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Romain VALIDIRE

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Capitulation for even K-groups in the cyclotomic $\mathbb{Z}_p$-extension.

par Romain VALIDIRE

Résumé. Soit p un nombre premier et F un corps de nombres. Depuis les travaux d'Iwasawa, le comportement de la p-partie du groupe des classes d'idéaux dans une $\mathbb{Z}_p$-extension de F est assez bien compris. M. Grandet et J.-F. Jaulent ont en outre donné un résultat précis concernant sa structure de groupe abélien.

Par ailleurs, le groupe des classes d'idéaux s'interprête comme la partie de torsion du K_0 de l'anneau des entiers de F. Les K-groupes pairs de l'anneau des entiers peuvent être vus comme des versions *supérieures* du groupe des classes et le comportement de ces K-groupes dans les $\mathbb{Z}_p$-extensions a déjà été étudié par de nombreux auteurs. Dans cet article, nous montrons que le résultat de Grandet et Jaulent sur les groupes de classes est encore vrai pour les K-groupes pairs dans la $\mathbb{Z}_p$-extension cyclotomique.

Abstract. Let p be a prime number and F be a number field. Since Iwasawa's works, the behaviour of the p-part of the ideal class group in the $\mathbb{Z}_p$-extensions of F has been well understood. Moreover, M. Grandet and J.-F. Jaulent gave a precise result about its abelian p-group structure.

On the other hand, the ideal class group of a number field may be identified with the torsion part of the K_0 of its ring of integers. The even K-groups of rings of integers appear as *higher* versions of the class group. Many authors have already studied the behaviour of the higher even K-groups in a $\mathbb{Z}_p$-extension. Here, we prove that Grandet and Jaulent's result on class group still holds for higher even K-groups in the cyclotomic $\mathbb{Z}_p$-extension.

Introduction

Let p be a prime number and F be a number field. We denote by $\mathcal{O}_F$ the ring of integers of F and by $Cl(\mathcal{O}_F)$ the ideal class group of $\mathcal{O}_F$.

Let F_∞/F be a $\mathbb{Z}_p$-extension, with finite layers F_n for all integers n and with the usual notations for the Galois groups $\Gamma := \mathrm{Gal}(F_\infty/F)$ and $\Gamma_n := \mathrm{Gal}(F_\infty/F_n)$. Iwasawa's theory of $\mathbb{Z}_p$-extensions is a way to investigate the behaviour of the p-primary part of the class groups $Cl(\mathcal{O}_{F_n})$. A well-known result is the famous Iwasawa's formula giving the order of the p-primary part of $Cl(\mathcal{O}_{F_n})$ for all n large enough. It is possible to obtain more precise results about this group by studying the vanishing of ideal classes in the $\mathbb{Z}_p$-extension. Let us consider the natural map :

$$Cl(\mathcal{O}_{F_n})\{p\} \to (Cl(\mathcal{O}_{F_\infty})\{p\})^{\Gamma_n}$$

between the p-primary part of $Cl(\mathcal{O}_{F_n})$ and the Γ_n-fixed points of the inductive limit:

$$Cl(\mathcal{O}_{F_\infty})\{p\} := \varinjlim Cl(\mathcal{O}_{F_n})\{p\}.$$

We denote by $\mathrm{Cap}(F_n)$ the kernel of this map. These *capitulation kernels* have been intensively studied (cp.[Iw], [Ku],...) and their asymptotical behaviour is well-known: for $m \geq n \gg 0$ the norm map between the class groups induces an isomorphism from $\mathrm{Cap}(F_m)$ to $\mathrm{Cap}(F_n)$ (cp. [Ku]); we say that the capitulation kernels *stabilize* for the norm map.

Consider the Iwasawa module $X_{F_\infty} := \varprojlim Cl(\mathcal{O}_{F_m})\{p\}$, where the limit is taken with respect to the norm maps. This group is a module over the complete group ring $\mathbb{Z}_p[[\Gamma]]$. Let μ and λ be respectively the p-valuation and the degree of the characteristic polynomial of X_{F_∞}. For n sufficiently large, the natural map from X_{F_∞} to $Cl(\mathcal{O}_{F_n})\{p\}$ induces an isomorphism from $(X_{F_\infty})^0$ to $\mathrm{Cap}(F_n)$, where $(X_{F_\infty})^0$ is the maximal finite submodule of X_{F_∞}.

