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Pure radiation fields in general relativity

by

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A METHOD FOR GENERATING PURE RADIATION FIELDS FROM EMPTY GRAVITATIONAL FIELDS IS GIVEN

The object of this investigation is to obtain a method for generating pure radiation fields from empty gravitational fields. Such methods were obtained earlier by Ehlers [1], Bonnor [2] and the present author [3] in other cases.

For this purpose we consider the three metrics

$$(1) \quad ds^2 = e^{2\sigma} d^*s^2 - \bar{e}^{2\sigma} (dx^3)^2,$$

$$(2) \quad d\bar{s}^2 = e^{2W} d^*s^2 - \bar{e}^{2W} (dx^3)^2,$$

$$(3) \quad d^*s^2 = *g_{ij} dx^i dx^j$$

$$(i, j, \dots = 1, 2, 4; \alpha, \beta, \dots = 1, 2, 3, 4)$$

where σ , W and $*g_{ij}$ are functions of x^i only.

Hereafter quantities defined with respect to (2) and (3) will be denoted by an overhead bar or an asterisk respectively.

The Ricci tensor for the metric (1) is given by

$$(4) \quad R_{ij} = *R_{ij} + 2\sigma_i \sigma_j + *g_{ij} * \Delta_2 \sigma$$

$$(5) \quad R_{33} = \bar{e}^{4\sigma} * \Delta_2 \sigma,$$

$$(6) \quad R_{3i} = 0,$$

where

$$(7) \quad \sigma_i = \frac{\partial \sigma}{\partial x^i},$$

and ${}^*\Delta_2$ is the Beltrami differential parameter of the second kind.

The empty space-time field equations for the metric (1) are therefore

$$(8) \quad {}^*\mathbf{R}_{ij} + 2\sigma_i\sigma_j = 0,$$

$$(9) \quad {}^*\Delta_2\sigma = 0,$$

and since (9) follows from (8) in view of the contracted Bianchi identities, we are left with only equations (8) for determining σ and ${}^*g_{ij}$.

Now the Ricci tensor for the metric (2) is given by

$$(10) \quad \bar{\mathbf{R}}_{ij} = {}^*\mathbf{R}_{ij} + 2W_iW_j + {}^*g_{ij}{}^*\Delta_2W,$$

$$(11) \quad \bar{\mathbf{R}}_{33} = e^{4W}{}^*\Delta_2W,$$

$$(12) \quad \bar{\mathbf{R}}_{3i} = 0,$$

where

$$(13) \quad W_i = \frac{\partial W}{\partial x_i}.$$

If we assume that W is a function of σ so that the level surfaces of W and σ coincide, then

$$(14) \quad W_i = W'\sigma_i, \quad W_{i;j} = W'\sigma_{i;j} + W''\sigma_i\sigma_j, \dots$$

where an overhead dash denotes ordinary differentiation with respect to σ and a semicolon followed by a lower index denotes covariant differentiation with respect to the metric (3).

From equations (11) and (14) we get

$$(15) \quad \bar{\mathbf{R}}_{33} = e^{4W}W''{}^*g^{ij}\sigma_i\sigma_j,$$

in view of (9). And if we assume that σ_i is a null vector then

$$(16) \quad \bar{\mathbf{R}}_{33} = 0.$$

Also from (10), (14), (8) and (9) and the fact that σ_i is a null vector we get

$$(17) \quad \bar{\mathbf{R}}_{ij} = 2(W'^2 - 1)\sigma_i\sigma_j.$$

Thus the Ricci tensor for the metric (2), in view of (12), (16) and (17), can be written as

$$(18) \quad \bar{\mathbf{R}}_{\alpha\beta} = \theta\sigma_\alpha\sigma_\beta,$$

where

$$(19) \quad \theta = 2(W^{12} - 1),$$

and $\sigma\beta$ is a null vector.

Now equations (18) are precisely the field equations for the unidirectional flow of pure radiation for the metric (2). Hence we have arrived at the following result:

For every solution of the empty space-time field equations corresponding to the metric (1) a solution of the field equations for the unidirectional flow of pure radiation, i. e. (18), is given by the metric (2) where W is an arbitrary function of σ and the gradient of σ is null vector.

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