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Topological charges and high dimensional instantons

by

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ABSTRACT. — We discuss topological properties of instantons of the theories with compact non-abelian gauge group. An example of the solution with the group SU(3) is considered. It is shown that the solutions with high topological charges defined by the Pontrjagin classes p_i, $i > 1$, have a non-finite action.

INTRODUCTION

The finite action solutions, i. e. instantons or pseudoparticles, of Jang-Mills theories have been extensively used in the field theory, especially for the 4-dimensional space and the compact gauge group SU(2), i. e. in the situation where they were first found in [*1*] [*2*]. The instantons for the SU(2)-group are described by only one topological invariant or topological charge, i. e. the Pontrjagin number p_1. The solution with $p_1 = 1$ were studied in [*2*]. Then Witten [*3*] and 't Hooft [*4*] found a large class of instanton solutions with the topological charge $p_1 = q$, where q is an integer and Belavin and Zaharov [*26*] found a method to construct a general solution depending on $8q - 3$ parameters. As we gathered M. Atiayh [*27*] reduced the study of SU(2)-instantons to a problem of algebraic geometry.

The SU(2)-instantons do not exhaust all the pseudoparticles interesting for the field theory. Recently Polyakov has proved the confinement hypothesis for the 3-dimensional QED with an abelian compact gauge group (cf. [*1*] and more detailed paper [*10*]) by the method which relies heavily on the computation of the contribution of the pseudoparticles to the functional integral representation of correlation functions. There is an opinion, that the pseudoparticles and some related things like fluctuations play the crucial role for the confinement problem, though for the group SU(N) with the large number of colours N the contribution of the instantons is exponentially small -

$$e^{-\frac{8\pi^2}{g^2}\nu}$$

here ν is a number of instantons, g^2 is the charge [7] [8] [9].

Among other applications of the instantons we want to point out the alleged U(1) problem, where they permit to explain the absence of massless meson [5] [6].

In this paper we want to discuss the possible types of the solutions of Jang-Mills theories from the topological point of view, not restricting ourselves to the confinement problem.

Following the ideas of paper [11], we are studying solutions with non-trivial topological charges. The solutions with charges different from p_1 emerge in the theories with the gauge group SU(3). They turn out to be of infinite action and so far seem to be physically uninteresting, but one ought to keep in mind that the action can be made finite by switching on the additional fields, e. g. the Fermy fields. Indeed, this is the situation for the monopole solutions.

This paper is arranged as follows: § 1 contains some informations on the « low brow » version of the theory of characteristic classes; § 2 is concerned with the topological invariants for the SU(N)-theory; in § 3 we compute an example for the SU(3)-theory.

§ 1

Let us consider a gauge field A_μ respectively to a gauge group G. The components of the field A_μ take the values in the Lie algebra LG of G. We shall consider the general case of the space of dimension n so the index μ may take the values $1, \ldots, n$, $n \geqslant 4$. The strength tensor $F_{\mu\nu}$ respectively to A_μ is of the usual form

$$F_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu + ie[A_\mu ; A_\nu] \tag{1}$$

To make clear the relation of the characteristic classes theory and the gauge formalism of field theory we want to compare the formulae of the two theories. Indeed, the gauge field A_μ is familiar in differential geometry as the linear connection, which is usually written as the differential form

$$\varphi = A_\mu dx^\mu \tag{2}$$

where x^μ, $\mu = 1, \ldots, n$ are the coordinates of a point of space or the local coordinates in the sense of differential geometry (1).

(1) The reader is suggested to understand that the generalisation to more sophisticated manifolds is strightforward. The proofs and all the rest of the theory are contained in [12] [13].

Then the curvature form is defined by the formula

$$\Phi = d\varphi - \frac{1}{2}[\varphi ; \varphi] \tag{3}$$

where $d\varphi$ is the exterior differential of the linear form φ

$$d\varphi = \partial_\rho A_\mu dx^\rho \wedge dx^\mu = \frac{1}{2}(\partial_\nu A_\mu - \partial_\mu A_\nu) dx^\mu \wedge dx^\nu \tag{4}$$

and $d\varphi$ is defined by the formula

$$[\varphi, \varphi] = [A_\mu ; A_\nu] dx^\mu \wedge dx^\nu \tag{5}$$

We see that

$$\Phi = \frac{1}{2} F_{\mu\nu} dx^\mu \wedge dx^\nu$$

where $F_{\mu\nu}$ is the strength tensor (1).

