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On the geometry of the relativistic electromagnetic fluid flows

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ABSTRACT. — The paper presents the geometrical aspects of the relativistic electromagnetic fluid flows involving the kinematical parameters associated with the congruences of time-like and space-like curves and explores certain theorems which are valid in the domain of the electromagnetic fluids.

1. INTRODUCTION

The modern investigations in astronomy and astrophysics have stimulated interest in relativistic matter fields which are more general than the familiar perfect fluid models. Lichnerowicz [*1*] initiated fundamental studies of more general hydrodynamical matter field solutions of Einstein field equations and later extended his investigation to relativistic magnetohydrodynamics (RMHD). His RMHD field equations are used by Yadzis [*2*] and Banerji [*3*] to infer the magnetic effect in galastic cosmogony, gravitational collapse and pulsar theory. Yodzis [*2*] deduced the relation $\varepsilon = -2\omega_i h^i$ for a medium with infinite conductivity, where ε is the charge density, ω_i the vorticity vector and h_i the magnetic field vector. Mason [*4*] however criticized the method of deduction of Yodzis on the ground that, while the electric field $e^i \to 0$ for a perfectly conducting medium, the conductivity $\bar{k} \to \infty$ so that $\bar{k}e^i$ may not vanish contrary to Yodzis assumption. The above relation had been also deduced by Raychaudhuri [*5*] *et al.* and Ellis [6] *et al.* Banerji [*3*] has derived some simple relations which generalize the well-known results of classical magnetohydrodynamics and obtained the relativistic analogue of Ferraro's theorem and Von Zeipel's theorem in steady state condition. Esposito [7] *et al.* have investigated the conservation of magnetic energy for shear free expanding motions and proved that the acceleration and magnetic field are orthogonal when the magnetic fields are « frozen-in ». RMHD field equations have been studied by Date [8] constructing the stress-energy-momentum tensor for thermally conducting, viscous, compressible fluid with infinite electrical conductivity and constant magnetic permeability. Recently several consequences of RMHD field equations have been investigated by the author [9-13] using kinematical parameters associated with the congruences of streamlines, electric and magnetic field lines.

The purpose of this paper is to deduce relations governing the general behaviour of electromagnetic fluids employing the theory of space-like congruences developed by Greenberg [14]. In particular, we shall study the RMHD field equations constructing the « magnetic » and « electric » vorticity vectors on the basis of Greenberg's theory of kinematical parameters associated with the congruence of space-like curves. We shall also establish certain theorems which hold in the domain of electromagnetic fluids in sec. 4 after writing field equations in sec. 2 and kinematical parameters associated with the congruences of time like and space-like curves in sec. 3.

2. FIELD EQUATIONS

The Maxwell field equations read as

$$\left(u^i B^j - u^j B^i + \eta^{ijkl}{}_{u_k e_l}\right)_{;i} = 0 \tag{2.1}$$

and

$$\left(u^i D^j - u^j D^i + \eta^{ijkl}{}_{u_k h_l}\right)_{;j} = - J^i, \tag{2.2}$$

where B^i is the magnetic induction vector, D^i the electric induction vector, J^i the electric current vector, e^i the electric field vector and h^i the magnetic field vector. The electric current vector is decomposed as

$$J^i = \varepsilon u^i + \bar{k} e^i, \tag{2.3}$$

where ε is the charge density and $\bar{k}$ the conductivity of the fluid.

Einstein field equations are

$$R_{ij} - \frac{1}{2} R g_{ij} = - T_{ij}, \tag{2.4}$$

where the stress-energy-momentum tensor [10], T_{ij}, for a self-gravitating, thermally conducting, viscous, compressible and charged fluid with constant magnetic permeability and electric permitivity is given by

$$T_{ij} = (\overset{*}{\rho} + p^*) u_i u_j - p^* g_{ij} + \nu \sigma_{ij} - (\lambda e_i e_j + \mu h_i h_j) + P_i u_j + P_j u_i, \tag{2.5}$$

where

$$\overset{*}{\rho} = \rho + \frac{1}{2}(\lambda \mid e \mid^2 + \mu \mid h \mid^2), \tag{2.6}$$

