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Infrared dimensional singularities of the massless $\lambda\varphi^4$ model

by

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ABSTRACT. — For the massless $\lambda\varphi^4$ model, defined perturbatively in dimension $D = 4 - \varepsilon$, we examine the analytic continuation of the Feynman amplitudes to lower dimensions. It is shown for a class of graphs, and conjectured for all graphs, that no singularity occurs at $D = 1$. We discuss the relevance of this study, in connection with the understanding of critical phenomena.

RÉSUMÉ. — On examine, pour le modèle $\lambda\phi^4$ sans masse défini de façon perturbative en dimension $D = 4 - \varepsilon$, le prolongement analytique des amplitudes de Feynman aux dimensions plus basses. On montre pour une classe de graphes, et on conjecture pour tous les graphes, l'absence de singularité à $D = 1$. On discute la pertinence de cette étude pour la compréhension des phénomènes critiques.

I. INTRODUCTION

The study of the infrared divergences is of interest for many phenomena, like the existence of an S-matrix in gauge theories (with the possible occur-

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rence of confinement or infrared slavery), or the critical phenomena (with the possible occurrence of phase transitions). The Ising model for example is related in the critical region to the massless $\lambda\varphi^4$ model [*1*].

It is not clear wether a perturbative approach to the infrared problems is sufficient to give a good insight on such phenomena, which can be specifically non perturbative and require a renormalization group analysis. Nevertheless one may hope that a complete description of the perturbative infrared singularities may give a clue for the non perturbative behaviour, and may even provide a way of summing up the perturbation [*2*].

In this paper we study the infrared properties of the $\lambda\varphi^4$ model by looking at the analytic continuation in the space-time dimension D of the Feynman amplitudes [*3*]. The infrared and ultraviolet divergences manifest themselves as poles located at real rational values of D. A general proof of this statement for arbitrary Feynman graphs, with an arbitrary number of vanishing masses, may be found in [*4*], using the CM-representation of Feynman amplitudes [*5*].

In order to explain the meaning of our results, let us recall the expansion of the Feynman amplitudes A in increasing powers of the mass [*2*]:

$$A(m, D) = A_0(D) + \sum_i A_i(D) m^{\gamma_i(D)}$$

where the exponents $\gamma_i(D)$ are linear in D.

The coefficients $A_0(D)$, $A_i(D)$ have (ultraviolet and infrared) poles for some rational values of D. The infrared poles occur for values of D where many $\gamma_i(D)$ coincide, giving a cancellation of these poles and the appearance of powers of $\ln m$, but no singularity of $A(m, D)$ if $m > 0$.

For $D > D_{IR}$, the lower bound D_{IR} being defined below, the exponents $\gamma_i(D)$ are positive and $A(O, D) = A_0(D)$. Then we perform the analytic continuation of $A_0(D)$ to $D = 1$. We show that $A_0(1)$ is finite for a class of graphs, and we conjecture that it is finite for all graphs of the $\lambda\varphi^4$ model, though the limit $m \to 0$ of $A(m, 1)$ does not exist (some $\gamma_i(1)$ being negative).

In section II we give our precise definition of the dimensionally regularized amplitudes for the massless $\lambda\varphi^4$ model. We fix the renormalization prescriptions needed to ensure the vanishing of the physical mass, and we establish the domain in the D complex plane where the Feynman integrals converge.

In Section III we study the analytic continuation of these amplitudes to lower dimensions. The location of the singularities is completely solved for the class of « computable » graphs, and we show in particular that for such graphs there is no pole at $D = 1$. It is also found that the poles at $D=3$ may come only from a restricted subset of graphs.

In section IV we discuss the extension of these preliminary results to the whole theory, and their eventual meaning in relation with the critical phenomena.

II. PERTURBATIVE DEFINITION OF THE MODEL

II.1. — The euclidean massless $\lambda\varphi^4$ model is defined from the free Lagrangian

$$\mathscr{L}_0 = 1/2\partial_\mu\varphi\partial^\mu\varphi \tag{1}$$

and the interaction Lagrangian

$$\mathscr{L}_{\mathrm{I}} = 1/2a\varphi^2 + \lambda\varphi^4 \tag{2}$$

where the mass counter-term a will be defined below as a power series in λ.

