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Section A :
Physique théorique.

Absolute bounds on vertices and couplings

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ABSTRACT. — We prove absolute upper and/or lower bounds on φ^4 and φ^6 dimensionless vertices and physical coupling constants.

1. DEFINITIONS AND ASSUMPTIONS

We derive absolute bounds on φ^4 and φ^6 dimensionless coupling constants g. For instance, in a pure φ^4 model we prove

$$0 \leq g \leq \text{const.}, \tag{1.1}$$

in the single phase region (no symmetry breaking). We also obtain bounds on associated vertex functions and on connected parts. Our general methods presumably have other consequences for n-particle amplitudes, $n \geq 8$, which we do not pursue here.

DEFINITIONS. — Let

$$\langle 1 \ldots n \rangle \equiv \langle \Phi(x_1) \ldots \Phi(x_n) \rangle \tag{1.2}$$

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denote the n-point (Euclidean) Schwinger function, and let $\langle 1 \dots n \rangle_T$ denote its connected part. We let

$$(1.3) \quad S_T^{(n)}(p) = S_T^{(n)}(p_1, \dots, p_{n-1}) = \int \langle 1 \dots n \rangle_T \exp\left[i \sum_{j=1}^{n=1} p_j x_j\right] dx_1 \dots dx_{n-1} \Big|_{x_n=0}$$

denote the Fourier transform of $\langle 1 \dots n \rangle_T$. We encounter $S_T^{(2)}(p)$ often, and denote it $\chi(p)$. We use the standard spectral representation

$$(1.4) \qquad \chi(p) = S_T^{(2)}(p) = \int \frac{d\rho(a)}{p^2 + a},$$

where $d\rho(a)$ is the spectral measure. Furthermore, we let

$$(1.5) \qquad \Gamma^{(n)}(p) = \Gamma^{(n)}(p_1, \dots, p_{n-1})$$

denote the n-point Euclidean vertex function (thus $\Gamma^{(n)}$ is the amputated, one particle irreducible part of $S_T^{(n)}(p)$). In an even theory, we have for example,

$$(1.6) \qquad \Gamma^{(4)}(p) = S_T^{(4)}(p) \prod_{i=1}^{4} \chi(p_i)^{-1},$$

where $p_4 = -(p_1 + p_2 + p_3)$. Also

$$(1.7) \quad \Gamma^{(6)}(p) = \left[S_T^{(6)}(p) - \sum_X S_T^{(4)}(X)\chi(P_X)^{-1} S_T^{(4)}(\sim X)\right] \prod_{i=1}^{6} \chi(p_i)^{-1},$$

where $p_1 + \dots + p_6 = 0$, where $(X, \sim X)$ is a partition of $(p_1, \dots, p_6)$ into two subsets of three elements, and where P_X is the sum of the momenta in X.

We let $d \leq 4$ denote the space-time dimension and let m denote the mass gap. Note that $\Gamma^{(n)}$ has the dimension of mass to the power $d - \frac{1}{2}n(d-2)$. We define the dimensionless amplitude $g^{(n)}(p)$ by

$$(1.8) \qquad g^{(n)}(p) = -m^{-d-n+\frac{1}{2}nd} \Gamma^{(n)}(p),$$

and the associated coupling constant $g^{(n)}$ by

$$(1.9) \qquad g^{(n)} = g^{(n)}(0).$$

For example,

$$(1.10) \qquad g^{(4)} = -m^{d-4} \chi^{-4} \int \langle 1234 \rangle_T dx_1 dx_2 dx_3,$$

where $\chi \equiv \chi(0)$.

ASSUMPTIONS. — We make some or all of the following assumptions. In the case of weakly coupled $\mathscr{P}(\varphi)_2$, assumptions (*a*)-(*f*) have been proved, while for single phase, even φ_2^4 models (*h*)-(*i*) have been proved. We expect (*a*)-(*f*) to hold for single phase φ_3^4, φ_3^6, and φ_4^4 models, while (*h*)-(*i*) are expected for all single phase even φ_d^4 models, $d \leq 4$. The evidence for the validity of (1.4) in φ_4^4 comes from perturbation theory. The proof of (*i*) in even, single phase φ_2^4 has been established by Cartier (private communication).

