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Covariant star-products^(*)

MOHSEN MASMOUDI⁽¹⁾

RÉSUMÉ. — On donne une démonstration élémentaire du théorème d'existence de produits-star sur les variétés symplectiques.

On montre l'existence de produit-star covariant sur les orbites coadjointes admettant une polarisation réelle.

ABSTRACT. — We give a direct and elementary proof of the well known existence theorem of $\star$ -products on a symplectic manifold.

Looking for covariant $\star$ -product on coadjoint orbits, we prove the existence of such a deformation when the orbit admits a real polarization.

1. Introduction

$\star$ -products were defined in [1] by Flato, Fronsdal, Lichnerowicz as a tool for quantizing a classical system, described with a symplectic manifold (M, ω) . Roughly speaking, a $\star$ -product is a (formal) deformation of the associative algebra $C^\infty(M)$ provided with usual (pointwise) product starting with the Poisson bracket. The quantum structure is then the deformed structure on the unchanged space of observables.

Each quantization procedure, when applied on a coadjoint orbit M of a Lie group G , gives some way to build up unitary irreducible representations of G . To use $\star$ -products for such a purpose, we need in fact a particular property, the covariance of the $\star$ -product:

$$[\tilde{X}, \tilde{Y}]_\star = \{\tilde{X}, \tilde{Y}\} = [\widetilde{X}, \widetilde{Y}],$$

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if $\tilde{X}$ is, for each X in the Lie algebra $\mathfrak{g}$ of G , the function on M defined by

$$x \mapsto \tilde{X}(x) = \langle x, X \rangle.$$

(See [2] for a discussion on invariance and covariance properties for $\star$ -products on a coadjoint orbit.)

In this paper, we recall first the theorem of existence of $\star$ -products on a symplectic manifold. This theorem is due to P. Lecomte and M. de Wilde [3]. Some recent new proofs were given by Maeda, Omori and Yoshioka [4] and Lecomte and de Wilde [5]. We expose here that last proof in a slightly different way, which is direct and totally elementary: we build a $\star$ -product by gluing together local $\star$ -products defined on domains of a chart of M . That proof follows the idea of Vey, Lichnerowicz, Neroslavsky and Vlassov ([6], [7]) and, of course, Maeda, Onori and Yoshioka. In these approaches, the obstruction to construct $\star$ -product lies in the third cohomology group $H^3(M)$ of the manifold M . Lecomte and de Wilde defined formal deformation of the Lie algebra $(C^\infty(M), \{\cdot, \cdot\})$, for such a deformation, the obstruction is in the group $H^3(C^\infty(M))$ for the adjoint action, which contains strictly $H^3(M)$. Let us finally mention the construction of Maslov and Karasev [8] who found an obstruction in $H^2(M)$ to construct simultaneously a deformation and a representation of the deformed structure on $C^\infty(M)$. Lecomte and de Wilde proved that all these obstructions can be surrounded, with the use of local conformal vector fields on M . We follow here that classical proof, using only local computations and Čech calculus.

Then we use this proof in the case of a coadjoint orbit in the dual $\mathfrak{g}^*$ of a Lie algebra $\mathfrak{g}$. More precisely, we consider a point x_0 in $\mathfrak{g}^*$ and suppose there exists in x_0 a real polarization. Under this assumption, we prove the existence of a covariant $\star$ -product on the coadjoint orbit of x_0 , endowed with its canonical symplectic structure.

2. Existence of $\star$ -products on a symplectic manifold

Let (M, ω) be a symplectic manifold. We denote by $\{\cdot, \cdot\}$ the Poisson bracket defined on $C^\infty(M)$ by the usual relations:

$$\{u, v\} = X_u v \quad \text{if} \quad i_{X_u} \omega = -du.$$

A $\star$ -product is by definition a formal deformation in the sense of Gerstenhaber [9] of the associative algebra $C^\infty(M)$, i.e. a bilinear map:

$$C^\infty(M) \times C^\infty(M) \longrightarrow C^\infty(M)[[\nu]], \quad (u, v) \mapsto v \star u = \sum_{r \geq 0} \nu^r C_r(u, v),$$

where $C^\infty(M)[[\nu]]$ is the space of formal power series in the variable ν with coefficients in $C^\infty(M)$, such that each C_r is a bidifferential operator and:

- (i) $C_0(u, v) = uv$, $C_1(u, v) = \{u, v\}$,
- (ii) $C_r(u, v) = (-1)^r C_r(v, u)$,
- (iii) $C_r(1, u) = 0, \forall r > 0$,
- (iv) $\sum_{r+s=t} C_r(C_s(u, v), w) = \sum_{r+s=t} C_r(u, C_s(u, w)), \forall t \geq 0$.