However, we have a more precise result; M. Grandet and J.-F. Jaulent prove in [GJ] that the capitulation kernel becomes a *direct summand* of the class group:

Theorem 0.1. *Assume that the invariant μ of X_∞ is trivial. Then there exists $(\alpha_1, \ldots, \alpha_\lambda) \in \mathbb{Z}^\lambda$ such that for all n large enough:*

$$Cl(\mathcal{O}_{F_n})\{p\} \simeq \mathrm{Cap}(F_n) \oplus \left(\bigoplus_{i=1}^{\lambda} \mathbb{Z}/p^{\alpha_i + n} \right), \ \textit{as abelian groups.}$$

On the other hand it is well-known that $Cl(\mathcal{O}_F)$ may be identified with the torsion part of $K_0(\mathcal{O}_F)$. As for class group we can consider the following *higher* capitulation kernels for all integers $i \geq 1$ and $n \geq 1$:

$$\mathrm{Cap}_i(F_n) := \ker\left(K_{2i}(\mathcal{O}_{F_n}) \otimes \mathbb{Z}_p \to (K_{2i}(\mathcal{O}_{F_\infty}) \otimes \mathbb{Z}_p)^{\Gamma_n} \right),$$

where $K_{2i}(\mathcal{O}_{F_n})$ denotes the Quillen K-groups associated with the ring $\mathcal{O}_{F_n}$ and

$$K_{2i}(\mathcal{O}_{F_\infty}) := \varinjlim K_{2i}(\mathcal{O}_{F_m}).$$

Using a general result due to T. Nguyen Quang Do, B. Kahn proved (cp. [Ka]) that the groups $\mathrm{Cap}_1(F_n)$ also stabilize for the norm map.

Now we assume that p is odd or $p = 2$ and F contains $\sqrt{-1}$. For a finite set S of primes containing the set of primes above p and the infinite primes of F, let $\mathcal{O}_F^S$ denote the ring of S-integers of F. Generalizing the result of T.

Nguyen Quang do and B. Kahn, M. Kolster and A. Movahhedi introduced (cp. [KM]) similar capitulation kernels $\mathrm{Cap}_i^{ét}(F_n)$ for all $i \geq 1$ using étale K-groups $K_{2i}^{ét}(\mathcal{O}_{F_n}^S)$ and proved for these groups the same stabilization property.

Our purpose in the present article is to prove that the theorem (0.1) also holds for higher étale capitulation kernels when F_∞ is the cyclotomic $\mathbb{Z}_p$-extension of F.

To prove the result, we first consider a particular subgroup of $K_{2i}^{ét}(\mathcal{O}_F^S)$: the *étale wild kernel* denoted by $WK_{2i}^{ét}(F)$. The definition of the wild kernels is given in section 2.

In section 3, we use a description of $WK_{2i}^{ét}(F)$ due to Schneider to prove that the capitulation kernels for the wild kernels also become direct summands.

In section 4, we prove that the p-quotients of the wild kernels and of the p-class group are *asymptotically* isomorphic (Proposition 4.1). Then we use this result to show that, when $\mu = 0$, the group $\mathrm{Cap}_i^{ét}(F_n)$ becomes a direct summand of the abelian p-group $K_{2i}^{ét}(\mathcal{O}_{F_n}^S)$. Finally, we show that we have a *non canonical* Galois descent for the even K-groups in the cyclotomic $\mathbb{Z}_p$-extension (Corollary 4.2).

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1. Preliminaries

In this section we introduce the objects studied in the rest of the paper. First, we fix some notations.