$$p^* = p + \frac{1}{2}(\lambda \mid e \mid^2 + \mu \mid h \mid^2), \tag{2.7}$$

$$\mathrm{P}_i = q_i - \mathrm{V}_i. \tag{2.8}$$

Here ρ is the matter energy density of the fluid, p the isotropic pressure, $\nu(\geqslant 0)$ the coefficient of viscosity, q^i the heat energy—flux vector and V^i the electromagnetic energy—flux vector. The matter energy density ρ is connected with the proper energy density i by the relation

$$\rho = \rho_0(1 + i) \tag{2.9}$$

The relations [*1*, *15*] connecting thermodynamical variables are

$$\mathrm{TDS} = \mathrm{D}i + p\mathrm{D}(1/\rho_0), \tag{2.10}$$

$$\mathrm{S}^i = \rho_0 \mathrm{S} u^i + q^i/\mathrm{T}, \tag{2.11}$$

$$q^i = \mathrm{K}(\mathrm{T}_{,j} - \mathrm{TD}u_j)\gamma^{ij}, \tag{2.12}$$

and

$$\chi = 1 + i + p/\rho_0, \tag{2.13}$$

where S, T, S^i, K and χ denote the entropy, the rest temperature, the entropy-flux vector the heat conduction coefficient and the fluid index respectively.

3. KINEMATICAL PARAMETERS

The kinematical properties of the fluid stream lines are characterized by the usual decomposition for the rate of change of the flow vector [*16*] u^i;

$$u_{i;j} = \sigma_{ij} + \omega_{ij} + \theta\gamma_{ij} + \mathrm{D}u_i u_j, \tag{3.1}$$

where σ_{ij}, ω_{ij}, θ denote the shear, rotation and expansion of the congruence of streamlines respectively. D stands for the directional derivative along the fluid flow.

The covariant derivative of the 4-vector n^i tangential to the space-like congruence is decomposed according to Greenberg [*14*] as follows;

$$n_{i;j} = \overset{*}{\sigma}_{ij} + \overset{*}{\omega}_{ij} + \overset{*}{\theta}\overset{*}{\gamma}_{ij} - \mathrm{D}^* n_i n_j + \mathrm{D}n_i u_j - (\mathrm{D}n_k u^k)u_i u_j + (\mathrm{D}^* n_k u^k)u_i n_j + n_{k;j}u^k u_i, \tag{3.2}$$

where the shear tensor $\overset{*}{\sigma}_{ij}$, rotation tensor $\overset{*}{\omega}_{ij}$ and scalar expansion $\overset{*}{\theta}$ of the space-like congruence are defined by

$$\overset{*}{\sigma}_{ij} = \overset{*}{\gamma}{}_i^k \overset{*}{\gamma}{}_j^l n_{(k;l)} - \overset{*}{\theta}\overset{*}{\gamma}_{ij}, \tag{3.3}$$

$$\overset{*}{\omega}_{ij} = \overset{*}{\gamma}{}_i^k \overset{*}{\gamma}{}_j^l n_{[k;l]}, \tag{3.4}$$

and

$$\overset{*}{\theta} = \frac{1}{2}\left(n^{i}_{;i} - n_{j;k}u^{j}u^{k}\right) \tag{3.5}$$

respectively. The projection tensor $\overset{*}{\gamma}_{ij}$ is defined as

$$\overset{*}{\gamma}_{ij} = g_{ij} - u_i u_j + n_i n_j \tag{3.6}$$

$D^*n_i = n_{i;j}n^j$ can be interpreted as the curvature vector associated with the congruence of space-like curves. The space-like curve is geodesic when D^*n_i vanishes.

4. GENERAL RELATIONS AND THEOREMS

Now the situation permits us to begin the study of various consequences of RMHD field equations involving the kinematical parameters associated with the congruence of time-like and space-like curves which are already mentioned in Sec. 3. In this section, we present the results of purely theoretical interest and extend the theory of RMHD relaxing the assumption of infinite conductivity. Following the concluding remarks made by Greenberg [14], we can apply his theory to study the behaviour of the congruences of electric and magnetic field lines. We may interpret the kinematical parameters $\overset{*}{\sigma}_{ij}$, $\overset{*}{\omega}_{ij}$, $\overset{*}{\theta}$ as the shear, rotation and expansion of the congruence formed by magnetic field lines (magnetic field tubes). It is known that the shear and rotation of the space-like congruence reside in 2-space of metric $\overset{*}{\gamma}_{ij}$. Further, this may be interpreted as the shear and rotation of the magnetic field tubes reside in 2-space quotient to the streamlines and magnetic field lines. Thus it would be reasonable to define an alternating tensor on the 2-space quotient to the streamlines and magnetic field lines by the relation

$$\overset{*}{\varepsilon}{}^{ij} = \eta^{ijkl}{}_{u_k n_l} \tag{4.1}$$

satisfying the properties

$$\overset{*}{\varepsilon}_{ij}\overset{*}{\varepsilon}{}^{ik} = \overset{*}{\gamma}_j{}^k \quad , \quad \overset{*}{\varepsilon}{}^{ij}u_j = \overset{*}{\varepsilon}{}^{ij}n_j = 0, \tag{4.2}$$

where n_i defines unit magnetic field vector.