With these properties, $\star$ defines an associative structure on $C^\infty(M)[[\nu]]$, of whom unity is 1 and:

$$[u, v]_\star = \sum_{r \geq 0} \nu^{2r} C_{2r+1}(u, v) = \frac{1}{2\nu} (u \star v - v \star u)$$

is a Lie bracket (it satisfies Jacobi identity) and a formal deformation of the Poisson bracket.

On a symplectic vector space $\mathbb{R}^{2n}$ and thus on any domain U of a canonical chart in M ; there exists $\star$ -products, for instance the Moyal $\star$ -product [1].

THEOREM [3]. — *On each symplectic manifold (M, ω) , there exists a $\star$ -product.*

Proof. — Let us first choose a locally finite covering $(U_\alpha)_{\alpha \in A}$ of the manifold such that each U_α is the domain of a canonical chart on M and all the intersections:

$$U_{\alpha_1 \dots \alpha_n} = U_{\alpha_1} \cap \dots \cap U_{\alpha_n}$$

are contractible. We fix a total ordering $\leq$ on A and a partition of the unity ψ_α subordinated to $(U_\alpha)_{\alpha \in A}$. If $\star_\alpha$ is a $\star$ -product on U_α and $\text{Der}(\star_\alpha)$ the space of derivation of $\star_\alpha$, there exists a canonical linear mapping:

$$\Phi_\alpha : F(U_\alpha) = C^\infty(U_\alpha) / \{\text{constants}\} \longrightarrow \text{Der}(\star_\alpha), \quad \Phi_\alpha([f])(v) = [f, v]_\alpha.$$

Let us denote by $\text{Con } f(U_\alpha)$ the space of (conformal) vector fields ξ_α on U_α such that:

$$L_{\xi_\alpha} \omega = \omega \quad \text{on } U_\alpha.$$

$\text{Con } f(U_\alpha)$ is an affine space on $F(U_\alpha)$.

Now we suppose, by induction on k , to have, on each U_α , a $\star$ -product $\star_\alpha$:

$$u \star_\alpha v = \sum_{r \geq 0} \nu^r C_{r,\alpha}(u, v),$$

with $C_{r,\alpha} = C_{r,\beta}$ on $U_{\alpha\beta}$, for all $r < 2k$ and an affine mapping D_α from $\text{Con } f(U_\alpha)$ into $\text{Der}(\star_\alpha)(D_\alpha(\xi_\alpha + X_f)) = D_\alpha(\xi_\alpha) + \Phi_\alpha([f])$ such that:

$$D_\alpha(\xi_\alpha) = \nu \partial_\nu + L_{\xi_\alpha} + \sum_{r > 0} \nu^{2r} D_\alpha^{2r}(\xi_\alpha),$$

the $D_\alpha^{2r}(\xi_\alpha)$ being differential operators, vanishing on constants. Of course, these assumptions hold for $k = 1$.

Now it is well known ([6], [7]) that for each $\alpha < \beta$, we can find a differential operator vanishing on constants $H_{\alpha\beta}$ such that, up to order $2k + 2$,

$$u \star'_\alpha v = \exp \nu^{2k} H_{\alpha\beta} (\exp -\nu^{2k} H_{\alpha\beta} u \star_\alpha \exp -\nu^{2k} H_{\alpha\beta} v)$$

coincide with $u \star_\beta v$. Thus:

$$\begin{aligned} (D_\alpha - D_\beta)(\xi_{\alpha\beta}) &= \\ &= \sum_{r=0}^{k-1} \nu^{2r} \Phi_\alpha([f_{\alpha\beta}^{2r}]) + \nu^{2k} \left(L_{\xi_{\alpha\beta}} H_{\alpha\beta} + 2k H_{\alpha\beta} + \Phi_\alpha([g_{\alpha\beta}(\xi_{\alpha\beta})]) \right) \end{aligned}$$

here $[f_{\alpha\beta}^{2r}]$ in $F(U_{\alpha\beta})$ do not depend of $\xi_{\alpha\beta}$ while:

$$[g_{\alpha\beta}(\xi_{\alpha\beta} + X_f)] = [g_{\alpha\beta}(\xi_{\alpha\beta}) + H_{\alpha\beta} f].$$