Let p be a fixed prime number and F be an algebraic number field. If $p = 2$ we also assume that $\sqrt{-1} \in F$. Let S be a finite set of primes in F, containing the set S_p of primes above p and the set S_∞ of infinite primes; let $\mathcal{O}_F^S$ denote the ring of S-integers of F and G_F^S denote the Galois group over F of the maximal algebraic extension of F which is unramified outside S. For any $\mathbb{Z}_p$-module M, we put $M^* = Hom(M, \mathbb{Q}_p/\mathbb{Z}_p)$, the Pontrjagin dual of M.

For integers $n \geq 0$ and $i \geq 1$ we denote by $\mu_n^{\otimes i}$ the ith twist of the group μ_n of roots of unity of order n and $\mathbb{Z}_p(i) := \varprojlim(\mu_{p^n}^{\otimes i})$, the ith twist of $\mathbb{Z}_p$. For any arbitrary $\mathbb{Z}_p\left[G_F^S\right]$-module M, we define the i-fold Tate twist of M by:

$$M(i) := M \otimes_{\mathbb{Z}_p} \mathbb{Z}_p(i).$$

For an abelian group A and a positive integer n we denote by:

- A/n, the quotient of A by the subgroup nA.

- ${}_nA$, the group of elements $a \in A$ such that $na = 0$.
- $A\{p\} = \bigcup_{k\geq 1}({}_{p^k}A)$, the p-primary part of A.

For $i \geq 1$ and $k = 1$ or 2, the higher étale K-theory groups $K_{2i+2-k}^{ét}(\mathcal{O}_F^S)$, introduced by Dwyer and Friedlander ([DF]), coincide with the (continuous) Galois cohomology groups:

$$\begin{aligned} K_{2i+2-k}^{ét}(\mathcal{O}_F^S) &\simeq H^k(G_F^S, \mathbb{Z}_p(i+1)) \\ &\simeq \varprojlim H^k(G_F^S, \mathbb{Z}/p^n(i+1)). \end{aligned}$$

Using Borel's results on algebraic K-groups, it can be shown that the even K-groups $K_{2i}^{ét}(\mathcal{O}_F^S)$ are finite and that the odd K-groups $K_{2i+1}^{ét}(\mathcal{O}_F^S)$ are finitely generated over $\mathbb{Z}_p$; their $\mathbb{Z}_p$-rank is $r_1 + r_2$ if i is even and r_2 if i is odd. As usual r_1 (resp. r_2) denotes the number of real (resp. pairs of conjugate complex) embeddings of F. Furthermore the odd étale K-groups do not depend on the choice of the set S. We have $K_{2i+1}^{ét}(\mathcal{O}_F^S) \simeq H^1(F, \mathbb{Z}_p(i+1))$, and we denote these groups by $K_{2i+1}^{ét}(F)$.

For any group G and any G-module M, we denote as usual by M^G the fixed points of M under the action of G and by M_G the quotient of M/I_GM, where I_G is the augmentation-ideal of $\mathbb{Z}[G]$.

Let L be a finite Galois extension of F with Galois group G, which is unramified outside S. We are interested in Galois descent and co-descent for étale odd K-groups in the extension L/F; we have two canonical morphisms between $K_{2i}^{ét}(\mathcal{O}_F^S)$ and $K_{2i}^{ét}(\mathcal{O}_L^S)$:

- the *extension map* $K_{2i}^{ét}(\mathcal{O}_F^S) \to K_{2i}^{ét}(\mathcal{O}_L^S)^G$, which may be identified with the restriction map in Galois cohomology.
- the *norm map* $K_{2i}^{ét}(\mathcal{O}_L^S)_G \to K_{2i}^{ét}(\mathcal{O}_F^S)$, which may be identified with the co-restriction map in Galois cohomology.