The Schwinger functions are perturbatively defined by the series of the amplitudes corresponding to the Feynman graphs G. The Feynman amplitudes, for any complex dimension D, are given by the integrals in the Schwinger parametric representation

$$\mathrm{A_G} = \int_0^\infty \prod_{i=1}^{l} d\alpha_i \mathrm{U}^{-\mathrm{D}/2} \exp(-\mathrm{V/U}) \tag{3}$$

where G is a Feynman graph with l_{G} internal lines, n_{G} four-legs vertices, $\mathrm{N_G}$ external lines, $\mathrm{L_G}$ independent loops. U and V are the usual Symanzik polynomials.

Let us define d_{G}, and more generally d_{S} for any subgraph S, as the naive dimension (or superficial degree of divergence) of the corresponding integral

$$d_{\mathrm{S}} = \mathrm{DL_S} - 2l_{\mathrm{S}}. \tag{4}$$

DEFINITION. — A subgraph S of a graph G is said to be essential if there is one connected component of S which contains all the external vertices of G (i. e. the vertices of G where external legs are linked). In other words, an essential subgraph S is such that the reduced graph G/S has vanishing external momenta.

With this definition we have the following theorem:

THEOREM. — The integral $\mathrm{A_G}$ is absolutely convergent if

a) $\mathrm{Re}\, d_{\mathrm{G}} < 0$;

b) $\mathrm{Re}\, d_{\mathrm{S}} < 0$ for any non trivial ($l_{\mathrm{S}} \geqq 1$) subgraph S;

c) $\mathrm{Re}\, d_{\mathrm{G}} - \mathrm{Re}\, d_{\mathrm{S}} > 0$ for any essential subgraph S, $\mathrm{S} \neq \mathrm{G}$.

Proof. — This theorem is given in [*6*]. But strictly speaking the proof of [*6*] uses the momentum representation and applies only for real integer dimensions. For completeness we recall in the following appendix a proof which uses the parametric representation and applies to any complex dimension. See also [*7*].

II.2. — Let us now apply this theorem to our model. First we consider graphs G without any self-energy insertion ($N_S \geqq 4 \ \forall S \subseteqq G$). Using the topological relation

$$L_S = l_S - n_S + c_S \tag{5}$$

where c_S is the number of connected components of S, and using the fact that 4 lines are linked to each vertex

$$N_S + 2l_S = 4n_S \tag{6}$$

formula (4) may be written as

$$d_S = (D - 4)\left(n_S - \frac{N_S}{2} + c_S\right) + 4c_S - N_S. \tag{7}$$

Since $n_S - N_S/2 + c_S = L_S \geqq 0$, and $N \geqq 4$ for each connected subgraph, we have $\mathrm{Re}\, d_S \leqq 0$ if $\mathrm{Re}\, D < 4$. Moreover $d_S = 0$ only if $L_S = 0$ and $N_S = 4c_S$, that is if S is a trivial subgraph without internal line. Therefore the ultraviolet convergence conditions *a*) and *b*) of the theorem are satisfied. On the other hand

$$d_G - d_S = (D - 4)(L_G - L_S) - N_G + N_S + 4c_G - 4c_S \tag{8}$$

It is sufficient to consider the convergence conditions for one-line irreducible graphs G. Then $L_G - L_S \geqq 1$ and condition *c*) of the theorem is satisfied if

$$\mathrm{Re}\, D > 4 - \frac{N_S - N_G + 4c_G - 4c_S}{L_G - L_S}. \tag{9}$$

If S is essential, S is a union of disjoint connected subgraphs S_j and one of them, say S_1, contains all the external vertices of G. Therefore