(*a*) The Wightman axioms for a scalar boson field, or the Osterwalder-Schrader axioms for the corresponding Euclidean theory. The two point function $\chi(p)$ satisfies (1.4). In particular we are assuming that $\int \frac{d\rho(a)}{a}$ is finite.

(*b*) A positive mass gap, $m > 0$.

(*c*) $\langle \varphi \rangle = 0$.

(*d*) The second Griffiths inequality,

$$\langle A_1 A_2 \rangle \geq \langle A_1 \rangle \langle A_2 \rangle,$$

where (A_1, A_2) is a partition of $(1, \ldots, n)$. For instance,

$$(1.11) \quad -\langle 1234 \rangle_T = -\{\langle 1234 \rangle - \langle 14 \rangle\langle 23 \rangle - \langle 12 \rangle\langle 34 \rangle - \langle 13 \rangle\langle 24 \rangle\} \leq \langle 12 \rangle\langle 34 \rangle + \langle 13 \rangle\langle 24 \rangle.$$

(*f*) The measure $d\rho(a)$ contains a delta function at $a = m^2$, of unit strength (This is a renormalization hypothesis).

(*h*) In the case of a φ^4 interaction, it is appropriate to assume the Lebowitz inequality $\langle 1234 \rangle_T \leq 0$, or the set of Lebowitz inequalities,

$$(1.12) \quad \langle 12U \rangle \leq \langle 12 \rangle \langle U \rangle + \sum_W \langle W, 1 \rangle \langle \sim W, 2 \rangle$$

where W is an (odd) subset of U.

(*i*) In the case of a pure φ^4 model, we assume

$$(1.13) \quad \langle 123456 \rangle_T \geq 0.$$

2. BOUNDS ON QUARTIC COUPLINGS

THEOREM 2.1.

(*i*) Assume (*a*)-(*d*) above. Then

$$(2.1) \quad g^{(4)} \leq \text{const.}\, \chi^{-2} m^{-4}.$$

(*ii*) Assume (*f*) also, then $\chi \geq m^{-2}$, so

$$g^{(4)} \leq \text{const.} \tag{2.2}$$

(*iii*) Assume (*h*) also, then

$$0 \leq g^{(4)} \leq \text{const.}\, \chi^{-2} m^{-4} \leq \text{const.} \tag{2.3}$$

Remark 1. — The constants are pure numbers, independent of all parameters. They can be determined explicitly by the proof.

Remark 2. — The upper bound on $g^{(4)}$ is related to the picture of « critical point dominance » discussed in [*1*]. According to this picture, for $d < 4$, $g^{(4)}$ assumes its maximum value at the critical point, *i. e.* at the onset of phase transitions. According to standard ideas, $m = 0$ at the critical point.

Remark 3. — For $d < 4$, the limit λ_0 (= bare φ^4 coupling constant) $\to \infty$ with m fixed plays a role in the application of the Callan-Symanzik equations to the study of critical phenomena [*2*]. We use Theorem 2.1 below, combined with bounds from [*3*], to show that either this limit exists, or that in this limit $g^{(4)} \to 0$ (see Corollary 2.4). If $g^{(4)}$ is monotone in the dimensionless charge $\lambda_0 m_0^{d-4}$, then the alternative $g^{(4)} \to 0$ is excluded. The possibility that $g^{(4)}$ is monotone is discussed in [*1*], and is a consequence of conventional assumptions made in the study of the Callan-Symanzik equations.

Remark 4. — Other bounds on quartic couplings have been established by Lukaszuk and Martin [*4*], see also Healy [*5*]. Their bounds require different assumptions and methods, and yield different conclusions.

In the case of a pure φ^4 model we also obtain bounds on $g^{(4)}(p)$ for $p \neq 0$.