These formulae suggest the close links of the geometrical and the field theory formalisms. To illustrate this general idea let us consider the Lagrangian density of the free gauge field in the Euclidean space of the dimension 4, i. e. we perform the Wick rotation

$$t \to it , \qquad x_l \to x_l , \qquad l = 1, 2, 3$$

changing the real Minkovsky time to the complex one. We have

$$\mathscr{L} = \mathrm{Tr}\{ - F_{\mu\nu}^2 \}$$

$$L = \int d^4x \cdot \mathscr{L} = \int d^4x \cdot \mathrm{Tr}\{ - F_{\mu\nu}^2 \} \tag{6}$$

Now we want to write Eq. (6) with the exterior forms. To this end we introduce the familiar duality operation $*$ which acts on the exterior forms of rank 2 by the formula

$$*(dx^\mu \wedge dx^\nu) = dx^\rho \wedge dx^\gamma = (= \varepsilon_{\mu\nu\rho\gamma} dx^\mu \wedge dx^\nu)$$

i. e. ρ, γ are such that the equation holds

$$dx^\mu \wedge dx^\nu \wedge dx^\rho \wedge dx^\gamma = dx^1 \wedge dx^2 \wedge dx^3 \wedge dx^4$$

The operation $*$ acts trivially on the coefficients of the forms $*f = f$. Then it is easy to verify that the following exterior product satisfies the equation

$$\Phi \wedge \Phi = \frac{1}{4} F_{\mu\nu}^2 dx^1 \wedge \dots \wedge dx^4 = \frac{1}{4} F_{\mu\nu}^2 dV \tag{7}$$

The coefficients of these exterior forms are matrices and the products of the coefficients are the products of matrices. Thus we have for the Lagrangian density

$$\mathscr{L} dV = \mathrm{Tr}\{ \Phi \wedge \Phi^* \} . \tag{8}$$

Formula (8) is true for any gauge group, if the gauge field is defined in the four-dimensional space.

Now we may define the topological charge with the help of exterior differential forms. To this end let us consider the following expression

$$F^*_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\rho\gamma}F_{\rho\gamma}$$

i. e. the tensor dual to the strength tensor $F_{\mu\nu}$. It is clear that

$$F_{\mu\nu}F^*_{\mu\nu}dV = \frac{1}{2}\Phi \wedge \Phi \tag{9}$$

Following [2], we may put the topological charge into the form

$$Q = \int dV \operatorname{Tr} (F_{\mu\nu}F^*_{\mu\nu}) = \frac{1}{2}\int \operatorname{Tr} (\Phi \wedge \Phi) \tag{10}$$

Eq. (10) permits a formulation for the topological charge of the SU(2)-gauge field, which transforms according to the adjoint representation of the group SU(2). To obtain the formula we may use the so-called A. Weil homomorphism (cf. for the details [13]), which defines the characteristic classes as the coefficients of the expansion in λ of the determinant

$$C(\Phi) = \det || \delta^{ij} + \lambda\Phi^{ij} || = 1 + \lambda \operatorname{Tr} \Phi + \ldots + \lambda^N \det \Phi$$

Here the curvature form is considered as a $n \times n$-matrix with the entries being exterior forms of the rank 2, i. e.

$$\Phi = || \Phi^{ij} || = \left\| \frac{1}{2} F^{ij}_{\mu\nu}dx^\mu \wedge dx^\nu \right\| \tag{11}$$

Let us consider the SU(2)-theory, i. e. the gauge fields A are 2×2 complex matrices with the zero trace. Then the curvature form Φ may be considered as 2×2-matrix with the entries being exterior forms of the rank 2. Hence eq. (11) may be written as

$$C(\Phi) = 1 + \lambda \operatorname{Tr} \Phi + \lambda^2 \det \Phi = 1 + \lambda^2 \det \Phi$$

We have used the equation

$$\operatorname{Tr} \Phi = 0, \qquad \operatorname{Tr} (A) = 0$$

since Tr (A) is a linear combination of matrices with zero trace:

$$F^{ii}_{\mu\nu} = \partial_\nu A^{ii}_\mu - \partial_\mu A^{ii}_\nu + ie[A_\mu ; A_\nu]^{ii} = 0$$

