Winter Braids Lecture Notes

Participants & Programme
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Participants & Programme

This volume of *Winter Braids Lecture Notes* contains the lecture notes for the four mini-courses given at Winter Braids XI, which took place in Dijon, from December 13th to 16th, 2021. We are grateful to Luis Paris, who did an amazing job as local organizer for this edition of the Winter Braids school.

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Abstracts of Courses

Gwénaël Massuyeau (Dijon)
Surgery equivalence relations for 3-manifolds

By classical results of Rochlin, Thom, Wallace and Lickorish, it is well-known that any two 3-manifolds (with diffeomorphic boundaries) are related one to the other by surgeries. In these lectures, we will present and compare several families of non-trivial equivalence relations on 3-manifolds, which are easily defined by restricting the type of the surgeries. We shall explain how certain filtrations of mapping class groups of surfaces (such as the lower central series of the Torelli subgroup) enter into the definitions of these surgery equivalence relations, and how concrete invariants of 3-manifolds (such as finite-type invariants) turn out to characterize such relations.

Arunima Ray (Bonn)

Slice knots and knot concordance

The goal of these lectures is to provide an introduction to slice knots and the equivalence relation of concordance, under which the set of knots forms the knot concordance group. Beyond the basic definitions, topics will include the effect of satellite operations on sliceness and concordance; detecting structure within the knot concordance group, for example, via the lens of metrics and filtrations; as well as relations with problems in 3- and 4-manifold topology, including the existence of exotic $\mathbb{R}^4$s. Open problems, big and small, will be mentioned throughout.

Ramanujan Santharoubane (Paris Saclay)
Applications of quantum representations of mapping class groups

This mini-course is an introduction to the Witten-Reshetikhin-Turaev representations of mapping class groups. We will show how to build these representations from Skein Theory. We will then describe its important properties such as its preserved Hermitian structure, integrability or the asymptotic faithfulness. The third part will focus on applications to geometric group theory via some induced surface groups representations. Lastly, we will talk about open problems regarding arithmetic groups or a conjecture from Ivanov. The mini course does not require any knowledge in quantum topology.

Special Lecture

Ivan Marin (Amiens)
Hyperplane arrangements : an introduction

Abstracts of Short Talks

Rhea Palak Bakshi (Zürich)
Kauffman bracket skein module of the connected sum of two solid tori

Skein modules were introduced by Jzef H. Przytycki as generalisations of the Jones and HOMFLYPT polynomial link invariants in the 3-sphere to arbitrary 3-manifolds. The Kauffman bracket skein module (KBSM) is the most extensively studied of all. However, computing the KBSM of a 3-manifold is known to be notoriously hard, especially over the ring of Laurent polynomials. With the goal of finding a definite structure of the KBSM over this ring, several conjectures and theorems were stated over the years for KBSMs. We show that some of these conjectures, and even theorems, are not true. In this talk I will briefly discuss a counterexample to Marche's generalisation of Witten's conjecture. I will show that a theorem stated by Przytycki in 1999 about the KBSM of the connected sum of two handlebodies does not hold. I will also give the exact structure of the KBSM of the connected sum of two solid tori.

Jorge Becerra (Groningen)
Solvable quantum invariants via perturbed Gaussian expressions

Quantum invariants of knots have become now a classical theme. Yet one of their main drawbacks is that the number of terms involved in computations grows exponentially in the complexity of the know. In this talk I would like to present a new strategy to tackle universal knot invariants that reduces the complexity of computations to polynomial time while remaining sufficiently powerful to capture topological features. The technique, developed by D. Bar-Natan and R. van der Veen, has its strength in turning the structure maps of a ribbon Hopf algebra into power series depending on a formal parameter ε where part of the series is encoded in the form of a Gaussian expression (ie a particular exponential). This approach allows to recover the Alexander polynomial for $\varepsilon = 0$ and gives a genus bound sharper than the Alexander bound for $\varepsilon^2 = 0$.

Giulio Belletti (Paris Saclay)
The maximum volume of hyperbolic polyhedra

In this talk I will discuss the problem of determining the maximum volume achieved by generalized hyperbolic polyhedra with a fixed 1-skeleton.

Boris Colombari (Marseille)
A diagrammatic characterization of Milnor invariants

The goal of this talk is to present some classification results on the wq-concordance over welded string links and welded links, which are a combinatorial extension of classical string links and links. These results can be seen as a welded version of the classification of clasper-concordance on classical links established by J. Conant, R. Schneiderman and P. Teichner. Introduced by J-B. Meilhan and A. Yasuhara, the notion of wq-concordance is indeed as a welded analogue of the notion of C_q-concordance on classical links.
I will show that welded string links (resp. welded links) are classified up to wq concordance by their Milnor invariants (resp. by their q-nilpotent peripheral system). This will provide, in turn, a combinatorial characterization of classical links having equivalent q-nilpotent peripheral systems.

Benjamin Haïoun (Toulouse)
Stated and internal skein algebras

I will define two generalisations of skein algebras to algebras of ‘tangles’on a surface : Le’s stated skein algebras and Gunningham-Jordan-Safronov’s internal skein algebras. I will quickly explain why they are isomorphic, namely what universal property stated skein algebras have with respect to the skein category associated with $U_q(sl_2)$-modules.

Giovanni Italiano (Pisa)
Hyperbolic 5-manifolds that fiber over the circle

We provide some examples of hyperbolic 5-manifolds fibering over the circle, showing that this phenomenon is not restricted to dimension 3. One consequence of this result is the existence of a hyperbolic group with a finite-type subgroup that is not hyperbolic. The main tool to build the fibration is Bestvina-Brady Morse theory applied to a hyperbolic n-manifold that decomposes into right-angled polytopes, enriched with a combinatorial game recently introduced by Jankiewicz, Norin and Wise. Joint work with B. Martelli and M. Migliorini.

Maria Marchwicka (Poznań)
On the slice genus of generalized algebraic knots

We give examples of a linear combination of algebraic knots and their mirrors that are algebraically slice, but whose topological and smooth four-genus is two. Our examples generalize an example of non-slice algebraically slice linear combination of iterated torus knots obtained by Hedden, Kirk and Livingston. Our main tool is a genus bound from Casson-Gordon invariants and a cabling formula that allows us to compute effectively these invariants.

Jules Martel (Dijon)
Homology of configuration spaces and quantum topology

We develop representations of mapping class groups in the spirit of R. Lawrence’s work for braid groups. By the time her construction aimed to recover some invariants of knots and braids arising from quantum groups. We wish to reconstruct quantum topological invariants from twisted homology of configuration spaces of surfaces.

Ivan Rasskin (Montpellier)
Construction of knots with sphere packings

How many spheres are needed to make a knotted necklace? And with spheres of different sizes? Behind these harmless questions lies a deep connection between knot theory, sphere-packings, polytopes, number theory and Lorentz geometry.
In this talk, we will quickly see how these theories are linked and we will use them to define two algorithms for the construction of knotted necklaces. The first method will allow us to establish that the minimum number of spheres necessary to build a necklace (or necklaces) in the shape of a given knot (or link) it is less than 5 times its crossing number, and the second method improves the previous bound for rational knots.
This is a joint work with Jorge Ramirez AlfonsÃn.