By virtue of (3.1), (3.2) and (4.1), we have

$$\overset{*}{\varepsilon}{}^{ij}_{;j} = 2\overset{*}{\omega}{}^{i} - 2u^i n_k \overset{*}{\omega}{}^{k} - \overset{*}{\varepsilon}{}^{ik}(Du_k - D^*n_k), \tag{4.3}$$

where

$$\overset{*}{\omega}{}^{i} = \frac{1}{2}\eta^{ijkl}{}_{u_j \overset{*}{\omega}_{kl}} \quad , \quad \overset{*}{\omega}{}^{i}u_i = 0 \tag{4.5}$$

Using (4.3) in (2.2), we get

$$\mathfrak{L}_u D^i + 3\theta D^i - u^i D^j_{;j} + |h| \overset{*}{\alpha}{}^i - 2u^i h_j \omega^j = J^i, \tag{4.6}$$

where $\mathfrak{L}_u$ denotes Lie differentiation with respect to the flow vector u^i and $\overset{*}{\alpha}{}^i$ is defined by

$$\overset{*}{\alpha}{}^i = 2\overset{*}{\omega}{}^i + \overset{*}{\varepsilon}{}^{ij}\{(\ln|h|)_{,j} + D^* n_j - Du_j\} \tag{4.7}$$

Similarly the counterpart of (4.6) may be deduced from (2.1) in the following form;

$$\mathfrak{L}_u B^i + 3\theta B^i - u^i B^j_{;j} - |e| \widehat{\alpha}{}^i - 2u^i e_j \omega^j = 0, \tag{4.8}$$

where overhead caret ($\wedge$) is used to denote the kinematical parameters associated with the congruence of electric field lines and $\widehat{\alpha}{}^i$ is defined as

$$\widehat{\alpha}{}^i = 2\widehat{\omega}{}^i + \widehat{\varepsilon}{}^{ij}\{(\ln|e|)_{,j} + \widehat{D}a_j - Du_j\}, \tag{4.9}$$

where a_i denotes unit electric field vector.

Splitting up (4.6) orthogonal to u^i and D^i with the help of Greenberg's [*14*] law of transport

$$\widehat{D}u_k = Da_k - (u^i Da_i)u_k + (u^i \widehat{D}a_i)a_k, \tag{4.10}$$

we get

$$\widehat{\gamma}{}^k_i \overset{*}{\alpha}{}^i = 0 \tag{4.11}$$

Similarly (4.8) yields

$$\overset{*}{\gamma}{}^k_i \widehat{\alpha}{}^i = 0 \tag{4.12}$$

Contracting (4.11) with n_k and (4.12) with a_k, we get

$$n_i \overset{*}{\omega}{}^i = 0, \tag{4.13}$$

and

$$a_i \widehat{\omega}{}^i = 0 \tag{4.14}$$

respectively. From (4.5), it is easy to verify that

$$\overset{*}{\omega}{}^i \overset{*}{\omega}_i = -\overset{*}{\omega}{}^2 \quad ; \quad 2\overset{*}{\omega}{}^2 = \overset{*}{\omega}_{ij} \overset{*}{\omega}{}^{ij} \tag{4.15}$$

Thus in view of (4.5), (4.13) and (4.15) we may conclude that there exists a space-like vector $\overset{*}{\omega}{}^i$ orthogonal to the flow vector and magnetic field vector having the magnitude equal to half of the magnitude of the rotation tensor associated with the congruence formed by magnetic field lines. Thus it seems reasonable to call the vector $\overset{*}{\omega}{}^i$ the « magnetic vorticity » vector. Similarly the « electric vorticity » vector will be denoted by $\widehat{\omega}{}^i$.