To compute det Φ we note that the equations

$$F^{11} + F^{22} = 0, \qquad F^{12} - F^{21} = 0.$$

are verified in the Lie algebra of SU(2). Hence we obtain

$$\begin{aligned}\det \Phi &= \Phi^{11}\Phi^{22} - \Phi^{21}\Phi^{12} \\ &= \frac{1}{4}\left[F^{11}_{\mu\nu}F^{22}_{\rho\gamma} - F^{21}_{\mu\nu}F^{12}_{\rho\gamma}\right]dx^{\mu} \wedge dx^{\nu} \wedge dx^{\rho} \wedge dx^{\gamma} \\ &= -\frac{1}{2}\Phi \wedge \Phi \end{aligned} \tag{12}$$

Thus we have obtained Eq. (9) for the SU(2)-topological charge.

Here we want to note that there exist two types of topological charges. The first one is pertinent to the theory of characteristic classes and is defined by exterior differential forms. We have just described it. The second one is defined by the homotopy class of a map of a sphere of an infinite radius into a factor space G/H where H is a subgroup of the gauge groupe G [*2*] [*11*] [*24*]

$$f : S^{n-1} \to G/H \tag{13}$$

i. e. the topological charge of the second type is defined by an element of the homotopy group $\pi_{n-1}(G/H)$. This situation happens in the problem of monopole solutions of Polyakov-'t Hooft, where H is a stationary subgroup of the vacuum vector. In some cases both types of the topological charges coincide, e. g. this happens when H is a subgroup consisting of only one element and the map defines the assymptotic conditions at infinity for the instanton problem of paper [*2*]. If we are interested in the gauge group SU(N), $N \geqslant 2$, and the space of dimension $n = 4$, then the condition at infinity provides us with the set of topological invariants, which are elements of the homotopy group $\pi_3(SU(N)) = Z$ where Z is the additive group of integers. For any integer considered as an element of $\pi_3(SU(N))$, there exists a finite-action solution of some SU(N)-Jang-Mills theory (cf. below § 2).

§ 2

Recently the instanton solutions have been studied in a number of papers, where they considered various types of compact semi-simple Lie groups as the gauge groups for the needs of elementary particles physics and gravitation [*14*] [*15*] [*16*] [*17*]. The embeddings of SU(2) into SU(N) have proved interesting for the elementary particles theory. There is a considerable progress in the field, the necessary mathematics being provided by the theory of simple subgroups of Lie groups [*18*]. We note, that the numbers of different vacuums for the SU(2)- and SU(N)-theories coincide since there exists an isomorphism [*19*],

$$\pi_3(SU(2)) \approx \pi_3(SU(N)), \qquad N \geqslant 2 \tag{14}$$

induced by the embedding of SU(2) into SU(N) (the so called stabilization theorem [19]). Therefore some physical effects, e. g. the nonconservation of chiral charge, the tunneling between different instanton states, are retained in SU(N)-theories, just as it is in the SU(2)-theory.

From the embedding of SU(2) into SU(N) and isomorphism (14) follows that for any value of the topological charge q belonging to $\pi_3(\mathrm{SU(N)})$ there exists a Jang-Mills theory, which has finite action solutions with the topological charge equal to q. Indeed, we may consider a SU(2)-Jang-Mills theory, take an imbedding of SU(2) into SU(N) and then reconsider our theory as a SU(N)-theory. Since in both theories the charge is defined by the map

$$f : \mathrm{S}^{n-1} \to \mathrm{SU(2)},\ \mathrm{SU(N)}$$

of the assymptotic values at infinity we have the coincidence of the charges by the stabilization theorem, but we know that in the SU(2)-theory any charge exists [3, 4].

In the gravitation theory there is the gauge group

$$\mathrm{SO(4)} \sim \mathrm{SO(3)} \times \mathrm{SO(3)}\,; \qquad \pi_3(\mathrm{SO(4)}) \approx \mathrm{Z} \oplus \mathrm{Z}$$

The 3-rd homotopy group of SO(4), $\pi_3(\mathrm{SO(4)})$, is isomorph to two copies of the additive group of integers

$$\pi_3(\mathrm{SO(4)}) \approx \pi_3(\mathrm{SO(3)}) \oplus \pi_3(\mathrm{SO(3)}) \approx \mathrm{Z} \oplus \mathrm{Z}$$

