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Section A :
Physique théorique.

Magnetic monopoles in curved spacetimes

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ABSTRACT. — Spherically symmetric solutions of the coupled Yang-Mills-Higgs system in a background metric are studied. We present several explicit solutions and comment briefly on their stability.

RÉSUMÉ. — Nous étudions certaines solutions des équations de Yang-Mills en présence d'un champ de matière dans une métrique externe. Généralisant les conditions de Bogomolny pour des métriques à symétrie sphérique nous ramenons les équations du mouvement à un système d'équations du premier ordre, ceci nous permet de construire certaines solutions explicites dont nous étudions la stabilité (au sens de Poincaré).

The search for classical solutions of Yang-Mills equations for gauge fields coupled to Higgs fields [*1*] is widely simplified by the following observation due to Bogomolny [*2*]. Consider gauge fields $A_\mu(x)$ coupled to scalar fields $\Phi(x)$ belonging to the adjoint representation of the group. In the static case the hamiltonian reads:

$$H = \int d^3x \left[\frac{1}{2}\vec{B}^2 + \frac{1}{2}(\vec{D}\Phi)^2 + \lambda V(\Phi) \right]$$

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where $\vec{B}$ is the magnetic field, $V(\Phi)$ is the self-interaction term of the Higgs field and

$$\vec{D}\Phi = \vec{\nabla}\Phi + i[\vec{A}, \Phi]$$

its covariant derivative. Then in the limit $\lambda \to 0$ any solution of the first order equation

$$\vec{B} = \pm \vec{D}\Phi \tag{1}$$

is a solution of the equations of motion. The converse not being necessarily true [3].

It is well known [4] that the above equation can be considered as a self duality constraint provided we reinterpret Φ as the time component of the static gauge field. In a previous paper [5] we have formulated the self duality constraints for gauge fields in static spherically symmetric metrics.

In this note we show that in curved spaces, the correspondence $A_0 \to \Phi$ turns out to be no longer true. However one can generalize in a quite simple fashion the usual Bogomolny conditions (1). Using these constraints in a systematic way we are led for external metrics to some new explicit solutions of the magnetic monopole type. Next we speculate about the possibility of coupling these equations with the gravitational field [6], we finally argue that in the Prasad-Sommerfield limit [7] the mass of the monopole comes out to be of the order of the Planck mass.

We consider the SU(2) Yang-Mills-Higgs model with the lagrangian

$$\mathscr{L} = \int d^3x\sqrt{g}\left[-\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} - \frac{1}{2}D_\mu\Phi^a D^\mu\Phi^a - \lambda V(\Phi)\right]$$

where

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - \varepsilon_{abc}A^b_\mu A^c_\nu$$

is the field strength and

$$D^\mu\Phi^a = \partial_\mu\Phi^a - \varepsilon_{abc}A^b_\mu\Phi^c$$

is the covariant derivative of an isotriplet scalar field.

We look for static spherically symmetric solutions of the Wu-Yang form [8]:

$$\Phi^a(r) = \frac{x^a}{r^2}H(r)$$

$$A^a_0(r) = 0$$

$$A^a_i = \varepsilon_{iab}\frac{x_b}{r^2}(K(r) - 1)$$

where $r = \sqrt{\sum_{i=1}^{3} x_i^2}$ is the three dimensional radius.

Furthermore we assume a spherically symmetric static metric

$$(ds)^2 = -A(r)(dt)^2 + B(r)dr^2 + r^2(d\theta)^2 + r^2\sin^2\theta(d\varphi)^2 .$$

The equations of motion turn out to be the following ones:

$$\left\{\begin{aligned} & H'' + \frac{H'}{2}\left(\frac{A'}{A} - \frac{B'}{B}\right) - \frac{H}{2r}\left(\frac{A'}{A} - \frac{B'}{B}\right) - \frac{2HK^2B}{r^2} - \lambda\frac{\delta V}{\delta\phi}rB = 0\,, \\ & K'' + \frac{K'}{2}\left(\frac{A'}{A} - \frac{B'}{B}\right) - \frac{KB}{r^2}(K^2 + H^2 - 1) = 0\,. \end{aligned}\right. \tag{2}$$

where prime denote the derivative with respect to r.

Throughout this paper we shall restrict ourselves to the particular case $\lambda = 0$ which is the Prasad-Sommerfield limit [7]. In order to construct explicit finite energy solutions our strategy is to convert the above equations (which are of second order and non-linear) into a pair of first order non-linear equations.

