Winter Braids Lecture Notes

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Introduction

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Introduction

PAOLO BELLINGERI, VINCENT FLORENS, JEAN-BAPTISTE MEILHAN AND EMMANUEL WAGNER

Winter Braids is an international school on low dimensional topology, with a particular emphasis on the multiple aspects of braids: algebraic, geometrical, topological or algorithmic. More generally, it is devoted to the various thematics connected with these objects, such as mapping class groups, singularity theory and plane algebraic curves, quantum topology and TQFTs, symplectic topology, categorification or theoretical physics.

The school is held each winter since 2010, in a different french university. The fourth edition, Winter Braids IV, took place in Dijon from February 10th to 13th, 2014. The previous editions were organised in Pau (Dec. 2010), Caen (Dec. 2011) and Grenoble (Dec. 2012). The fourth edition counted 64 participants, including 50 Phd students or postdocs, coming from universities in 14 countries.

The school is mainly aimed at young researchers. It typically offers four mini-courses (of usually three hours) on classical or emerging topics of our research area, and gives in addition the opportunity to the young participants to present their own work. The goal of the Winter Braids school is to bring together young researchers in low-dimensional topology and, through these mini-courses, provide them a foundation of common knowledge, which would generate interactions and collaborations amoung the coming generations of researchers in France and Europe on braids and their ramifications.

Following a recurring request from the participants, it was decided to publish lecture notes for the mini-courses : the *Winter Braids Lecture Notes* were created for this purpose. This is a ‘gratis’ open access electronic journal, providing peer-reviewed surveys of the topics adressed in the school. With this journal, we hope to provide, over the years and successive editions of the school, a collection of introductory texts on classical and emerging topics in low dimensional topology and its ramifications. As such, the aim of the Winter Braids Lecture Notes is not only to present new topics, but more generally to provide a body of original introductory texts for young researchers in topology.

This volume contains lecture notes for the four mini-courses given at Winter Braids IV. These courses were dedicated to mapping class groups, Khovanov homology, Lefschetz fibrations and interactions with mathematical physics, covering a wide range of topics on low dimensional topology and interactions. Below is a short description for each of these courses.

- The mini-course by Benjamin Audoux (Marseille) was devoted to the categorification of polynomial link invariants and their applications. More precisely, the main goal was to give a complete construction of the Rasmussen invariant, from Khovanov homology, and to prove, in a purely combinatorial way, some of its celebrated applications on the slice genus of knots (Milnor conjecture).

- David Cimasoni (Geneva) gave three lectures on dimers models, which are combinatorial objects which arise in various fields, such as geometry and Teichmuller theory, or more recently knot theory. One of the main goal of the lectures was to show how purely geometrical tools, such as (co)homology, spin structures, or real algebraic curves, can be applied in very natural problems in combinatorics and statistical physics. The fundamental work of Kastelyn has also been covered in details.

- The aim of Luis Paris (Dijon) was to introduce mapping class groups for non orientable surfaces. A strong emphasis was put on the basics of mapping class group (including proofs of some key results in the orientable case), and on pointing out the basic new features and objects that arise in the non orientable settings. The structure of the mapping class group of non orientable surfaces of low genus has been fully described.

- Nermin Salepci (Lyon) explored topological and geometrical properties of Lefschetz fibrations. The first half of the lectures was devoted to topological properties and known classification results, while the latter half focused on relations to symplectic manifolds and open book decompositions, as well as contact structures on 3-dimensional manifolds.

The organisers also selected 21 short talks, providing a wide spectrum of topics in topology and interactions. The main objects, as in previous editions, were braid groups, related algebras, generalisations and applications in low dimensional topology. In particular the talks covered notions such as signatures of (positive) braids, braid equivalences in 3-manifolds, Temperley Lieb algebras and (symplectic) Burau generalisations, surface braid groups, virtual and welded braids. Other short talks addressed topics such as classical and quantum invariants of knots and 3-manifolds, TQFT, mapping class groups (coverings and quantum representations), hyperplane arrangements, categorification (Heegaard Floer and Khovanov homologies) and applications of dimers to knot theory. The result is an overview of the current research activity in low-dimensional topology. The abstracts of these short talks can be found at the end of this introduction.

We are deeply indepted to the four authors of these lecture notes, for their remarkable work and for helping us lauching this opening volume of the Winter Braids Lecture Notes.
We also wish to ackowledge the support of CNRS, which provided Winter Braids IV with the status of *École Thématique*. We also thank the GDR Tresses, for its constant support since the first edition. This edition was also supported by the ESF program ITGP, the ANR projects INTERLOW and VasKho, as well as the Fédération de Recherche SMT, the Institut Mathématique de Bourgogne and Université de Bourgogne.

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Abstracts of Short Talks

Pauline Bailet (Nice)
On the monodromy of Milnor fibers of hyperplane arrangements

We describe a general setting where the monodromy action on the first cohomology group of the Milnor fiber of a hyperplane arrangement is the identity.