Hence we have two topological charges, i. e. the Euler class and the Pontrjagin class

$$\chi = \int \mathrm{R}^{\gamma\delta}_{\alpha\beta}\mathrm{R}^{\tau\nu}_{\rho\sigma}\varepsilon^{\alpha\beta\rho\sigma}\varepsilon_{\gamma\delta\tau\nu}\sqrt{g}\, d^4x$$

$$p_1 = \int \mathrm{R}_{\mu\nu\alpha\beta}\mathrm{R}^{\mu\nu}_{\gamma\delta}\varepsilon^{\alpha\beta\gamma\delta}\sqrt{g}\, d^4x$$

In paper [14] F. Wilczek asked a question, whether there is any relations between the possible values of χ and p_1, e. g. is it possible that $\chi = 0$ and $p_1 = 1$. For the gauge fields on the 4-dimensional sphere, which under some circumstances can provide the physical space for the problem, there exists the theorem

$$p_1 \quad \text{is always even}$$

$$\text{if} \quad \chi = 0\,, \quad \text{then} \quad p_1 = 4k$$

But the gravitation theory requires that χ and p_1 must be the classes of the tangent bundle of the physical space. Therefore the requirements on χ and p_1 put restrictions on the topological structure of the manifold of the physical space. Indeed, if the Pontrjagin class of a 4-dimensional manifold M^4 is not equal to zero then the second homology group of this manifold is not trivial. Whether manifolds with sophisticated topological structure are suitable for the theory of gravitation remains to be seen.

§ 3

In this context it is interesting to try to find topological invariants which stem from the topological structure of gauge groups SU(N), $N \geqslant 3$. The first invariant of this type is the 3-rd Chern class, C_3. Let us consider the case of SU(3). Then C_3 is defined by the formula

$$\det \| \Phi^{ij} \| = \det \| F^{ij}_{\mu\nu} dx^\mu \wedge dx^\nu \| = \varepsilon^{i_1 i_2 i_3}_{j_1 j_2 j_3} F^{i_1 j_1}_{\mu_1 \nu_1} F^{i_2 j_2}_{\mu_2 \nu_2} F^{i_3 j_3}_{\mu_3 \nu_3} dx^{\mu_1} \wedge dx^{\nu_1} \wedge \ldots \wedge dx^{\nu_3} \quad (15)$$

We see that C_3 is defined by an exterior differential form of rank 6. The 3-rd Chern number, $[C_3]$, is an integral of this form over tha space $x^1, \ldots, x^6$ and equals to

$$[C_3] = \int_{R^n} C_3$$

This number to be non-zero we need the dimension of the space be equal to 6. This follows from Eq. (15) being a symmetric polynomial of the 3-rd degree in the components of F^{ij}. Another corollary of Eq. (15) is that the topological charge $[C_3]$ cannot provide the lower bounds for the estimate of the energy since the energy density is a symmetric polynomial of the 2-nd degree (cf. [*23*]).

Now we want to consider a simple example of a SU(3)-theory with the non-trivial topological charge. To solve the equations for the gauge fields we need a symmetry condition on our problem. A condition of the kind is the existence of an embedding of the gauge group into the group of the space symmetries. This embedding couples space-like symmetries with internal symmetries of the gauge group. Because we need the 6-dimensional space for our problem we may take the space of the $D(0, 2) = 6$ representation of SU(3), i. e. symmetric matrices, and consider its real part, which is of dimension 6, as the space of our problem. Our theory being the static one we shall omit the time variable. Then the gauge field is a tensor

$$A^a_k, \qquad a = 1, \ldots, 8; \qquad k = 1, \ldots, 6 \qquad (16)$$

belonging to the space of the tensor product $8 \otimes \bar{6}$, where $8 = D(1, 1)$ is the adjoint representation of SU(3). We have the familia formula

$$8 \otimes \bar{6} = D(1, 0) \oplus D(0, 2) \oplus D(2, 1) \oplus D(1, 3) \qquad (17)$$

It will be convinient to write the space coordinates as matrix elements of a symmetric 3×3 matrix

$$z_{ij} = z_{ji}, \qquad i, j = 1, 2, 3$$

Formulae (16) and (17) suggest the following.