THEOREM 2.2. — Assume (*a*)-(*h*) above. There exists $\delta > 0$, such that if $|p_i| < \delta m$, then $g^{(4)}(p)$ is analytic in p and bounded uniformly by

$$|g^{(4)}(p)| \leq \text{const.}\, \chi^{-2} m^{-4} \leq \text{const.} \tag{2.4}$$

Proof of Theorem 2.1. — By the positivity of $d\rho$ (Wightman positivity condition)

$$\langle 12 \rangle \geq 0, \tag{2.5}$$

so the right side of (1.11) is positive. By symmetry we then have

$$\begin{aligned} -\langle 1234 \rangle_T &\leq (\langle 12 \rangle \langle 34 \rangle + \langle 13 \rangle \langle 24 \rangle)^{\frac{1}{2}} (\langle 14 \rangle \langle 23 \rangle \\ &\quad + \langle 12 \rangle \langle 34 \rangle)^{\frac{1}{2}} (\langle 13 \rangle \langle 24 \rangle + \langle 14 \rangle \langle 23 \rangle)^{\frac{1}{2}} \\ &= \left(\sum_{i=1}^{8} A_i \right)^{\frac{1}{2}} \leq 2 \sup A_i^{\frac{1}{2}}. \end{aligned} \tag{2.6}$$

Here each A_i is either

$$\langle 12 \rangle \langle 13 \rangle \langle 14 \rangle \langle 23 \rangle \langle 24 \rangle \langle 34 \rangle = \prod_{1 \le i < j \le 4} (ij)$$

or else a permutation of

$$\langle 12 \rangle^2 \langle 34 \rangle^2 \langle 13 \rangle \langle 24 \rangle .$$

We bound each A_i using Proposition A2.1, of Appendix 2,

$$\langle x0 \rangle \le O(1) \mid x \mid^{-d_e - m(1-\varepsilon)|x|} \chi .$$

An elementary estimate shows that each $A_i^{\frac{1}{2}}$ is integrable over $dx_1 dx_2 dx_3$. By homogeneity, and (A2.1),

$$\int A_i^{\frac{1}{2}} dx_1 dx_2 dx_3 \le O(1) m^{-d} \chi^2 ,$$

from which we conclude

$$-S_T^{(4)}(0) \le O(1) m^{-d} \chi^2 . \tag{2.7}$$

By (1.8), with $n = 4$, and (1.6) we have (2.1). The theorem then follows.

Proof of Theorem 2.2. — Assuming (*h*) and (2.6),

$$| \langle 1234 \rangle_T | \le 2 \sup_i A_i^{\frac{1}{2}} . \tag{2.8}$$

We bound the power series coefficients of $S_T^{(4)}(p)$, expanding about $p = 0$. The derivatives $\partial / \partial p_j$ become multiplication by ix_j in the Fourier transform, and are dominated by the exponential decrease of (2.8), estimated by (A2.1). Thus the power series in momenta converges for $| p_i | < m/3$, and establishes the analyticity and boundedness of $S_T^{(4)}(p)$,

$$| S_T^{(4)}(p) | \le O(1) m^{-d} \chi^2 . \tag{2.9}$$

Finally, the analyticity and boundedness of $\chi(p)^{-1}$ follows from (1.4), for $| p | < m$, yielding

$$| \chi(p) |^{-1} \le 2\chi(0)^{-1} .$$

Thus $g^{(4)}(p)$ is analytic in p and

$$| g^{(4)}(p) | \le O(1) m^{-d} \chi^{-2} \le O(1) m^{-d+4} ,$$

with the constant independent of p, m, λ for $| p_i | \le \delta m$. This completes the proof. We have also proved.

COROLLARY 2.3. — Assuming (*a*)-(*h*), $S_T^{(4)}(p)$ is analytic for $| p_i | \le \delta m$ and bounded uniformly by (2.9).

We now let $S_j^{(n)}$ be the n-point Schwinger function for a model satisfying (*a*)-(*h*) and labelled by the index j (for instance, $S^{(n)}_{\lambda_0,m}$ may be the φ_2^4 model, labelled by the bare coupling λ_0 and mass m).

COROLLARY 2.4. — Assume m_j is bounded away from zero. Then either there exists a convergent subsequence of Schwinger functions

$$S^{(n)} = \lim_i S_j^{(n)}, \tag{2.10}$$

with $S^{(n)}$ satisfying (*a*)-(*h*), or else $g_T^{(4)}(p) \to 0$.