$$c_G - c_{S_1} = 1 - 1 = 0 \tag{10}$$

$$N_{S_1} - N_G \geqq 2 \tag{11}$$

$$N_{S_j} - 4 \geqq 0 \quad \forall j \tag{12}$$

$$L_G - L_S = n_G - \frac{N_G}{2} + 1 - L_S \leqq n_G - 1 \tag{13}$$

and condition *c*) is satisfied if

$$\mathrm{Re}\, D > 4 - \frac{2}{n_G - 1}. \tag{14}$$

II.3. — Next we consider primitive proper self-energies, that is one-line irreducible graphs G with

$$N_G = 2$$

$$N_S \geqq 4 \qquad \forall S \subset G.$$

In that case the ultraviolet convergence condition *a*) becomes

$$\operatorname{Re} D < D_{UV} = 4 - \frac{2}{n_G} \tag{15}$$

and the infrared condition *c*) becomes

$$\operatorname{Re} D > D_{IR} = 4 - \frac{2 + N_S - 4c_S}{n_G - L_S}. \tag{16}$$

We may have $N_S = 4c_S$ and $L_S = 0$ only if there exists a trivial essential subgraph S with $l_S = 0$, that is if G is a generalized tadpole with only one external vertex. In this case we perform a mass renormalization by putting

$$A^R_{\text{tadpole}} = 0 \tag{17}$$

which is the usual prescription of the dimensional renormalization for purely homogeneous integrals.

In the other cases, we may exclude essential subgraphs having disconnected isolated vertices and we have $L_S \geqq 1$, or $N_S \geqq 6c_S$. Therefore $D_{IR} < D_{UV}$ and A_G is defined for $D_{IR} < \operatorname{Re} D < D_{UV}$. In this domain, we can perform a homogeneity integration to find

$$A_G(k) = (k^2)^{d_G/2} F_G(D) \tag{18}$$

$$F_G(D) = \Gamma\left(-\frac{d_G}{2}\right) f_G(D) \tag{19}$$

where $f_G(D)$ is analytic for $D_{IR} < \operatorname{Re} D < 4$.

The function $\Gamma(-d_G/2) = \Gamma((2 - D/2)n_G - 1)$ has its first ultraviolet pole at $D = 4 - \frac{2}{n_G}$. Our renormalization prescription is to take for the renormalized self-energy A^R_G the analytic continuation of formula (18) in the domain $4 - \frac{2}{n_G} < \operatorname{Re} D < 4$

$$A^R_G(k) = (k^2)^{d_G/2} F_G(D)\,. \tag{20}$$

It corresponds to a mass counter-term such that $A^R_G(k)$ vanishes for vanishing k and $\operatorname{Re} D > 4 - \frac{2}{n_G}$, ensuring the vanishing of the physical mass in this domain. The lowest ultraviolet pole of the unrenormalized self-energy, at $D = 4 - \frac{2}{n_G}$, becomes the highest infrared pole of the renormalized self-energy.

II.4. — Finally for a general graph G, we perform the preceding renormalization following the Bogoliubov induction. Once the lower order self-energies have been renormalized, a self-energy insertion of order n_i on the line i replaces the bare propagator $(k^2)^{-1}$ by

$$F_i(D)(k^2)^{d_i/2-1} = \frac{F_i(D)}{\Gamma(1-d_i/2)} \int_0^\infty d\alpha_i \alpha_i^{d_i/2} e^{-\alpha_i k^2} \tag{21}$$

and we can perform the same analysis as in the preceding cases, leading to a new mass counter-term of higher order if G itself is a self-energy graph. In such a way, the whole renormalized perturbation series, up to order n, is defined for $4 - \frac{2}{n} < \operatorname{Re} D < 4$.

III. THE CLASS OF COMPUTABLE GRAPHS

Any Feynman amplitude is meromorphic in the space-time dimension [4]: the only singularities which appear in the complex plane are poles located at real rational values of D. We are interested in the infrared singularities which appear in the half-plane Re D < 4, and we study in this section the « computable » graphs depicted in figure 1. The definition of this class is an inductive one: in figure 1, each bubble represents a lower order graph in the same class.

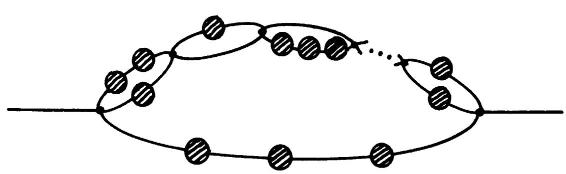

FIG. 1.