Ansatz:

$$A^a_{ij} = \frac{1}{2}(\lambda^a_{iu} z_{uj} + \lambda^a_{ju} z_{ui}) \cdot h(r); \qquad r = \sqrt{z^2_{uv}} \qquad (18)$$

where λ^a_{iu}, λ^a_{ju} are the entries of Gell-Man's λ-matrices and the double lower indices are due to the space coordinates. The Ansatz of this type have been made in a number of papers [*20*] [*21*] [*22*]. The particular form of our Ansatz is determined by the spherical SU(3)-symmetry of our solution.

On substituting Eq. (18) for the gauge field into the equation for the field tensor

$$F^a_{ij,kl} = A^a_{ij,kl} - A^a_{kl,ij} + ef_{abc}A^b_{ij}A^c_{kl} \tag{19}$$

we obtain

$$F^a_{ij,kl} = U^a_{ij,kl}\dot{h} + V^a_{ij,kl}\frac{h}{2r} + W^a_{ij,kl}\cdot h^2 \tag{20}$$

where

$$U^a_{ij,kl} = \frac{z_{kl}}{r}\cdot\frac{1}{2r}\left(\lambda^a_{iu}z_{uj} + \lambda^a_{ju}z_{ui}\right) - \frac{z_{ij}}{r}\cdot\frac{1}{2r}\left(\lambda^a_{ku}z_{ul} + \lambda^a_{lu}z_{uk}\right)$$

$$V^a_{ij,kl} = \left(\lambda^a_{iu}\delta^{kl}_{uj} + \lambda^a_{ju}\delta^{kl}_{ui}\right) - \left(\lambda^a_{ku}\delta^{ij}_{ul} + \lambda^a_{lu}\delta^{ij}_{uk}\right)$$
$$- \left[\frac{z_{kl}}{r}\cdot\frac{1}{2r}\left(\lambda^a_{iu}z_{uj} + \lambda^a_{ju}z_{ui}\right) - \frac{z_{ij}}{r}\frac{1}{2r}\left(\lambda^a_{ku}z_{ul} + \lambda^a_{lu}z_{uk}\right)\right]$$

$$W^a_{ij,kl} = \frac{1}{2r}ef_{abc}\left(\lambda^b_{iu}z_{uj} + \lambda^b_{ju}z_{ui}\right)\cdot\frac{1}{2r}\left(\lambda^c_{kv}z_{vl} + \lambda^c_{lv}z_{vk}\right)$$

after some computations we obtain the Lagrangian density satisfying the Ansatz in the form

$$L = \int_0^{+\infty} dr\cdot\mathscr{L},$$

$$\mathscr{L} = a_1r^5\cdot\dot{h}^2 + a_2r^4h\dot{h} + a_3r^5h^2\dot{h} + a_4r^3h^2 + a_5r^4h^3 + a_6r^5h^4, \tag{21}$$

$$a_1 = \int_{S^5} |\,U^a\,|^2(\Pi d\alpha) \qquad a_4 = \frac{1}{4}\int_5 |\,V\,|^2(\Pi d\alpha)$$

$$a_2 = \int_{S^5} \mathrm{Re}\,(U\overset{*}{V})\,(\Pi d\alpha) \qquad a_5 = \int |\,W\,|^2(\Pi d\alpha),$$

$$a_3 = 2\int_{S^5} \mathrm{Re}\,(U\overset{*}{W})\,(\Pi d\alpha) \qquad a_6 = \int_{S^5} |\,W\,|^2(\Pi d\alpha)$$

The Euler equations for functional (21) can be written as

$$\ddot{h} + \frac{5}{r}\dot{h} + \frac{\alpha_1}{r^2}h + \frac{\alpha_2}{r}h^2 - \alpha_3h^3 = 0 \tag{22}$$

$$\alpha_1 = \frac{2a_2 - a_4}{a_1}, \qquad \alpha_2 = \frac{5a_3 - 3a_5}{2a_1}, \qquad \alpha_3 = \frac{2a_6}{a_1}$$

$$\alpha_3 > 0, \qquad \text{since} \qquad a_1, a_6 > 0.$$

Note, that $\alpha_3 > 0$ since $a_1, a_6 > 0$.