For each $\alpha < \beta$, we choose a vector field $\xi_{\alpha\beta}$ in $\text{Con } f(U_{\alpha\beta})$, a C^∞ function $g_{\alpha\beta}(\xi_{\alpha\beta})$ and put for any $[f]$ in $F(U_{\alpha\beta})$:

$$\begin{aligned} g_{\alpha\beta}(\xi_{\alpha\beta} + X_f) &= g_{\alpha\beta}(\xi_{\alpha\beta}) + H_{\alpha\beta} f, \\ f_{\alpha\alpha}^{2r} &= 0, \quad f_{\beta\alpha}^{2r} = -f_{\alpha\beta}^{2r}, \\ g_{\alpha\alpha} &= 0, \quad g_{\beta\alpha} = -g_{\alpha\beta}. \end{aligned}$$

Now, on $U_{\alpha\beta\gamma}$ ($\alpha < \beta < \gamma$), $H_{\alpha\beta\gamma}$ ($= H_{\alpha\beta} + H_{\beta\gamma} + H_{\gamma\alpha}$) can be written as $\Phi([h_{\alpha\beta\gamma}])$. The problem is to choose simultaneously the C^∞ functions $h_{\alpha\beta\gamma}$. As in [3], we choose the unique C^∞ solution of all the equations:

$$L_{\xi_{\alpha\beta\gamma}} h_{\alpha\beta\gamma} + (2k - 1)h_{\alpha\beta\gamma} = -g_{\alpha\beta\gamma}(\xi_{\alpha\beta\gamma}),$$

for each $\xi_{\alpha\beta\gamma}$ in $\text{Con } f(U_{\alpha\beta\gamma})$. $h_{\alpha\beta\gamma}$ is totally antisymmetric in α, β, γ and $h_{\alpha\beta\gamma} - h_{\alpha\beta\delta} + h_{\alpha\gamma\delta} - h_{\beta\gamma\delta}$ vanishes on $U_{\alpha\beta\gamma\delta}$. We define then:

$$s_{\alpha\beta} = \sum_{\gamma} h_{\alpha\beta\gamma} \psi_{\gamma} \quad \text{in } C^\infty(U_{\alpha\beta}),$$

$$G_{\alpha\beta} = H_{\alpha\beta} - \{s_{\alpha\beta}, \cdot\},$$

$$K_{\alpha} = \sum_{\beta} G_{\alpha\beta} \psi_{\beta}.$$

$G_{\alpha\beta\gamma}$ vanishes on $C^\infty(U_{\alpha\beta\gamma})$, K_{α} is well defined and, for each α ,

$$\begin{aligned} u \star'_{\alpha} v &= \exp \nu^{2k} K_{\alpha} (\exp -\nu^{2k} K_{\alpha} u \star_{\alpha} \exp -\nu^{2k} K_{\alpha} v), \\ D'_{\alpha}(\xi_{\alpha}) &= \exp \nu^{2k} K_{\alpha} \circ D_{\alpha}(\xi_{\alpha}) \circ \exp -\nu^{2k} K_{\alpha} \end{aligned}$$

satisfy the induction hypothesis at order $2k + 2$.

If the second Čech cohomology group of M vanishes, there exists a global conformal vector field ξ on M and a global derivation $D'(\xi)$ of the $\star$ -product therefore we refind here the proof of [11]. In the general case, our proof by building directly a $\star$ -product does not need the theorem of [6] which allows to construct $\star$ -product, starting with particular deformation of the Poisson bracket.

3. Parametrization of coadjoint orbits

Let G be a connected and simply connected Lie group, $\mathfrak{g}$ its Lie algebra and $\mathfrak{g}^*$ the dual of $\mathfrak{g}$. G acts on $\mathfrak{g}^*$ by the coadjoint action, denoted here by:

$$\langle g \cdot x, X \rangle = \langle x, \text{Ad } g^{-1}(X) \rangle, \quad \forall X \in \mathfrak{g}, \quad \forall x \in \mathfrak{g}^*, \quad \forall g \in G.$$

Let x_0 be a point of $\mathfrak{g}^*$ and M its coadjoint orbit $G \cdot x_0$, endowed with the canonical 2-form:

$$\omega_x(X^-, Y^-) = \langle x, [X, Y] \rangle (= B_x(X, Y)), \quad \forall X, Y \in \mathfrak{g},$$

here X^- is the vector field defined on M by:

$$X^- f(x) = \frac{d}{dt} f(\exp -tX \cdot x) \Big|_{t=0}.$$

From now on, we suppose there exists a real polarization $\mathfrak{h}$ in x_0 . This means $\mathfrak{h}$ is a maximal isotropic subspace in $\mathfrak{g}$ for the bilinear form B_{x_0} , $\mathfrak{h}$ is a subalgebra of $\mathfrak{g}$ and, if $G(x_0)$ is the stabilizer of x_0 , $\text{Ad } G_{x_0}(\mathfrak{h}) \subset \mathfrak{h}$.