Using (3.5) in the resulting equation obtained by the contraction of (4.8) with u_i, we get

$$\mathrm{D}^* \mid \mathrm{B} \mid + 2\overset{*}{\theta} \mid \mathrm{B} \mid - 2e_i\omega^i = 0, \tag{4.16}$$

where $\mid \mathrm{B} \mid (> 0)$ is the magnitude of the magnetic induction vector B^i. With the help of definition [*14*]

$$2\overset{*}{\theta} = \mathrm{D}^* \ln \mathrm{A}^*, \tag{4.17}$$

where A^* is the proper area subtented by the magnetic field lines as they pass through the screen in the 2-surface dual to the surface formed by u_i and h_i, (4.16) reduces to

$$\mathrm{D}^* \ln \mid \mathrm{B} \mid \mathrm{A}^* = 2e_i\omega^i/\mid \mathrm{B} \mid, \qquad \mid \mathrm{B} \mid > 0 \tag{4.18}$$

The magnetic fields are said to be « frozen-in » if the magnetic flux through an area bounded by the particles of the fluid remains constant i. e. $\mathrm{D}^* \ln \mid \mathrm{B} \mid \mathrm{A}^* = 0$. Thus in view of (4.18) we state the following theorem:

THEOREM (4.1). — The electric field and vorticity of the fluid are orthogonal when the magnetic fields are « frozen-in ».

Similarly the counterpart of (4.18) can be obtained from (4.6) as follows:

$$\widehat{\mathrm{D}} \ln \mid \mathrm{D} \mid \widehat{\mathrm{A}} + (\varepsilon + 2h_i\omega^i)/\mid \mathrm{D} \mid = 0, \qquad \mid \mathrm{D} \mid > 0 \tag{4.19}$$

The electric fields are said to be « frozen-in » if the electric flux through an area bounded by the particles of the fluid remains constant i. e. $\widehat{\mathrm{D}} \ln \mid \mathrm{D} \mid \widehat{\mathrm{A}} = 0$. When the electric fields are « frozen-in », (4.19) assumes the form

$$2h_i\omega^i = -\varepsilon, \qquad \mid \mathrm{D} \mid > 0 \tag{4.20}$$

which explains the orientation of magnetic field in a charged rotating stars provided the electric fields (due to the presence of charge) are « frozen-in ».

The « steady rigid rotation » is characterized by the conditions

$$\theta = \sigma_{ij} = 0 \tag{4.21}$$

and with respect to a Fermi triad the rate of change of the magnetic and electric induction vectors must vanish i. e. $\gamma_k{}^i\mathrm{DB}^k = \gamma_k^i\mathrm{DD}^k = 0$. Splitting up (4.6) and (4.8) orthogonal to u^i, we get

$$\gamma_k{}^i\mathrm{DD}^k - (\sigma_k{}^i + \omega_k{}^i)\mathrm{D}^k + 2\theta\mathrm{D}^i + \mid h \mid \overset{*}{\alpha}{}^i = \bar{k}e^i \tag{4.22}$$

and

$$\gamma_k{}^i\mathrm{DB}^k - (\sigma_k{}^i + \omega_k{}^i)\mathrm{B}^k + 2\theta\mathrm{B}^i - \mid e \mid \widehat{\alpha}{}^i = 0 \tag{4.23}$$

respectively. Assuming the electromagnetic fluid to be in « steady rigid rotation » one can obtain from (4.22) and (4.23) in view of (4.11) and (4.12) the following result

$$V^k \omega_k = 0 \tag{4.24}$$

which states the following theorem:

THEOREM (4.2). — The electromagnetic energy—flux vector and vorticity of the fluid are orthogonal when the electromagnetic fluid is in « steady rigid rotation ».

When the electromagnetic fluid is in « steady rigid rotation » then (4.23) yields

$$2\bar{k} \mid e \mid \widehat{\omega}^i B_i = \eta^{jlik}{}_{J_l B_i u_k} \{ (\ln \mid e \mid)_{,j} + \widehat{D} a_j - D u_j \} \tag{4.25}$$

which gives the theorem:

THEOREM (4.3). — The « electric vorticity » and magnetic field are orthogonal when the electromagnetic fluid is in « steady rigid rotation » and the electric field is parallel or antiparalled to the magnetic field.