For any such graph G, the renormalized amplitude A_G^R is given by formula (20), with

$$d_G = (D-4)n_G + 2. \tag{22}$$

We show now that any such graph has no pole at $D = 1$. To do so, we take as our induction hypothesis

$$F_G(1) \quad \text{is finite.} \tag{23}$$

$$F_G(1) = 0 \quad \text{if } n_G \text{ is odd.} \tag{24}$$

This is true at the starting point of the induction since for the graphs G_2 and G_3 given in figures 2 and 3

$$F_2(D) = \Gamma(3 - D)\left[\Gamma\left(\frac{D}{2} - 1\right)\right]^3\left[\Gamma\left(\frac{3D}{2} - 3\right)\right]^{-1} \tag{25}$$

$$F_3(D) = \frac{\Gamma\left(2n_3 - 1 - \frac{n_3 D}{2}\right)F_0^{n_3-1}\Gamma\left(\frac{n_3 D}{2} - 2n_3 + 2\right)\Gamma\left(\frac{D}{2} - 1\right)}{\Gamma\left(2n_3 - 2 - \frac{n_3 D}{2} + \frac{D}{2}\right)\Gamma\left(\frac{n_3 D}{2} + \frac{D}{2} - 2n_3 + 1\right)} \tag{26}$$

with

$$F_0 = \Gamma\left(2 - \frac{D}{2}\right)\left[\Gamma\left(\frac{D}{2} - 1\right)\right]^2[\Gamma(D - 2)]^{-1} \tag{27}$$

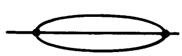

FIG. 2.

FIG. 3.

Moreover, for any chain of self-energies as given in figure 4, the same hypothesis holds since there is at least one proper self-energy with an odd number of vertices if the whole chain has an odd number of vertices.

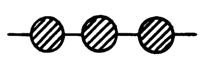

FIG. 4.

We can then compute the graphs shown in figure 5, and we find

$$A_5 = (q^2)^{\frac{n_5-1}{2}(D-4)}H_5(D) \tag{28}$$

$$H_5(D) = \Gamma\left(\frac{n_5-1}{2}(4-D)\right)\frac{F_{S_1}(D)F_{S_2}(D)}{\Gamma\left(1+2n_{S_1}-n_{S_1}\frac{D}{2}\right)\Gamma\left(1+2n_{S_2}-n_{S_2}\frac{D}{2}\right)} \cdot \frac{\Gamma\left(\frac{D}{2}-1-2n_{S_1}+n_{S_1}\frac{D}{2}\right)\Gamma\left(\frac{D}{2}-1-2n_{S_2}+n_{S_2}\frac{D}{2}\right)}{\Gamma\left(n_5\frac{D}{2}-2n_5+2\right)}. \tag{29}$$

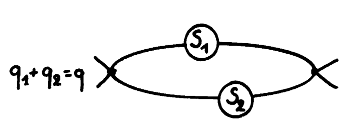

FIG. 5.

It is easy to verify that

$$F_{S_i}(D)\Gamma\left(\frac{D}{2} - 1 - 2n_{S_i} + n_{S_i}\frac{D}{2}\right)\Bigg|_{D=1} \quad \text{is finite} \tag{30}$$

$$\left[\Gamma\left(n_5\frac{D}{2} - 2n_5 + 2\right)\right]^{-1}\Bigg|_{D=1} = 0 \quad \text{if } n_5 \text{ is even}. \tag{31}$$

Therefore $H_5(1)$ is finite, and vanishes if n_5 is even.

Moreover, for any chain G_6 of such graphs (see figure 6), the same properties hold since there is at least one component with an even number of vertices if the chain has an even number of vertices.

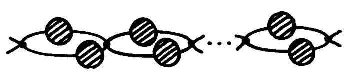

FIG. 6.