Proof. — Assume for some p, with $|p_i| \leq \delta m$, there is a subsequence of $g_r(p)$ bounded away from zero. Then by (2.4)

$$\chi_r^2 \leq \text{const. } m_r^{-4}(g_r^{(4)}(p))^{-1} \leq \text{const.}$$

This bound (uniform in r) on

$$\int \langle 12 \rangle_{T,r} dx_1 = \int \langle 12 \rangle_r dx_1 \leq \text{const.}$$

proves a uniform bound

$$|\langle fg \rangle| \leq |f|_{\mathcal{S}} |g|_{\mathcal{S}} \tag{2.11}$$

for a suitable Schwartz space norm $| \, . \, |_{\mathcal{S}}$. Thus by Theorem 1 of [*3*], a convergent subsequence of Schwinger functions $S_{j_i}^{(n)}$ exists, and $S^{(n)}$ is a tempered distribution. The properties (*a*)-(*h*) follow from the corresponding properties in the approximate theories.

Conversely, if $g_j(p) \to 0$ for each p with $|p_i| < \delta m$, the uniform bound (2.4) ensures that the family of analytic functions $g_j(p)$ converges to the analytic function zero.

3. BOUNDS ON $S_T^{(6)}(p)$

In this section we prove preliminary bounds on S_T^6.

THEOREM 3.1.

(*i*) Assume (*a*)-(*h*). Then there exists $\delta > 0$ such that if $|p_i| \leq m\delta$, then $S_T^{(6)}(p)$ is analytic and bounded by

$$m^{2d}\chi^{-3} |S_T^{(6)}(p)| \leq \text{const.} \tag{3.1}$$

(*ii*) Assume (*a*)-(*i*). Then

$$0 \leq m^{2d}\chi^{-3} S_T^{(6)}(0) \leq \text{const.} \tag{3.2}$$

The constants are pure numbers, independent of the parameters.

Proof. — We consider $p = 0$. The extension to $p \neq 0$ follows the proof of Theorem 2.2. Using (f), we have

$$\langle 123456 \rangle - \langle 12 \rangle \langle 3456 \rangle \geq 0. \tag{3.3}$$

The definition of the six point connected part is

$$\langle 123456 \rangle_T = \langle 123456 \rangle - \sum_Y \langle Y_1 \rangle \langle Y_2 \rangle + 2 \sum_Z \langle Z_1 \rangle \langle Z_2 \rangle \langle Z_3 \rangle. \tag{3.4}$$

Here $Y = (Y_1, Y_2)$ ranges over the fifteen partitions of $(1, 2, \ldots, 6)$ into two and four element subsets, while $Z = (Z_1, Z_2, Z_3)$ ranges over the fifteen partitions of $(1, \ldots, 6)$ into three pairs. Thus

$$- \langle 123456 \rangle_T \leq \sum_Y{}' \langle Y_1 \rangle \langle Y_2 \rangle - 2 \sum_Z \langle Z_1 \rangle \langle Z_2 \rangle \langle Z_3 \rangle. \tag{3.5}$$

where Σ' omits the partition $\langle 12 \rangle \langle 3456 \rangle$. We use (h) and the definition of $\langle 1234 \rangle_T$ to obtain

$$- \langle 123456 \rangle_T \leq \sum_Z{}' \langle Z_1 \rangle \langle Z_2 \rangle \langle Z_3 \rangle, \tag{3.6}$$

where Σ' omits the three partitions which contain the factor $\langle Z_1 \rangle = \langle 12 \rangle$. By symmetry,

$$- \langle 123456 \rangle_T \leq 12 \sup \prod_{j=1}^{15} \prod_{i=1}^{3} \langle Z_i^{(j)} \rangle^{1/15}, \tag{3.7}$$

where $j = 1, 2, \ldots, 15$ labels the two element subsets of $(1, \ldots, 6)$, and $(Z_1^{(j)}, Z_2^{(j)}, Z_3^{(j)})$ one of the twelve partitions of $(1, \ldots, 6)$ into three pairs, no one of which is j.