We have the following (see [Ka] and [KM]):

Theorem 1.1. *Let L/F be a Galois p-extension with Galois group G. Let S be a finite set of primes, containing the primes above p and the primes which ramify in L. Then for $i \geq 1$ there is an exact sequence induced by the extension map:*

$$\begin{aligned} 0 \to H^1(G, K_{2i+1}^{ét}(L)) \to K_{2i}^{ét}(\mathcal{O}_F^S) \\ \to K_{2i}^{ét}(\mathcal{O}_L^S)^G \to H^2(G, K_{2i+1}^{ét}(L)) \to 0 \end{aligned}$$

and an isomorphism induced by the norm map:

$$K_{2i}^{ét}(\mathcal{O}_L^S)_G \simeq K_{2i}^{ét}(\mathcal{O}_F^S).$$

We deduce the following corollary:

Corollary 1.1. *The kernel of the extension map*

$$\mathrm{Cap}_i(L/F) := \ker\left(K_{2i}^{\acute{e}t}(\mathcal{O}_F^S) \to K_{2i}^{\acute{e}t}(\mathcal{O}_L^S)\right)$$

does not depend on the set S containing $S_p \cup S_\infty$ and the ramified primes in L/F.

Let F_∞/F be an arbitrary $\mathbb{Z}_p$-extension with finite layers F_n. For integers $m \geq n \geq 0$, we put $G_{m,n} = \mathrm{Gal}(F_m/F_n)$ and $\Gamma_n = \mathrm{Gal}(F_\infty/F_n)$. The main objects studied here are the kernels of the extension maps:

$$\mathrm{Cap}_i(F_m/F_n) := \ker\left(K_{2i}^{\acute{e}t}(\mathcal{O}_{F_n}^S) \to K_{2i}^{\acute{e}t}(\mathcal{O}_{F_m}^S)^{G_{m,n}}\right).$$

Remark. Since a $\mathbb{Z}_p$-extension of number field is p-ramified, it is enough to consider the case $S = S_p \cup S_\infty$; we put $\mathcal{O}'_F := \mathcal{O}_F^{S_p \cup S_\infty}$.

We consider the kernel: $\mathrm{Cap}_i(F_n) := \ker\left(K_{2i}^{\acute{e}t}(\mathcal{O}_{F_n}^S) \to K_{2i}^{\acute{e}t}(\mathcal{O}_{F_\infty}^S)^{\Gamma_n}\right)$, with :

$$K_{2i}^{\acute{e}t}(\mathcal{O}_{F_\infty}^S) := \varinjlim K_{2i}^{\acute{e}t}(\mathcal{O}_{F_m}^S).$$

We may deduce from theorem 1.1 the short exact sequence (see [Ka] and [KM]):

$$0 \to \mathrm{Cap}_i(F_n) \to K_{2i}^{\acute{e}t}(\mathcal{O}_{F_n}^S) \to K_{2i}^{\acute{e}t}(\mathcal{O}_{F_\infty}^S)^{\Gamma_n} \to 0. \tag{1.1}$$

Now, we focus on the asymptotical behaviour of the higher capitulation kernels. We have the following proposition:

Proposition 1.1. *For all $m \geq n \gg 0$, the Galois group $G_{m,n}$ acts trivially on $\mathrm{Cap}_i(F_m)$ and the norm map induces an isomorphism:*

$$\mathrm{Cap}_i(F_m) = \mathrm{Cap}_i(F_m)_{G_{m,n}} \simeq \mathrm{Cap}_i(F_n).$$

Our purpose is to prove that the exact sequence (1.1) is a split exact sequence of abelian groups when F_∞ is the cyclotomic $\mathbb{Z}_p$-extension.

We need the following lemma on abelian groups:

Lemma 1.1. *Let M be a finite abelian p-group and N be a subgroup. Let e be an integer such that p^e annihilates N. If for all integers n, $0 \leq n \leq e$, the inclusion map from N to M induces an injection $N/p^n \hookrightarrow M/p^n$ then N is a direct summand in M.*

2. Localisation kernels and étale wild kernels

In the following, we will consider the localisation kernels for $i \in \mathbb{Z}$ and $n \geq 1$:

$$Ш_S^2(F, \mathbb{Z}/p^n(i)) := \ker\left(H^2(G_F^S, \mathbb{Z}/p^n(i)) \overset{loc.}{\to} \oplus_{v \in S} H^2(F_v, \mathbb{Z}/p^n(i))\right),$$

$$Ш_S^2(F, \mathbb{Z}_p(i)) := \ker\left(H^2(G_F^S, \mathbb{Z}_p(i)) \overset{loc.}{\to} \oplus_{v \in S} H^2(F_v, \mathbb{Z}_p(i))\right).$$

P. Schneider studied these localisation kernels in [S]. He conjectured that for all $i \in \mathbb{Z}$, the groups $Ш^2_S(F, \mathbb{Z}_p(i))$ are finite (indeed, it is true for $i \geq 1$). For $i = 0$, the finiteness of $Ш^2_S(F, \mathbb{Z}_p)$ is equivalent to the famous Leopoldt Conjecture.

