_+-((\partial B(n))_\varepsilon\cup(\Sigma_0)_\varepsilon)}(\varphi)(x) : x\in B(n) \\ \qquad\qquad \cap R^d_+ - (\Sigma_0\cup\partial B(n))_\varepsilon \\ \Pi^*_{\partial(B(n)R^d_- -((\partial B(n))_\varepsilon\cup(\Sigma_0)_\varepsilon)}(\varphi)(x) : x\in B(n) \\ \qquad\qquad \cap R^d_- - (\Sigma_0\cup\partial B(n))_\varepsilon \\ \varphi(x) \quad : x\in(\Sigma_0)_\varepsilon\cup(\partial B(n))_\varepsilon \\ \Pi^*_{\partial(B(n)^c\cap(R^d_-((\Sigma_0)_\varepsilon\cup\partial B(n))_\varepsilon)}(\varphi)(x) : x\in B(n)^c \\ \qquad\qquad \cap(R^d_+ - ((\Sigma_0)_\varepsilon\cup\partial B(n))_\varepsilon) \\ \Pi^*_{\partial B(n)^c\cap(R^d-((\Sigma_0)_\varepsilon\cup(\partial B(n))_\varepsilon)}(\varphi)(x) : x\in B(n)^c \\ \qquad\qquad \cap(R^d_- - ((\Sigma_0)_\varepsilon\cup(\partial B(n))_\varepsilon)) \end{cases}$$

and let

$$\varphi_{(\Sigma_0)_\varepsilon\cup(\partial B(n))_\varepsilon}(x) = \begin{cases} 0 : x\in(\Sigma_0)_\varepsilon\cup(\partial B(n))_\varepsilon \\ \text{the free field with the Dirichlet boundary condition on} \\ \partial((\Sigma_0)_\varepsilon\cup(\partial B(n))_\varepsilon) \quad \text{for} \quad x\notin(\Sigma_0)_\varepsilon\cup(\partial B(n))_\varepsilon. \end{cases}$$

Then, the formula

$$\varphi = \varphi_{(\Sigma_0)_\varepsilon\cup(\partial B(n))_\varepsilon} + \Pi^*_{\Sigma_0,\partial B(n)}(\varphi)$$

gives the stochastic orthogonal decomposition with respect to the σ-algebra $\mathscr{F}((\Sigma_0)_\varepsilon\cup(\partial B(n))_\varepsilon)$.

Assuming certain local decay properties of the kernels K associated to the decompositions written above one is able to prove the following Markov property of the unique (for small sufficientlyz) thermodynamic limit $\mu^\varepsilon_\infty(d\varphi)$ of the measure.

THEOREM 5.1. — Let the kernels $K_{\Sigma_0, \partial B}$ have certain local decay properties. Assume that $|z|$ is sufficiently small. Then, the measure $\mu^{\varepsilon}_{\infty}(d\varphi)$ has the following ε-Markov property.

Let $R^{d,\varepsilon}_{\pm}\{x \in R^d \mid x_0 > (<) \varepsilon\}$ and let F, G are two strictly localized (respectively in $R^{d,\varepsilon}_{+}$ and $R^{d,\varepsilon}_{-}$) functionals of the field $\mu^{\varepsilon}_{\infty}$. Then, we have

$$E_{\mu^{\varepsilon}_{\infty}}\{F \cdot G \mid \mathscr{F}((\Sigma_0)_{\varepsilon})\} = E_{\mu^{\varepsilon}_{\infty}}\{F \mid \mathscr{F}((\Sigma_0)_{\varepsilon}\} \, E_{\mu^{\varepsilon}_{\infty}}\{G \mid \mathscr{F}(\Sigma)_{\varepsilon}\} \quad (4.13)$$

where the equality (4.13) holds $\mu^{\varepsilon}_{\infty}(z)$-almost everywhere.

In particular taking the U−V regularization (by this we violate the rotational invariance) to be local in time, we can also prove that the global Markov property holds also with $\varepsilon = 0$ and with respect to Σ_0. Hence there follows the existence of the transfer matrix in the x_0-direction for the corresponding gases.

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