We now follow the proof of Theorem 2.1 to obtain

$$- S_T^{(6)}(0) \leq \text{const. } m^{-2d} \chi^3.$$

proving one side of (3.1).

We next use (1.12) (see [3]) with $U = (3456)$ to obtain

$$\langle 123456 \rangle \leq \langle 12 \rangle \langle 3456 \rangle + \sum_W \langle W, 1 \rangle \langle \sim W, 2 \rangle,$$

where W ranges over the eight odd subsets of U. Thus by (3.4),

$$\langle 123456 \rangle_T \leq - \sum_Y{}'' \langle Y_1 \rangle \langle Y_2 \rangle + 2 \sum_Z \langle Z_1 \rangle \langle Z_2 \rangle \langle Z_3 \rangle$$

where Σ'' ranges over the six partitions containing both 1 and 2 in the four element subset. By the definition of $\langle 1234 \rangle_T$,

$$\langle 123456 \rangle_T \leq - \sum_{Y}'' \langle Y_1 \rangle_T \langle Y_2 \rangle_T + \sum_{Z}' \langle Z_1 \rangle \langle Z_2 \rangle \langle Z_3 \rangle .$$

By (1.11) we note that

$$\langle 123456 \rangle_T \leq 2 \sum_{Z}' \langle Z_1 \rangle \langle Z_2 \rangle \langle Z_3 \rangle ,$$

which is just a bound by twice the right hand side of (3.6). We now proceed as above to obtain the bound (3.2) for $p = 0$.

Finally, we remark that assuming (*i*) yields $0 \leq S_T^{(6)}(0)$, to complete the proof.

4. BOUNDS ON $g^{(6)}$

We use the bounds of Chapter 3 to establish bounds on the six point vertex function in the φ^4 model.

THEOREM 4.1. — Assuming (*a*)-(*h*), for $| p_i | \leq m\delta$, $g^{(6)}(p)$ is analytic and bounded by

$$| g^{(6)}(p) | \leq \text{const.}$$

where δ is the smaller of the constants given by Theorems 2.2 and 3.1.

Proof. — By Theorem 3.1, $m^{2d}\chi^{-3}S_T^{(6)}(p)$ is analytic and bounded. By Theorem 2.2 $g^{(4)}(p)$ is analytic and bounded. Note

$$g^{(4)}(p) = m^{d-4} \prod_{i=1}^{4} \chi(p_i)^{-1} S_T^{(4)}(p).$$

The desired bound follows by

$$g^{(6)}(p) = - m^{2d-6} \Gamma^{(6)}(p)$$

and the representation (1.7).

APPENDIX 1

THE FREE EUCLIDEAN PROPAGATOR

We establish the elementary bounds on

$$c(m^2 ; x) = \frac{1}{\pi} \int \frac{e^{-ipx} dp}{p^2 + m^2} = m^{d-2} c(1 ; mx)$$

used below. By Euclidean invariance, we may evaluate c for $x = x_t = (t, \vec{0}), t > 0$, and

$$f(t) = c(1 ; x_t) = \int \frac{e^{-t\mu}}{\mu} d\vec{p}:$$

where $\mu = \mu(\vec{p}) = (\vec{p}^2 + 1)^{\frac{1}{2}}$.

PROPOSITION A1.1. — There is a constant O(1) depending only on d, such that

$$f(t) \leq \begin{cases} O(1) t^{(1-d)/2} e^{-t} & t \geq \frac{1}{2} \\ O(1) t^{-(d-2)} & t \leq \frac{1}{2}, d \geq 3 \\ O(1) \mid \ln t \mid & t \leq \frac{1}{2}, d = 2 \end{cases}$$

REMARK. — The long and short range exponents of t differ.