Let us give some interpretations for these kernels.

Proposition 2.1. (see [NSW, Lemma (8.6.3)]) *The groups* $Ш^2_S(F, \mathbb{Z}/p^n(i))$ *are finite and for* $i = 1$ *we have the isomorphism:*

$$Cl(\mathcal{O}^S_F)/p^n \simeq Ш^2_S(F, \mu_{p^n}).$$

Remark. By finiteness of class group $Ш^2_S(F, \mathbb{Z}_p(1)) \simeq Cl(\mathcal{O}^S_F)\{p\}$.

Let n be a positive integer. Since p is an odd prime (or $\sqrt{-1} \subset F$ if $p = 2$) we have $cd_p(G^S_F) \leq 2$ (cohomological p-dimension); the exact cohomology sequence of the short exact sequence

$$0 \to \mathbb{Z}_p(i+1) \to \mathbb{Z}_p(i+1) \to \mathbb{Z}/p^n(i+1) \to 0$$

yields the isomorphism:

$$H^2(G^S_F, \mathbb{Z}_p(i+1))/p^n \simeq H^2(G^S_F, \mathbb{Z}/p^n(i+1)),$$

whence follows (see [Ta, Theorem (6.2)]):

Proposition 2.2. *Assume that* F *contains* μ_{p^n}. *Then for* $i \geq 1$, *we have a canonical isomorphism:*

$$K^{\acute{e}t}_{2i}(\mathcal{O}^S_F)/p^n \simeq H^2(G^S_F, \mu_{p^n})(i), \tag{2.1}$$

and an exact sequence:

$$0 \to Cl(\mathcal{O}^S_F)/p^n(i) \to K^{\acute{e}t}_{2i}(\mathcal{O}^S_F)/p^n \overset{(\oplus l_v)}{\to} \bigoplus_{v \in S} \mu_{p^n}(i-1) \overset{\Sigma}{\to} \mu_{p^n}(i-1) \to 0,$$

where l_v *comes from the localisation map at the prime* $v \in S$ *and* Σ *is the product map.*

Tate's results ([Ta]) on K_2 and Galois cohomology give a canonical isomorphism:

$$WK_2(F)\{p\} \simeq Ш^2_S(F, \mathbb{Z}_p(2)),$$

where $WK_2(F)$ is the *classical* wild kernel (i.e. the kernel of all Hilbert symbols on $K_2(F)$) which appears in Moore's exact sequence:

$$0 \to WK_2(F) \to K_2(F) \to \oplus_v \mu(F_v) \to \mu(F) \to 0,$$

where v runs through all finite and real infinite primes of F, and $\mu(F)$ (resp. $\mu(F_v)$) denotes the group of roots of unity of the number field F (resp. the local field F_v).

The groups $Ш^2_S(F, \mathbb{Z}_p(i+1))$ do not depend on the choice of the set S containing $S_p \cup S_\infty$. For all $i \geq 1$, they can be identified with subgroups of $K_{2i}^{\acute{e}t}(\mathcal{O}_F^S)$. Thus, these remarks and the description of the classical wild kernel leads to the definition of the *higher étale wild kernels* (cp. [Ba], [N2]).