Finally we compute the graph G of figure 1 and we find

$$A_G(p) = (p^2)^{1-2n_G+n_G\frac{D}{2}}F_G(D) \tag{32}$$

$$F_G(D) = \Gamma\left(n_G\frac{D}{2} - 2n_G + 1\right)\frac{H_6(D)F_4(D)}{\Gamma\left(\frac{n_6-1}{2}(4-D)\right)\Gamma\left(1+2n_4-n_4\frac{D}{2}\right)} \cdot \frac{\Gamma\left(n_6\frac{D}{2} - 2n_6 + 2\right)\Gamma\left(\frac{D}{2} - 1 - 2n_4 + n_4\frac{D}{2}\right)}{\Gamma\left(\frac{D}{2} - 1 - 2n_G - n_G\frac{D}{2}\right)}. \tag{33}$$

It is easy to verify on formula (33) that the induction hypothesis holds at the following step, which proves the general assertions (23) and (24) for any self-energy in the computable class.

Remark. — The same kind of analysis may be performed at $D = 3$, with weaker results since $F_0(3) \neq 0$, $F_2(3)$ is infinite, $F_3(3)$ vanishes for n_3 odd only when $n_3 \geqq 5$, and $H_5(3)$ vanishes for n_5 even only when $n_5 \geqq 4$. But we see that only a finite and small set of subgraphs is responsible for the appearance of poles at $D = 3$, at least in the computable class.

IV. DISCUSSION

We cannot refrain to conjecture that the results of section III hold for all the graphs of our model: we hope to prove in a later paper that no pole occurs at D = 1 in the analytic continuation of the perturbative $\lambda\varphi^4$ model. For example we have examined the graphs of figure 7, which do not belong to the computable class, by the general methods of [5]. And we have proved that they remain finite at D = 1. It would then be interesting to prove (or disprove?) the general conjecture.

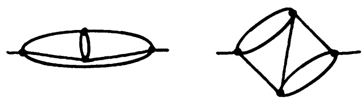

FIG. 7.

We would also like to clarify the situation at D = 3, with the hope that the residue of the pole may be simply expressed, this pole being generated by a limited number of subgraphs.

If these conjectures can be worked out, one is left with the meaning of such results. Looking at the assumed regularity at D = 1, the dense accumulation of poles at D = 2, the seldomness of poles at D = 3, it is tempting to relate these facts with the known properties of the phase transitions in the Ising model. But such an interpretation is quite unclear to us, for at least two reasons:

1) First the non perturbative behaviour may be very different from the perturbative one, even if the perturbation series can be summed, or Borel summed. As a very trivial example, consider the graph G_2 which has a pole at D = 3. By summing the geometric series corresponding to an arbitrary number of G_2 insertions, at D = 4 − ε, one obtains a « dressed » propagator $(1 - A_{G_2})^{-1}$ whose analytic continuation vanishes at D = 3.

2) Moreover the process of analytic continuation does not necessarily provide an acceptable field theory. In particular, the analytic continuation to D = 1, 2 or 3 of the massless $\lambda\varphi^4_{4-\varepsilon}$ model does probably not coincide with the limit of vanishing mass for the massive $\lambda\varphi^4_1$, $\lambda\varphi^4_2$ or $\lambda\varphi^4_3$ model.

For example the euclidean two-point function of the free scalar field is given by

$$P(m, D) = \int d^D k \, \frac{e^{ik.(x-y)}}{k^2 + m^2} \tag{34}$$

At D = 1,

$$P(m, 1) \sim \frac{1}{2m} e^{-m|x-y|} \underset{m\to 0}{\longrightarrow} \infty . \tag{35}$$

But if we take $m = 0$ for D > 2 we get

$$P(O, D) \sim \pi^{D/2} 2^{2D-2} \Gamma\left(\frac{D}{2} - 1\right) [(x-y)^2]^{1-\frac{D}{2}} = P_0(D) \tag{36}$$

which has a finite value at $D = 1$:

$$P_0(1) = -2\pi |x - y|. \tag{37}$$

However this is a negative two-point function, whose absolute value increases linearly at large distances: it does not correspond to a field theory fulfilling sensible axioms.