If H_0 is the analytic subgroup of G , with Lie algebra $\mathfrak{h}$, we denote by H the subgroup $G(x_0)H_0$ of G . Then M becomes a fibre bundle over G/H :

$$\pi : M = G/G(x_0) \longrightarrow G/H.$$

In this part, we recall the results of Pedersen [10].

Let $\mathcal{E}^0$ be the subspace $\pi_*(C^\infty(G/H))$. It is an abelian subalgebra of $(C^\infty(M), \{\cdot, \cdot\})$. Let $\mathcal{E}^1$ be the algebra:

$$\mathcal{E}^1 = \{u \in C^\infty(M) \text{ such that } \{u, \mathcal{E}^0\} \subset \mathcal{E}^0\}.$$

For each open subset V in G/H , we define $\mathcal{E}^0(V)$ as $\pi_*(C^\infty(V))$ and $\mathcal{E}^1(V)$ as

$$\left\{ u \in C^\infty(\pi^{-1}(V)) \text{ such that } \{u, \mathcal{E}^0(V)\} \subset \mathcal{E}^0(V) \right\}.$$

The space $\mathcal{E}^1$ is sometimes called the space of quantizable functions. It is easy to verify that the functions $\tilde{X}$, for X in $\mathfrak{g}$ are in $\mathcal{E}^1$. Now let $\mathfrak{m}$ be a supplementary space of $\mathfrak{h}$ in $\mathfrak{g}$ and V be a sufficiently small neighborhood in G/H such that $\mathfrak{m}$ is a supplementary space of $\text{Ad } g\mathfrak{h}$ for each g in G such that $g \cdot x_0$ belongs to V . Pedersen proved that, if $(X_1, \dots, X_k)$ is a basis of $\mathfrak{m}$, then, on $\pi^{-1}(V)$, we can write each function u of $\mathcal{E}^1(V)$ in the form:

$$u|_{\pi^{-1}(V)} = \left(\sum_{i=1}^n \alpha_i \tilde{X}_i + \alpha_0 \right) \Big|_{\pi^{-1}(V)},$$

where the α_i are in $\mathcal{E}_0(V)$. Moreover, the α_i are uniquely determined on $\pi^{-1}(V)$ by that relation. Now we define a "local" induced representation. First there exists a local character χ of H : let $\mathcal{V}$ be a neighborhood of 0 in $\mathfrak{h}$ such that $\exp$ is a diffeomorphism on $\mathcal{V}$, we put:

$$\chi(\exp X) = e^{i\langle x_0, X \rangle} \quad \text{if } X \in \mathcal{V}.$$

Then, if $\mathcal{U}$ is a neighborhood of 0 in $\mathfrak{m}$ and $\mathcal{V}$, $\mathcal{U}$ sufficiently small, the neighborhood $\mathcal{G} = \exp(\mathcal{U})\exp(\mathcal{V})$ of unity in G is diffeomorphic to $\mathcal{U} \times \mathcal{V}$, we choose V to be $\exp(\mathcal{U})H$ and define the local representation (E, ρ) by:

$$E = \{ \phi \in C^\infty \text{ such that } \phi(xh) = \chi(h)^{-1}\phi(x) \text{ if } h \in \exp(\mathcal{V}), x, xh \in \mathcal{G} \}$$

and

$$(\rho(a)\phi)(x) = \phi(a^{-1}x) \quad \text{if } a, x, a^{-1}x \in \mathcal{G}.$$

Of course, we can identify E with $C^\infty(V)$ by putting, for each f in $C^\infty(V)$,

$$\phi(xh) = \chi(h)^{-1}f(xH) \quad \text{if } x \in \exp(\mathcal{U}), h \in \exp(\mathcal{V}).$$

Generally, ρ cannot be extended to a representation of G . But infinitesimally,

$$d\rho(X)\phi(x) = \frac{d}{dt} (\rho(\exp tX)\phi)(x) \Big|_{t=0}$$

is a representation of $\mathfrak{g}$ on the space $C^\infty(V)$. Moreover, by construction, the $d\rho(X)$ are differential operators of order 1 on V , we write:

$$d\rho(X) \in \text{Diff}^1(V).$$

Finally, we call U the set $\pi^{-1}(V)$ and define a map δ from $\mathcal{E}^1(V)$ to $\text{Diff}^1(V)$ by:

$$\delta(u) = \delta \left(\sum_{i=1}^k \tilde{X}_i + \alpha_0 \right) = \sum_{i=1}^k \alpha_i d\rho(X_i) + \alpha_0.$$

THEOREM [10]. — δ is an isomorphism of Lie algebras between $\mathcal{E}^1(V)$ and $\text{Diff}^1(V)$.