Fathi Ben Aribi (Paris)
The L^2-Alexander invariant detects the unknot

The L^2-Alexander invariant is a knot invariant introduced by Li and Zhang in 2006. It can be seen as a L^2-torsion of a certain L^2-chain complex derived from the knot complement. It can also be built from the knot group, with Fox calculus, similarly as the Alexander polynomial, except that the operators act on infinite-dimensional Hilbert spaces. In my talk I will present this construction and show that this invariant detects the trivial knot.

Maxime Bourrigan (Lyon)
Braids, signatures and the Blanchfield form

Through the closure operation, any link invariant defines a function on braid groups. In 2005, motivated by the quest for quasimorphisms on diffeomorphism groups, Gambaudo and Ghys studied the case of the (Levine-Tristram) signatures. More precisely, they gave a formula linking this topological invariant to the Burau symplectic representation. In fact, link signatures are numerical counterparts of the Blanchfield form, an algebraic invariant describing Poincaré duality in the infinite cyclic cover of a link. In this talk, I will describe how classical results linking the signature of a 4-manifold and the linking form of its boundary extend to this context to give a generalisation of the Gambaudo-Ghys formula.

Bruno Cisneros (Dijon)
The genus of a virtual braid

Virtual braids are combinatorial generalizations of classical braids that arises from the finite type invariant theory. Usually they are defined via their diagrams, and they can be seen also as non planar graphs, identified up to certain relations. I will present a topological interpretation of these objects as a natural generalization of the classical geometric braids. Finally I will present the genus of a virtual braid.

Moshe Cohen (Haifa)
Dimers from knots

We consider dimers or perfect matchings on a graph Gamma obtained from a knot diagram. We use these to compute knot polynomial invariants. We also re-cast Kauffman's clock lattice as the graph of perfect matchings of Gamma and give a formula for its height. Based on joint works with O. Dasbach, H. M. Russell, and M. Teicher.

Ester Dalvit (Trento)
Visualization of welded knots and ribbon 2-knots

Welded knots are defined diagrammatically via Reidemeister-type moves. Satoh defined a construction to obtain a ribbon torus knot from a welded knot diagram or a 2-knot in the case of a welded arc.

The map is known to be surjective but its injectivity remains an open question. In this talk I will tell this story showing movies to visualize the objects involved and their interpretations in various dimensions.

Carolina De Miranda e Peirero (Caen)
The Lower Central and Derived Series of the Braid Groups of the Torus and of the Klein bottle

We are interested in studying the lower central series and the derived series of the braid groups (resp. pure braid groups) of the torus, $B_n(T)$ (resp. $P_n(T)$), and of the Klein bottle, $B_n(K)$ (resp. $P_n(K)$), one of our aims being to decide whether these groups are residually nilpotent or residually soluble. For the braid groups of surfaces, these series have been studied in the case of the disc, sphere and the projective plane. Besides that, Bellingeri, Gervais and Guaschi showed that $B_n(T)$ is residually nilpotent if and only if $n \leq 2$. In this work, we analyse the derived series of $B_n(T)$, and the lower central series and the derived series of $B_n(K)$ and $P_n(K)$. We showed that $B_n(T)$ is residually soluble if and only if $n \leq 4$ and $B_n(K)$ is residually nilpotent if and only if $n \leq 2$.

Ioannis Diamantis (Athens)
Braid equivalence in 3-manifolds with rational surgery description

In this talk we describe braid equivalence for knots and links in a 3-manifold M obtained by rational surgery along a framed link in S^3. We first prove a sharpened version of the Reidemeister theorem for links in M. We then give geometric formulations of the braid equivalence via mixed braids in S^3 using the L-moves and the braid band moves. We finally give algebraic formulations in terms of the mixed braid groups $B_{m,n}$ using cabling and the techniques of parting and combing for mixed braids. We also provide concrete formulae of the braid equivalence in the case where M is a lens space, a Seifert manifold or a homology sphere obtained from the trefoil.

Jens Kristian Egsgaard (Aarhus)
The Jones representations of braid groups at $q = -1$ (Part I)

We show that the Jones representation of Bn evaluated at $q = -1$ is equivalent to the action on a particular quotient of an exterior power of the homology of a ramified double cover of the n-punctured sphere via the Birman-Hilden theorem. This generalizes a discovery by Kasahara made for the sphere with six punctures.

Peter Feller (Bern)
The signature of positive braids is linearly bounded by their genus

We provide linear lower bounds for the signature of positive braids in terms of their length. This yields linear bounds for the topological 4-ball genus of knots that are closures of positive braids.

Thomas Gobet (Amiens)
Temperley-Lieb algebras and Zinno's basis

The images of the simple elements of the dual braid monoid of type A in the Temperley-Lieb algebra form a basis of it as shown by Zinno. We want to understand the coefficients of the change base matrix between that basis and the diagram basis.