Using (2.4) and (2.5) in the relation [16]

$$\gamma^{ij} R_{jk} u^k = \gamma^i_j \left(\omega^{jk}_{;k} - \sigma^{jk}_{;k} + 2\theta^{,j} \right) + \left(\omega^i_k + \sigma^i_k \right) D u^k, \tag{4.26}$$

we get

$$P^i = \gamma^i_j \left(\omega^{jk}_{;k} - \sigma^{jk}_{;k} + 2\theta^{,j} \right) + \left(\omega^i_k + \sigma^i_k \right) D u^k. \tag{4.27}$$

Contracting (4.27) with ω_i and using theorem (4.2) and kinematical relation (3.2) for $\omega^i = \mid \omega \mid l^i$, where l^i is unit vorticity vector, we get

$$q^i \omega_i = - 2 \mid \omega \mid \underset{\vee}{\omega_i} \omega^i, \tag{4.28}$$

where $\underset{\vee}{\omega_i}$ denotes the rotation of vortex tubes. This explains the orientation of heat—flux tubes in thermodynamically imperfect stars. If the rotation of the vortex tubes happens to be zero, the heat flux tube will be orthogonal to the axis of rotation of a rotating star. Similarly (4.27) yields

$$q^i h_i = - 2 \mid h \mid \overset{*}{\omega_i} \omega^i \tag{4.29}$$

which gives the theorem:

THEOREM (4.4). — The heat-flux tubes and magnetic fields are orthogonal when the magnetic field tubes are parallel or antiparallel to the axis of rotation of the electromagnetic fluid which is in « steady rigid rotation ».

Similarly the counterpart of theorem (4.4) can be stated as follows.

THEOREM (4.5). — The heat flux tubes and electric fields are orthogonal when electric field tubes are parallel or antiparallel to the axis of rotation of the electromagnetic fluid which is in « steady rigid rotation ».

From (4.3), we obtain divergence identity for the « magnetic vorticity » vector,

$$\overset{*}{\omega}{}^{i}_{;i} + \overset{*}{\omega}{}^{i}\mathrm{D}u_i - \overset{*}{\omega}{}^{i}\mathrm{D}^*n_i - (n_k\omega^k u^i)_{;i} + u^k\mathrm{D}^*n_k n_i\omega^i + \frac{1}{2}\overset{*}{\varepsilon}{}^{ki}(\mathrm{D}u_{k;i} - \mathrm{D}^*u_{k;i}) = 0 \quad (4.30)$$

which demands further investigation to understand the behaviour of « magnetic vorticity » vector on the basis of kinematical prameters associated with the congruence formed by « magnetic vorticity » lines. Though this congruence is a space-like congruence but differs from the congruence formed by fluid vortex lines in the sense that the « magnetic vorticity » is orthogonal to the flow lines and magnetic field lines while the fluid vorticity is orthogonal to the flow lines only. With this remark we postpone above discussion at present.

By means of (2.4), (2.5), (3.1), (3.2) and the Ricci identity for space-like vector n^i,

$$n^i_{;ij} - n^i_{;ji} = \mathrm{R}_{ij}n^i, \quad (4.31)$$

we obtain

$$2\mathrm{D}^*\overset{*}{\theta} + \mathrm{D}^*(\mathrm{D}n_k u^k) - 2\overset{*}{\omega}{}^2 + 2\overset{*}{\sigma}{}^2 + 2\overset{*}{\theta}{}^2 - 3\theta^2 + \nu\sigma_{ij}n^i n^j - (\mathrm{D}^*n^i)_{;i} - 2(\sigma_{ij} + \omega_{ij})n^i\mathrm{D}n^j - (n_i\mathrm{D}u^i)^2 + \frac{1}{2}(\rho + \lambda\,|\,e\,|^2 - p - \mu\,|\,h\,|^2) = 0 \quad (4.32)$$

which may be regarded as space-like ⟨ magnetic ⟩ counterpart of Raychaudhuri's [17] equation in the context of electromagnetic fluids.