The proof of this in [10], indeed, it is a direct consequence of the fact that $d\rho$ is a representation. Now, we define canonical coordinates on U : let $(y_1, \dots, y_k)$ be a coordinate system on V in G/H , we define:

$$q_i = \pi_* y_i, \quad p_i = \delta^{-1}(\partial_{y_i}).$$

(p_i, q_i) is a canonical system of coordinates on U , the q_i belong to $\mathcal{E}^0(V)$ and the p_i to $\mathcal{E}^1(V)$. Then by construction, we have the following theorem.

THEOREM .— *On the intersection of two such chart U and U' , the coordinates satisfy:*

$$q'_i = Q_i(q), \quad p'_i = \sum_{j=1}^k \alpha_{ij}(q)p_j + \alpha_{i0}(q). \quad (*)$$

Endowed with that atlas, M is an open subset of an affine bundle L over G/H , whose transition functions are defined by the relations $()$.*

Remarks

The functions $\tilde{X}$ being in $\mathcal{E}^1$, they have the following form in our coordinate system:

$$\tilde{X} = \sum_{i=1}^k \alpha_i(q)p_i + \alpha_0(q).$$

If $\mathfrak{h}$ satisfies the Pukanszky condition, then M is exactly the bundle L .

4. Construction of covariant $\star$ -product

We consider now our orbit M as an open submanifold of the fibre bundle $\pi : L \rightarrow G/H$. L is canonically polarized with the tangent spaces $T_x L_x$ of its fibres L_x . Then we build up a $\star$ -product on L as in the second part. We still denote by $\mathcal{E}^0$ (resp. $\mathcal{E}^0(V)$) the space $\pi_*(C^\infty(G/H))$ (resp. $\pi_*(C^\infty(V))$). Moreover, we choose our canonical charts with domain $\pi^{-1}(V_\alpha)$ where V_α is one of the local domains of chart defined in the third part and the partition of unity ψ_α subordinated to U_α in $\mathcal{E}^0$. Finally, we add to our induction hypothesis that, for each α , $C_{r,\alpha}$ vanishes on $\mathcal{E}^1(V_\alpha)$ for $r > 2$ and $C_{2,\alpha}(\mathcal{E}^1(V_\alpha), \mathcal{E}^1(V_\alpha)) \subset \mathcal{E}^0(V_\alpha)$.

If we choose the (p, q) coordinates of the preceeding part on our neighborhood U_α and begin with Moyal product with these coordinates, then for $k = 1$, the induction hypothesis holds.

Now, it is not very difficult to choose $H_{\alpha\beta}$ such that $H_{\alpha\beta}$ vanishes on $\mathcal{E}^1(V_{\alpha\beta})$ (we choose first $H'_{\alpha\beta}$ such that $H'_{\alpha\beta}$ vanishes on $\mathcal{E}^1(V_{\alpha\beta})$, then we prove the existence of a C^∞ function $\varphi_{\alpha\beta}$ such that $H_{\alpha\beta} = H'_{\alpha\beta} + \partial\varphi_{\alpha\beta}$ vanishes on $\mathcal{E}^1(V_{\alpha\beta})$). With that choice, $H_{\alpha\beta\gamma}$ is a Hamiltonian vector field vanishing on $\mathcal{E}^1(V_{\alpha\beta\gamma})$ so it is identically zero. Hence we can construct directly the family $(K_\alpha)_{\alpha \in A}$.

Now, because K_α vanishes on $\mathcal{E}^1(V_{\alpha\beta})$, our induction hypothesis is still true for $\star'_\alpha$. In this way, we obtain a $\star$ -product on L , after restriction to M , we have a covariant $\star$ -product on M , since each $\tilde{X}$ is in $\mathcal{E}^1$.

THEOREM . — *Let x_0 be an element in the dual $\mathfrak{g}^*$ of a Lie algebra $\mathfrak{g}$ such that there exists in x_0 a real polarization $\mathfrak{h}$. Then on the coadjoint orbit M of x_0 , there exists a covariant $\star$ -product.*

Let us recall [2] that for each covariant $\star$ -product on M , there exists a representation of G into the group of automorphisms of $(C^\infty(M)[[\nu]], \star)$, which is a deformation of the geometric action of G on M and $C^\infty(M)$.

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