In flat space eq. (1) reads explicitly:

$$\left\{\begin{aligned} & K' = \mp\frac{KH}{r}\,, \\ & H' = \frac{H}{r} \mp \frac{1}{r}(K^2 - 1)\,. \end{aligned}\right. \tag{3}$$

(Both upper or both lower sign are to be taken).

Now the simplest generalization of those equations would be:

$$\left\{\begin{aligned} & K' = \alpha(r)KH \\ & H' = \beta(r)(K^2 - 1) + \gamma(r)H \end{aligned}\right. \tag{4}$$

where α, β, γ are unknown functions of r.

Eqs. (4) are consistent with the equations of motion (2) if and only if

$$\begin{aligned} \alpha(r) = \beta(r) &= \pm\frac{\sqrt{B}}{r} \\ \gamma(r) &= \frac{1}{r} - \frac{A'(r)}{2A(r)} \\ \frac{1}{2}\left(\frac{A'}{A}\right)' + \frac{A'}{Ar} &- \frac{A'B'}{4AB} = 0\,. \end{aligned} \tag{5}$$

The last equation represents a constraint on the metric and it is trivially solved: either $A = 1$ and B is arbitrary or $A = e^{\varphi}$, $B = \left(\frac{d\varphi}{dr}\right)^2 r^4\alpha^2$ (α is a constant and $\varphi(r)$ is an arbitrary function of r).

We discuss below these two cases.

CASE I. — The metric is of the form

$$(ds)^2 = -(dt)^2 + B(r)(dr)^2 + r^2(d\theta)^2 + r^2\sin^2\theta(d\varphi)^2$$

and eqs. (4) read

$$\left\{\begin{aligned} K' &= \frac{\sqrt{B}}{r} KH \\ H' &= \frac{H}{r} + \frac{\sqrt{B}}{r}(K^2 - 1) \end{aligned}\right. \tag{6}$$

(the + sign has been chosen for convenience).

Defining $K = e^{\chi}$ and

$$r_* = \int dr \sqrt{B}$$

we obtain

$$\frac{d^2\chi}{dr_*^2} = \frac{1}{r^2}(e^{2\chi} - 1).$$

Let

$$\chi = \rho(r) - \int^r dr \sqrt{B} \int dr \frac{\sqrt{B}}{r^2}.$$

Then the above equation reduces to

$$\frac{d^2\rho}{dr_*^2} = \frac{1}{r^2} \exp - 2 \int dr \sqrt{B} \int dr \frac{\sqrt{B}}{r^2} \exp 2\rho. \tag{7}$$

For an arbitrary metric the solution cannot in general be obtained simply by quadrature.

The requirement of explicit solvability

$$\frac{1}{r^2} \exp - 2 \int dr \sqrt{B} \int dr \frac{\sqrt{B}}{r^2} = K \tag{8}$$

leads to

$$B = 1/(1 + \lambda r^2).$$

We have to distinguish two possibilities: $\lambda = \pm a^2$.

For positive $\lambda = a^2$

$$(ds)^2 = - (dt)^2 + \frac{(dr)^2}{1 + a^2 r^2} + r^2 d\Omega$$

describes a space of constant negative curvature.

We get

$$r_* = \frac{1}{a} \operatorname{Arg\,sh} ar.$$

Eq. (7) then takes the form

$$\frac{d^2\rho}{dr_*^2} = a^2 e^{2\rho}. \tag{9}$$

The general solution

$$\rho = \ln \frac{C}{a \operatorname{sh}(Cr_* + \beta)}$$

depends on two parameters C, β; however the condition $\beta = 0$ turns out to be necessary in order to get a finite energy.

Therefore one obtains

$$\begin{cases} K = \dfrac{C}{a}\dfrac{\operatorname{sh} ar_*}{\operatorname{sh} Cr_*} \\ H = \operatorname{ch} ar_* - \dfrac{C}{a}\coth Cr_* \operatorname{sh} ar_* . \end{cases} \tag{10}$$

Next, the standard boundary condition

$$\frac{H}{r} \to -\mu \quad \text{for} \quad r \to \infty$$

leads to

$$C = a + \mu$$

where μ is the vacuum expectation value of the Higgs field. Thus the solution (10) depends on two parameters a and μ. However it turns out that the mass of this configuration is independent of a. In order to calculate the mass it is particularly convenient to observe that the hamiltonian density is a total derivative [eqs. (6) are used in the course of the derivation below].