The bounded solutions of Eq. (22), when $r \to \infty$ can be found from the equation

$$\ddot{h} - \alpha_3 h^3 = 0 \tag{23}$$

(cf. the phase picture of this equation on Fig. 1). To find the solution decreasing as $r \to \infty$ we may search it in the form of a series in powers of r^{-1}

$$h = \frac{b_1}{r} + \frac{b_2}{r^2} + \dots \tag{24}$$

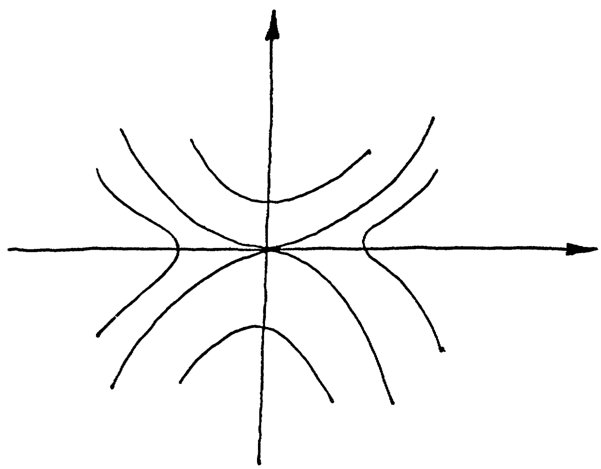

FIG. 1.

We may restrict ourselves only to the first term which has the coefficient b_1 equal to

$$b_1 = \frac{5a_3 - 3a_5 \mp \sqrt{(5a_3 - 3a_5)^2 + 32a_6(3a_1 - 2a_2 + a_4)}}{4a_6}$$

Thus we see that the solution decreasing as $r \to \infty$ has the assymptotic r^{-1}. Therefore the integral of the energy diverges as r^2 and the topological charge as $\ln r$.

Here we want to note that formulae like (15) are rigorously applied only to compact manifolds. In the field theory they write them also for the non-compact ones the solutions of the field equations decreasing sufficiently rapidly. In the present situation a way can be suggested to get a reasonable value for the topological charge by applying the conformal transformation of the 6-dimensional sphere S^6 onto the Euclidean space R^6

$$f : S^6 \to R^6$$

Then we obtain an exterior differential form on the compact manifold S^6, which is the image of the Chern class C_3 by the transformation. Now the integral

$$\int_{S^6} f^*C_3$$

is taken over the compact manifold S^6. The value of this integral is defined as the topological charge of the problem.

CONCLUSIONS

Finally we want to make the following remarks:

(1) The existence of finite-action solutions of Jang-Mills theories with the gauge group SU(N) has purely topological nature and is based on the dimension 4 of the physical space;

(2) The asymptotic conditions at infinity play the crucial role for the existence of such solutions. It is interesting to consider the problem of condition at infinity directly in the Minkovsky space. To this end it is possible to apply the following construction. Let us consider the domain

$$V = \{ y_1^2 - y_2^2 - y_3^2 - y_4^2 > 0,\ y_1 > 0 \}$$

The following domain is an analogue of the upper complex semi-plane

$$\mathscr{D} = \{ Z = X + iY \,;\, X \in R^4,\ Y \in V \}$$

This domain can be put into the form of a bounded domain Ω in $\mathbb{C}^4$. The conformal group SO(4, 2) is transitive on $\Omega \approx \mathscr{D}$ and

$$\mathscr{D} \sim SO(4, 2)/SO(4) \times SO(2)$$

The Bergmann-Shilov (see of [*25*]) boundary of $\mathscr{D}$ is the set S. We have

$$S \sim SO(4) \times SO(2)/SO(3) \sim S^3 \times S^1$$

This boundary can be viewed upon as the set of end points of geodesics starting in the interior of $\mathscr{D}$.

Arguments based on the conformal invariance show that the asymptotic conditions at infinity provide the topological charges, which are determined by the homotopy classes of maps of the manifold $S = S^1 \times S^3$ into the gauge group G. It is not hard to see, that these classes constitute the homotopy group $\pi_3(G)$ and therefore we have only one topological charge for this problem.

(3) To obtain additional topological charges the dimension of physical space must be taken bigger than 4. It is likely to have such situation, when the coordinates describing interior degrees of freedom are included in the physical space. Then a possible analogue of the space R^4 is a manifold $\tilde{R} = R^4 \times M$. Topological properties of such fields would be quite sophisticated.

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When this paper was finished we received the preprint by C. J. Isham « Topological currents for arbitrary groups in $\mathbf{R}^3$ », where he studies homotopical and homological properties of topological charges of 3-dimensional chiral theories, i. e. the connections between $\pi_3(M)$ and $H^3(M)$. The results of our papers do not intersect, but we share Dr. Isham's expectations of topological methods.

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