Definition. Let p be a prime number. For a number field F and $i \geq 1$, we define the $2i$th étale wild kernel:

$$WK_{2i}^{\acute{e}t}(F) := Ш_S^2(F, \mathbb{Z}_p(i+1)).$$

The Poitou-Tate duality sequence yields the short exact sequence:

$$0 \to WK_{2i}^{\acute{e}t}(F) \to K_{2i}^{\acute{e}t}(\mathcal{O}_F^S) \to \oplus_{v \in S} H^2(F_v, \mathbb{Z}_p(i+1)) \\ \to H^0(F, \mathbb{Q}_p/\mathbb{Z}_p(-i))^* \to 0.$$

There exist some relations between the étale wild kernels when i varies over the positive integers. For example we have (cp. [N3])

Proposition 2.3. *Assume that F contains μ_{p^n}. For all positive integers i and j there is a canonical isomorphism:*

$$WK_{2i}^{\acute{e}t}(F)/p^n \simeq WK_{2j}^{\acute{e}t}(F)/p^n(i-j).$$

Let us point out another map which will be useful in the following. For a number field F containing μ_{p^n} and for $i \geq 1$, the isomorphism (2.1) yields the commutative diagram:

$$\begin{array}{ccc} WK_{2i}^{\acute{e}t}(F)/p^n & \simeq & Ш_S^2(F, \mathbb{Z}_p(i+1))/p^n \\ \downarrow & & \downarrow \\ Cl(\mathcal{O}_F^S)/p^n(i) & \simeq & Ш_S^2(F, \mu_{p^n})(i) \end{array}$$

The vertical maps are in general not bijective; in the last section we will give conditions for bijectivity. For the moment, let us give a condition for surjectivity (see [KM, Lemma 2.8] and [V, Proposition 1.3.8]).

Proposition 2.4. *Assume that F contains μ_{p^n} and that at least one p-adic prime in F totally ramifies in F_∞/F. For all $i \geq 1$, the maps*

$$WK_{2i}^{\acute{e}t}(F)/p^n \to Cl(\mathcal{O}_F^S)/p^n(i)$$

are onto.

Remark. It is also possible to construct these maps passing through the logarithmic valuations and the logarithmic class group $\widetilde{\mathcal{C}\ell}(F)$ introduced by Jaulent (cp. [J1] and [J2]).

3. Iwasawa theory for wild kernels and capitulation

In this section we study the capitulation kernel for étale wild kernels when F_∞/F is the *cyclotomic* $\mathbb{Z}_p$-extension of F. As usual we put:

$$WK_{2i}^{ét}(F_\infty) := \varinjlim WK_{2i}^{ét}(F_n).$$

For all $m \geq n \geq 0$ we have the following equalities:

$$\mathrm{Cap}_i(F_m/F_n) = \ker(WK_{2i}^{ét}(F_n) \to WK_{2i}^{ét}(F_m)),$$
$$\mathrm{Cap}_i(F_n) = \ker(WK_{2i}^{ét}(F_n) \to WK_{2i}^{ét}(F_\infty))$$

Then the capitulation kernels for wild kernels stabilize for the norm map in the same manner as for odd K-groups.

We may deduce from proposition(1.1) (see also [LMN, Lemma 1.1]) that for $i \geq 1$ there is a short exact sequence:

$$0 \to \mathrm{Cap}_i(F_n) \to WK_{2i}^{ét}(F_n) \to WK_{2i}^{ét}(F_\infty)^{\Gamma_n} \to 0. \tag{3.1}$$

Following the ideas of Grandet and Jaulent we prove that, under certain assumptions, (3.1) is a split exact sequence of abelian groups.

We still assume that p is odd and $\sqrt{-1} \in F$, if $p = 2$. Let $E = F(\mu_{2p})$ and E_∞ be the cyclotomic $\mathbb{Z}_p$-extension of E. We still denote by Γ (resp. Γ_n) the Galois group of E_∞/E (resp. E_n/E). Let $\Delta = \mathrm{Gal}(E/F)$ and let d be the order of Δ.

Now let us give the description of étale wild kernels using Iwasawa theory (cp.[N3]).

We put $X'_\infty := \varprojlim Cl(\mathcal{O}'_{E_n})\{p\}$, where the limit is taken for the norm map. The $\mathbb{Z}_p$-module X'_∞ is naturally a module over the complete group ring $\Lambda := \mathbb{Z}_p[[\Gamma]] \simeq \mathbb{Z}_p[[1-\gamma]]$, for any chosen topological generator γ of Γ.