Thus one has

$$E = 4\pi \int r^2 dr \sqrt{B}\left[\frac{K^{2\prime}}{Br^2} + \frac{1}{2r^4}(K^2 - 1)^2 + \frac{1}{2B}\left[\left(\frac{H}{r}\right)'\right]^2 + \frac{K^2H^2}{r^4}\right]$$

$$E = 4\pi\left[(K^2 - 1)\frac{H}{r}\right]_{r_*=0}^{r_*=+\infty} = 4\pi\mu . \tag{11}$$

From eq. (11) it is obvious that the above feature is general as long as the standard boundary conditions

$$\begin{cases} \dfrac{H}{r} \to -\mu \\ K \to 0 \end{cases} \quad \text{for } r \to \infty$$

hold.

For $a \to 0$ we get back the Prasad-Sommerfield case:

$$\begin{cases} K = \dfrac{Cr}{\operatorname{sh} Cr} \\ H = 1 - Cr \coth Cr . \end{cases} \tag{12}$$

Next we pass to the case where $\lambda = -a^2$, the metric now describes a space of constant positive curvature. The derivation of the solutions goes through the same steps as before. In the interior region $r < \frac{1}{a}$ one is led to

$$\begin{cases} \dfrac{H}{r} = a \operatorname{cotg} ar_* - C \operatorname{cotg} Cr_* \\ K = \dfrac{C}{a} \dfrac{\sin ar_*}{\sin Cr_*} \end{cases} \tag{13}$$

where $r_* = \frac{1}{a} \operatorname{Arc}\sin ar$.

Now C is to be regarded as a function of μ defined implicitly by

$$\mu = C \operatorname{cotg} \frac{C\pi}{2a} \qquad (C < a)$$

Eq. (11) leads to a total energy

$$E = 4\pi \left(1 - \frac{C^2}{a^2 \sin^2 \dfrac{C\pi}{2a}} \right) \mu \tag{14}$$

which is now metric dependent because the gauge field on the boundary $r = \frac{1}{a}$ has not the usual behaviour $K \to 0$.

Indeed one has

$$K \to \frac{C}{a \sin \dfrac{C\pi}{2a}}.$$

CASE II. — We now investigate the second case where $A \neq 1$. The first order equations (4) become [9]

$$\begin{cases} K' = \dfrac{\sqrt{B}}{r} KH & (15) \\ H' = \dfrac{\sqrt{B}}{r}(K^2 - 1) + H\left(\dfrac{1}{r} - \dfrac{A'}{2A}\right) & (16) \end{cases}$$

Defining $r_* = \int \sqrt{\frac{B}{A}}\, dr$ and $K = e^\chi$ we get

$$\frac{d^2\chi}{dr_*^2} = \frac{A}{r^2}(e^{2\chi} - 1).$$

If we further set

$$\chi = \rho - \int^r \sqrt{\frac{B}{A}}\, dr \int \frac{\sqrt{AB}}{r^2}\, dr$$

we obtain

$$\frac{d^2\rho}{dr_*^2} = \frac{A}{r^2} \exp - 2 \int \sqrt{\frac{B}{A}} dr \int \frac{\sqrt{AB}}{r^2} \exp 2\rho \tag{17}$$

As before the assumption of explicit solvability implies

$$\frac{A}{r^2} \exp - 2 \int \sqrt{\frac{B}{A}} dr \int \frac{\sqrt{AB}}{r^2} = \text{Constant} \tag{18}$$

It turns out that this condition is equivalent to the requirement of conformal flatness at the level of the metric. Indeed by differentiating eq. (18) twice we get

$$4(1 - B) + \frac{2rB'}{B} - \frac{2rA'}{A} + \frac{2r^2A''}{A} - \frac{r^2A'^2}{A^2} - \frac{r^2A'B'}{AB} = 0 \tag{19}$$

which expresses nothing but the vanishing of the Weyl tensor:

$$C_{\mu\nu\rho\sigma} = R_{\mu\nu\rho\sigma} - \frac{1}{2}(g_{\mu\rho}R_{\nu\sigma} - g_{\mu\sigma}R_{\nu\rho} - g_{\nu\rho}R_{\mu\sigma} + g_{\nu\sigma}R_{\mu\rho}) + \frac{R}{6}(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho}).$$