Proof. — Let $\mu_t(\vec{p}) = (\vec{p}^2 + t^2)^{\frac{1}{2}}$, so

$$f(t) = t^{-(d-2)} e^{-t} \mathrm{I}(t)$$

where

$$\mathrm{I}(t) = \int \mu_1^{-1} \exp\left[-(\mu_t - t)\right] d\vec{p}.$$

For $t \geq \frac{1}{2}$, and $t \geq \mid \vec{p} \mid$, we use

$$\mu_t - t \leq \text{const.}\ \vec{p}^2 t^{-1}$$

to establish

$$\mathrm{I}(t) \leq O(t^{(d-3)/2}).$$

Furthermore, for $\frac{1}{2} \leq t \leq \mid \vec{p} \mid$, we use $\mu_t - t \geq \text{const.} \mid \vec{p} \mid$ to give

$$\mathrm{I}(t) \leq O(t^{-1}).$$

Thus for $t \geq \frac{1}{2}$, we have established the proposition.

For $t \leq \frac{1}{2}$, and $d \geq 3$, we have

$$\mathrm{I}(t) \leq \int \mid \vec{p} \mid^{-1} \exp\left[-\mid \vec{p} \mid + \frac{1}{2}\right] d\vec{p} \leq O(1).$$

To bound the case $d = 2$, with $t \leq \frac{1}{2}$, we use

$$\mathrm{I}(t) \leq \text{const.} \int_0^\infty e^{-u} \frac{du}{(t^2 + u^2)^{\frac{1}{2}}} \leq O(\mid \ln t \mid).$$

APPENDIX 2

THE EUCLIDEAN PROPAGATOR

The Euclidean propagator $\langle xy \rangle$ has the spectral representation given by (1.4),

$$\langle xy \rangle = \langle \Phi(x)\Phi(y) \rangle = \int d\rho(a)c(a\,;\,x-y).$$

PROPOSITION A2.1. — Assume (*a*)-(*c*) above, and let $\varepsilon > 0$. Then

$$\langle x0 \rangle \leq O(1) \mid x \mid^{-d} e^{-m(1-\varepsilon)|x|}\chi, \tag{A2.1}$$

where O(1) depends only on the dimension d and on ε.

PROOF. — We use proposition A1.1. Let $d \geq 3$. Then

$$\begin{aligned}\langle x0 \rangle &= \int d\rho(a)a^{(d-2)/2}c(1\,;\,\sqrt{a}x). \\ &\leq O(1)\int \frac{d\rho(a)}{a}\, a \mid x \mid^{-d+2} e^{-\sqrt{a}|x|(1-\varepsilon/2)} \\ &\leq O(1)e^{-m(1-\varepsilon)|x|}\left(\int \frac{d\rho(a)}{a}\right) \mid x \mid^{-d+2} \left(\sup_a ae^{-\sqrt{a}|x|\varepsilon/2}\right).\end{aligned}$$

The sup occurs for $a = 4(\varepsilon x)^{-2}$. Thus (A2.1) follows. For $d = 2$,

$$\begin{aligned}\langle x0 \rangle &= \int d\rho(a)c(1\,;\,\sqrt{a}x) \\ &\leq O(1)\int_{\mathrm{I}} \frac{d\rho(a)}{a}\, a \mid \ln\sqrt{a}x \mid + O(1)\int_{\mathrm{II}} \frac{d\rho(a)}{a}\, ae^{-\sqrt{a}|x|},\end{aligned}$$

where

$$\mathrm{I} = \left\{ a : m \leq \sqrt{a} \leq \mid 2x \mid^{-1} \right\} \quad \text{and} \quad \mathrm{II} = \left\{ a : \sqrt{a} \geq \mid 2x \mid^{-1} \right\}.$$

The integral over II is bounded as above, by (A2.1). The integral over I is bounded using

$$\sup_{a\in \mathrm{I}} a \left| \ln \sqrt{a} \mid x \mid \right| \leq \mid x \mid^{-2} \sup_{0\leq u\leq 1} u^2 \mid \ln u \mid \leq O(1) \mid x \mid^{-2}.$$

This yields the bound $O(1) \mid x \mid^{-2}\chi$. Since $\mid mx \mid \leq \frac{1}{2}$, for $a \in \mathrm{I}$, the integral over I is bounded by (A2.1).

REMARK. — The increase in the bound on the short range singularities (from $x^{(2-d)}$ to x^{-d}) is associated with an allowed divergence of the field strength renormalization integral

$$Z^{-1} = \int d\rho(a).$$

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