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APPENDIX

We sketch here a proof of the convergence theorem given in section II. We use the Hepp sectors [8]

$$h_\sigma = \{ \alpha \mid O \leqq \alpha_{\sigma_1} \leqq \ldots \leqq \alpha_{\sigma_l} \} \tag{A1}$$

where σ is any permutation of $\{ 1, \ldots, l \}$. In a given sector, we perform the usual change of variables

$$\begin{cases} \alpha_{\sigma_1} = \beta_1 \beta_2 \ldots \beta_l \\ \alpha_{\sigma_2} = \beta_2 \ldots \beta_l \\ \ldots \\ \alpha_{\sigma_l} = \beta_l \end{cases} \tag{A2}$$

and we have for any Feynman amplitude

$$\mathrm{A} = \sum_\sigma \mathrm{A}_\sigma \tag{A3}$$

$$\mathrm{A}_\sigma = \int_0^\infty \beta_l^{l-1} d\beta_l \int_0^1 \prod_{i=1}^{l-1} (d\beta_i \beta_i^{i-1}) \mathrm{U}(\beta)^{-\mathrm{D}/2} \exp\left\{ - \frac{\mathrm{V}(\beta)}{\mathrm{U}(\beta)} \right\} \tag{A4}$$

Let us call R_i the subgraph $\{ \sigma_1, \ldots, \sigma_i \}$ made from the first i lines of the sector. It is well known, and easy to verify from the definition of U and V in terms of one-trees and two-trees, that

$$\mathrm{U}(\beta) = \prod_i \beta_i^{\mathrm{L}(\mathrm{R}_i)} (1 + \mathrm{P}(\beta)) \tag{A5}$$

where $\mathrm{P}(\beta)$ is a polynomial in $\beta_1, \ldots, \beta_{l-1}$ with positive coefficients, and $\mathrm{P}(0) = 0$.

$$\mathrm{V}(\beta) = s_j \mathrm{U}_{\mathrm{R}_1} \mathrm{U}_{\mathrm{R}_2/\mathrm{R}_1} \ldots \mathrm{U}_{\mathrm{R}_{j-1}/\mathrm{R}_{j-2}} \prod_{i=j}^{l} \beta_i \mathrm{U}_{\mathrm{R}_{j+1}/\mathrm{R}_j} \ldots \mathrm{U}_{\mathrm{G}/\mathrm{R}_{l-1}} [1 + \mathrm{Q}] \tag{A6}$$

where R_j is the smallest essential subgraph among the R_i's of the sector, s_j the cut-invariant corresponding to $\mathrm{G}/\mathrm{R}_{j-1} - \{ j \}$, and $\mathrm{Q}(\beta)$ a polynomial in $\beta_1, \ldots, \beta_{l-1}$ with $\mathrm{Q}(0) = 0$. From (A5) and (A6) we find

$$\frac{\mathrm{V}}{\mathrm{U}} = s_j \prod_i \beta_i^{v_i} [1 + \mathscr{R}(\beta)] \tag{A7}$$

where $\mathscr{R}(\beta)$ is a bounded real function of $\beta_1, \ldots, \beta_{l-1}$,

$$v_i = 1 \quad \text{if } \mathrm{R}_i \text{ is essential} \quad (i \geqq j),$$
$$v_i = 0 \quad \text{if } \mathrm{R}_i \text{ is not essential} \quad (i < j).$$

Now if $d_\mathrm{G} < 0$, the β_l integration can be performed, giving

$$\mathrm{A}_\sigma = \Gamma\left(- \frac{d_\mathrm{G}}{2} \right) s_j^{\frac{d_\mathrm{G}}{2}} \int_0^1 \left(\prod_{i=1}^{j-1} \beta_i^{-\frac{d_{\mathrm{R}_i}}{2} - 1} d\beta_i \right) \left(\prod_{i=j}^{l-1} \beta_i^{\frac{d_\mathrm{G}}{2} - \frac{d_{\mathrm{R}_i}}{2} - 1} d\beta_i \right) (1 + \mathscr{R}) \tag{A8}$$

which achieves the proof of the theorem.

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