Among this class of conformally flat metrics we shall now keep the subclass for which the constraint (5) is fulfilled. Let us recall that this is a necessary condition ensuring that the equations of motion are satisfied. In fact defining $A = e^{\varphi}$ and setting $B = \left(\frac{d\varphi}{dr}\right)^2 r^4\alpha^2$ into eq. (19) one obtains the general solution [*10*]

$$e^{\varphi/2} = \frac{- \alpha\alpha' r^2 \pm \sqrt{\alpha^2\alpha^{2'} r^4 + \alpha'' r^2(1 - 4\alpha^2 r^2)}}{1 - 4\alpha^2 r^2} \tag{20}$$

where α, α', α'' are constants.

As an illustration, the choice $\alpha' = 4\alpha$, $\alpha'' = 4\alpha^2$ yields

$$A = B = \left(\frac{2r\alpha}{1 \pm 2r\alpha}\right)^2$$

One finally obtains

$$K = \frac{C\left(r \pm \frac{1}{2\alpha}\right)}{\text{sh}\, C\left(r \pm \frac{1}{2\alpha}\right)}$$

$$\frac{H}{r \pm \frac{1}{2\alpha}} = \left[\frac{1}{r \pm \frac{1}{2\alpha}} - C \coth C\left(r \pm \frac{1}{2\alpha}\right)\right]$$

It is quite amusing to notice how the introduction of an external metric manifests itself minimally, namely by a single translation $\frac{1}{2\alpha}$ on the flat space solution. However in that case the energy density diverges at the origin.

The general discussion involves quite complicated expressions which are not illuminating, thus we shall not pursue further in this direction. In the following we briefly comment on some interesting properties of our solutions. To explore their physical significance let us consider the energy momentum tensor which is the response of the system to a change in the metric.

For the stress tensor one finds

$$T^r_r = \frac{1}{Br^2}(K')^2 - \frac{1}{2r^4}(K^2 - 1)^2 - \frac{K^2H^2}{r^4} + \frac{1}{2B}\left(\frac{H'}{r} - \frac{H}{r^2}\right)^2$$

$$T^\theta_\theta = T^\varphi_\varphi = \frac{1}{2r^4}(K^2 - 1)^2 - \frac{1}{2B}\left(\frac{H'}{r} - \frac{H}{r^2}\right)^2 .$$

From eq. (15) we obtain

$$T^r_r = - T^\theta_\theta = - T^\varphi_\varphi = - \frac{1}{r^2\sqrt{AB}}\left[\frac{H^2A'}{4\sqrt{AB}}\right]' . \tag{22}$$

Consequently when $A = 1$ all components of the stress tensor vanish identically. Such a feature has been analysed in detail by Bialynicki-Birula [*11*] who called this property « Poincaré stability ». The Higgs field and the gauge field contribute to the stress tensor with opposite signs and cancellation occurs if the first order equations (6) are satisfied. When $A \neq 1$ this stability condition is no longer fulfilled. However as we shall discuss below the stresses can be counter-balanced by gravity.

For our purpose we shall need the following identity [See Appendix A]

$$T^0_0 = - \frac{1}{r^2\sqrt{AB}}\left[(K^2 - 1)\frac{H}{r}\sqrt{A} - \frac{A'H^2}{4\sqrt{AB}}\right]' \tag{23}$$

Introducing again the coupling constant eq. (16) reads

$$H' = \frac{\sqrt{B}}{er}(K^2 - 1) + H\left(\frac{1}{r} - \frac{A'}{2A}\right)$$

The asymptotic behaviour of the field

$$H \to - \mu r = - \frac{mr}{e}$$

$$K \to e^{-mr}$$

(in accordance with the Higgs mechanism $m = e\mu$ is the mass of the vector field) is consistent with the metric

$$A = e^{\varphi} \qquad B = \left(\frac{d\varphi}{r}\right)^2 r^4 \alpha^2 \tag{24}$$

provided that $\alpha = \frac{m}{2}$.

The energy momentum tensor (eqs. 22, 23) is asymptotically such that

$$T^0_0 = + T^r_r = - T^{\varphi}_{\varphi} = - T^{\theta}_{\theta} = - \frac{1}{2e^2 r^4}.$$

One recognizes the energy momentum tensor of a magnetic charge $g = \frac{1}{e}$ sitting at the origin.