The electromagnetic energy-flux vector is defined by

$$\mathrm{V}^i = \eta^{ijkl}{}_{e_j h_k u_l}, \quad (4.33)$$

which yields the divergence identity

$$\mathrm{V}^i_{;i} = (\ln|\,\mathrm{V}\,|)_{,i}\mathrm{V}^i + 2\{\,|\,h\,|\,e_i\overset{*}{\omega}{}^i - |\,e\,|\,h_i\widehat{\omega}^i\} + \mathrm{V}^i(\widehat{\mathrm{D}}a_i + \mathrm{D}^*n_i - \mathrm{D}u_i) \quad (4.34)$$

Using (4.34) in the resulting equations obtained by the contraction of (4.6) with e_i and (4.8) with h_i, we get

$$\mathrm{DW} + 4\theta\mathrm{W} + \sigma_{ij}(\lambda e^i e^j + \mu h^i h^j) + \mathrm{V}^i\mathrm{D}u_i - \mathrm{V}^i_{;i} = k\,|\,e\,|^2, \quad (4.35)$$

where

$$W = \frac{1}{2}(\lambda \mid e \mid^2 + \mu \mid h \mid^2).$$

Assuming the conservation of particles number i. e. $(nu^i)_{;i} = 0$, one can express (4.35) as below;

$$\mathfrak{L}_u(n^{-4/3}W) + n^{-4/3}[\sigma_{ij}(\mu h^i h^j + \lambda e^i e^j) + V^i Du_i - V^i_{;i} - \bar{k} \mid e \mid^2] = 0 \quad (4.36)$$

which may be regarded as electromagnetic energy equation which shows that the presence of differential rotation with electromagnetic field causes the electromagnetic energy to be exchanged back and forth for the fluid energy and such process of exchange of energy with its surroundings is responsible for the production of Joule's heat. It is easy to show that $T^{ij}_{;j}u_i = 0$ is equivalent to

$$\overset{*}{D}\rho + 3\theta(\rho + p) + \sigma_{ij}(\lambda e^i e^j + \mu h^i h^j) + 2\theta(\lambda \mid e \mid^2 + \mu \mid h \mid^2) \\ - 2\nu\sigma^2 - P^i Du_i + P^i_{;i} = 0 \quad (4.37)$$

Combining (4.35) and (4.37), we get

$$D\rho + \bar{k} \mid e \mid^2 + 3\theta(\rho + p) - 2\nu\sigma^2 + q^i_{;i} - q^i Du_i = 0 \qquad (4.38)$$

Using (2.9)-(2.13) and the matter conservation law i. e. $(\rho_0 u^i)_{;i} = 0$ in (4.38), we have

$$S^i_{;i} = \frac{1}{T}[2\nu\sigma^2 + \mid q \mid^2/kT - \bar{k} \mid e \mid^2] \qquad (4.39)$$

which shows that the generation of entropy depends upon Joule's heat besides shear viscosity and thermal conductivity of the fluid. The generation of entropy is positive only when the following inequality

$$2\nu\sigma^2 + \mid q \mid^2/kT > \bar{k} \mid e \mid^2$$

is satisfied. The entropy is not constant when the fluid is usually intractable. For example, plasmas are often reactive gases with elaborate equations of state and that heat radiation can be dominant due to high effective heat capacity associated with their chemical reactivity and their isothermal flows. Thus entropy generation should always be positive.

One can show that $h_i T^{ij}_{;j} = 0$ is equivalent to

$$(\overset{*}{\rho} + p^*)h^k Du_k - p^*_{,j}h^j - \{\lambda e_{i;j}h^i e^j + \mu h_{i;j}h^i h^j - \mu \mid h \mid^2 h^j_{;j}\} \\ + \nu\sigma^j_{i;j}h^i + h^i DP_i + 4\theta h^i P_i + (\sigma_{ij} + \omega_{ij})P^i h^j = 0 \quad (4.40)$$

Substituting the resulting equation obtained by the contraction of (4.8) with u_i and using (3.2) and theorem (4.4) in (4.40), we finally obtain

$$(\rho + p + \lambda \mid e \mid^2 h^k \mathrm{D}u_k - \left(p + \frac{1}{2}\lambda \mid e \mid^2\right)_{,j} h^j + \lambda \mid h \mid (\overset{*}{\sigma}_{ij} + \overset{**}{\theta}\gamma_{ij})e^i e^i = 0 \quad (4.41)$$

which yields the theorem:

THEOREM (4.6). — The fluid acceleration and magnetic fields are orthogonal when parallel or antiparallel magnetic field tubes (*) to the vortex tubes and the electromagnetic fluid are in « steady rigid rotation » and the magnetic fields lie in the surfaces of constant partial pressure (isotropic pressure + supplementary pressure due to electric fields).

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(*) The magnetic field tubes are in « steady rigid rotation » i. e.

$$\overset{*}{\sigma}_{ij} = \overset{*}{\theta} = 0.$$