Dimos Goundaroulis (Athens)
Framization of the Temperley-Lieb Algebra

In this talk we propose a framization of the Temperley-Lieb algebra. The framization is a procedure that can briefly be described as the adding of framing to a known knot algebra in a way that is both algebraically consistent and topologically meaningful. Here, our framization is defined as a quotient of the Yokonuma-Hecke algebra. The main theorem provides necessary and sufficient conditions for the Markov trace defined on the Yokonuma-Hecke algebra to pass through to our framization. Finally, we discuss the possible knot invariants that can be defined through this procedure.

Benoit Guerville (Pau)
A topological invariant of line arrangements

In a joint work with E. Artal Bartolo and V. Florens, we construct a topological invariant of line arrangements - linked with Alexander's module and characteristic varieties. It is based on the inclusion of the boundary manifold in the complement, and it is simply computable. It is a strong invariant since it allows to detect new examples of Zariski pairs ; that is a pair of arrangements with the same combinatorics but with different topologies. It can be viewed as an analogy of linking matrix of a link.

Carl Hammarsten (Washington)
Heegaard Floer Homology and Branched Spines

A 3-dimensional closed manifold Y represented by its branched spine has a canonical Heegaard decomposition. We present this decomposition graphically in the form of a Strip Diagram. We show that strip diagrams have nice properties which greatly simplify the calculation of Heegaard Floer homology for "most" manifolds. Motivated by this work, we present a combinatorial definition of a chain complex which we expect to be homotopically equivalent to the Heegaard Floer one, yet significantly smaller. Finally, we consider the presentation of a branched spine by its O-graph and show how to reformulate our definition in these terms.

Soeren Fuglede Joergensen (Uppsala)
The Jones representations of braid groups at $q = -1$ (Part II)

We discuss a conjecture by Andersen, Masbaum, and Ueno stating that for all large enough k, the level k quantum representation of a given pseudo-Anosov mapping class has infinite order. As an application of the results described in the first part of the talk, we prove this conjecture for a large family of pseudo-Anosov mapping classes of punctured spheres.

Julien Korinman (Grenoble)
Decomposition of quantum representations

I will describe representations of the mapping class group of surfaces arising in TQFTs associated to gauge groups $U(1)$, $SU(2)$ and $SO(3)$ and present their decomposition into irreducible factors.

Miguel Maldonado (Caen)
Mapping class groups and coverings

We consider the punctured mapping class group of a surface and analyze the relation between this group and that of its covering surface. The case for non-orientable surfaces is emphasized.

Jean-Mathieu Magot (Grenoble)
Combinatorial review of the relation $\Theta = \lambda_{CW} + \frac{p_1}{4}$

The Θ invariant of parallelized rational homology 3-spheres was first defined by M. Kontsevich in the nineties, as a certain count of embeddings of the Theta graph. G. Kuperberg, D. Thurston and C. Lescop showed that this invariant is equal to $\lambda_{CW} + \frac{p_1}{4}$, where λ_{CW} is the Casson-Walker invariant and p_1 is an invariant of parallelizations defined as a relative Pontrjagin class. In this talk, we present an alternative, purely combinatorial proof of this result. The proof builds on recent works of C. Lescop, who gave a combinatorial formula from Heegaard diagrams for the Θ invariant, and relies on the theory of finite type invariants. Our talk will review all prerequisites.

Yuliya Mikhalchishina (Novosibirsk)
Local Representations of braid groups

The local linear representations of braid group B_3 are under investigation as well as the local homogeneous representations of braid group B_n, $n \geq 2$. The connection of these representations with the Burau one is under study. Using the Wada representations of B_n in the automorphism group Aut(F_n) of a free group the linear representations of B_n are constructed.

Delphine Moussard (Pisa)
Equivariant triple intersections

Given a null-homologous knot K in a rational homology sphere M, and the standard infinite cyclic covering X of (M, K), we define an invariant of triples of curves in X, by means of equivariant triple intersections of surfaces. We prove that this provides a map Φ on $A^{\otimes 3}$, where A is the Alexander module of (M, K), whose isomorphism class is an invariant of the homeomorphism class of the pair (M, K). For a fixed Blanchfield module (A, b), i.e. an Alexander module A endowed with a Blanchfield form b, we consider pairs (M, K) whose Blanchfield modules are isomorphic to (A, b), equipped with a fixed isomorphism from the Blanchfield module of (M, K) to (A, b). In this setting, we compute the variation of Φ under borromean surgeries, and we describe the set of all maps Φ.

Louis-Hadrien Robert (Strasbourg)
Grothendieck groups of Khovanov-Kuperberg algebras

The sl3 homology is a variant of the Khovanov homology. The construction starts with sl_3instead of sl_2. The geometrical counterpart of the sl_3-homology involves foams rather than surfaces. The Khovanov homology, and the sl_3 homology, have a version for tangles. It involves some algebras called H_n in the first case and K_ϵ, the projective indecomposable modules over these algebra decategorify on dual-canonical bases. While in the sl_2 case, this modules are very easy to identity, in the sl_3 case it is much more difficult (and still open).