Therefore the metric will be asymptotically of the Reissner-Nordström form

$$A = \frac{1}{B} = 1 - \frac{2MG}{r} + \frac{4\pi G}{e^2 r^2}. \tag{25}$$

The main feature is that now eqs. (24) are quite restrictive.

Indeed from eqs. (24), (25) we are led to a degenerate Reissner-Nordström metric where all the parameters are now fixed

$$A = \frac{1}{B} = \left(1 - \frac{\sqrt{4\pi G}}{er}\right)^2$$

$$M = \frac{\sqrt{4\pi}}{e\sqrt{G}} \qquad m = \frac{1}{\sqrt{4\pi}} \frac{e}{\sqrt{G}}. \tag{26}$$

Obviously such a feature occurs only in the Prasad-Sommerfield limit where the arbitrary scale μ is now fixed by the gravitational constant. Interpreting G as Newton's constant the mass of the monopole M turns out to be of the order of the Planck mass. Another alternative would be to consider G as a strong gravitational coupling constant; in this case eqs. (26) might be relevant in the context of strong gravity [*12*].

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APPENDIX A

Proof of eq. (23)

Varying the lagrangian with respect to the metric yields the energy momentum tensor

$$T_{\alpha\beta} = F_{\alpha}^{a\nu}F_{\beta\nu}^{a} - \frac{1}{4}g_{\alpha\beta}F_{\mu\nu}^{a}F^{a\mu\nu} + D_{\alpha}\Phi^{a}D_{\beta}\Phi^{a} - \frac{1}{2}g_{\alpha\beta}D_{\mu}\Phi^{a}D^{\mu}\Phi^{a}$$

After some calculation one finds

$$T_0^0 = -\frac{(K')^2}{Br^2} - \frac{K^2H^2}{r^4} - \frac{1}{2B}\left(\frac{H'}{r} - \frac{H}{r^2}\right)^2 - \frac{1}{2r^4}(K^2 - 1)^2 \qquad \text{(A.1)}$$

(note that T_{00} is positive definite as it should be).

From eqs. (15), (16) one obtains

$$\frac{1}{Br^2}\left(K' - \frac{\sqrt{B}}{r}KH\right)^2 + \frac{1}{2B}\left[\frac{H'}{r} - \frac{H}{r^2} - \frac{\sqrt{B}}{r^2}(K^2 - 1) + \frac{HA'}{2Ar}\right]^2 = 0$$

Therefore

$$\begin{aligned}\frac{K^{2\prime}}{Br^2} + \frac{K^2H^2}{r^4} + \frac{1}{2B}\left(\frac{H'}{r} - \frac{H}{r^2}\right)^2 + \frac{1}{2r^4}(K^2 - 1)^2 \\ = \frac{1}{r^2\sqrt{AB}}\left[\frac{2KK'H\sqrt{A}}{r} - \frac{H^2(A')^2}{8A^{3/2}\sqrt{B}} + \frac{\sqrt{A}}{r}\left[H' - \frac{H}{r}\right](K^2 - 1)\right. \\ \left. - \frac{1}{2\sqrt{AB}}HA'\left(H' - \frac{H}{r}\right) + \frac{1}{2r\sqrt{A}}(K^2 - 1)HA'\right] \\ = \frac{1}{r^2\sqrt{AB}}\left\{\left[\frac{(K^2 - 1)H\sqrt{A}}{r}\right]' - \left[\frac{H^2A'}{4\sqrt{AB}}\right]'\right\} \\ + \frac{H^2}{2Br^2}\left[\frac{1}{2}\left(\frac{A'}{A}\right)' + \frac{A'}{Ar} - \frac{A'B'}{4AB}\right]\end{aligned} \qquad \text{(A.2)}$$

Using eq. (5) the last term disappears. Thus one is led to formula (23)

$$\begin{aligned}T_0^0 = -\frac{K^{2\prime}}{Br^2} - \frac{K^2H^2}{r^4} - \frac{1}{2B}\left(\frac{H'}{r} - \frac{H}{r^2}\right)^2 - \frac{1}{2r^4}(K^2 - 1)^2 \\ = -\frac{1}{r^2\sqrt{AB}}\left\{\left[\frac{(K^2 - 1)H\sqrt{A}}{r}\right]' - \left[\frac{H^2A'}{4\sqrt{AB}}\right]'\right\}\end{aligned} \qquad \text{(A.3)}$$

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