Let L'_∞ be the maximal abelian unramified pro-p-extension of E_∞, in which all primes above p are completely decomposed. By class field theory X'_∞ is isomorphic to the Galois group $\mathrm{Gal}(L'_\infty/E_\infty)$.

One shows that X'_∞ is a finitely generated Λ-torsion module. Let $f(1-\gamma)$ be its characteristic polynomial. We denote by μ (resp. λ) the p-adic valuation (resp. the degree) of $f(1-\gamma)$. They are respectivelly called μ-invariant and λ-invariant.
Finally we denote by $(X'_\infty)^0$ the maximal finite submodule of X'_∞.

P. Schneider prove (see [S, §6 lemma 1]) that the localisation kernels can be described as co-descent modules.

Theorem 3.1. *For $i \in \mathbb{Z}$ and $i \neq 0$, we have a canonical isomorphism:*

$$Ш_S^2(F, \mathbb{Z}_p(i+1)) \simeq \left(X'_\infty(i)\right)_{\Gamma\times\Delta}.$$

Remark. For $i=0$, the co-invariant $(X'_\infty)_\Gamma$ is not isomorphic to the p-part of the p-class group $Cl(\mathcal{O}'_E)\{p\}$; it has been described by J.-F. Jaulent in *logarithmic* terms and it is isomorphic to the logarithmic class group $\widetilde{\mathcal{C}\ell}(E)$ (cp. [J2]). The Gross conjecture asserts that $\widetilde{\mathcal{C}\ell}(E)$ is finite.

Using Schneider's theorem, we can describe the extension maps in F_∞/F (see [N3] or [LMN]). For all $m \geq n \geq 0$, we denote $\gamma^{p^n}-1$ by ω_n and ω_m/ω_n by $\nu_{m,n}$. Consider the natural morphisms $i_{m,n}$:

$$\begin{array}{rcl} X'_\infty(i)_{\Gamma_n} & \to & X'_\infty(i)_{\Gamma_m} \\ x \bmod \omega_n & \mapsto & \nu_{m,n}x \bmod \omega_m. \end{array}$$

We have a commutative diagram (with natural map for étale wild kernels):

$$\begin{tikzcd} WK_{2i}^{\acute{e}t}(E_n) \arrow[r] \arrow[d, "\simeq"] & WK_{2i}^{\acute{e}t}(E_m) \arrow[d, "\simeq"] \\ X'_\infty(i)_{\Gamma_n} \arrow[r, "i_{m,n}"] & X'_\infty(i)_{\Gamma_m} \end{tikzcd}$$

Proposition 3.1. *For $i \geq 1$, and for n sufficiently large, we have a canonical isomorphism:*

$$\mathrm{Cap}_i(E_n) \simeq (X'_\infty)^0(i).$$

As an easy consequence we find:

Corollary 3.1. *For $i \geq 1$, and for all $m \geq n \gg 0$:*

$$\mathrm{im}(\mathrm{Cap}_i(E_n) \to \mathrm{Cap}_i(E_m)) \simeq p^{m-n}(X'_\infty)^0(i).$$

Proof. It follows from the description of the extension map and the fact that Γ_n acts trivially on $(X'_\infty)^0(i)$ for $n \gg 0$. □

We can also describe the cokernel of the extension map. However we have to suppose that the μ-invariant of X'_∞ is trivial. This is true when the extension $E/\mathbb{Q}$ is abelian (cf. [FW]); it is conjectured to be true for all number fields. In that case X'_∞ is finitely generated over $\mathbb{Z}_p$.

Proposition 3.2. *Assume that μ is trivial. For all $m \geq n \gg 0$, we have the equality:*

$$\mathrm{im}(WK_{2i}^{\acute{e}t}(E_n) \to WK_{2i}^{\acute{e}t}(E_m)) = p^{m-n}(WK_{2i}^{\acute{e}t}(E_m)).$$

Proof. Let $T = 1-\gamma$ thus $\Lambda \simeq \mathbb{Z}_p[[T]]$. Since $\mu = 0$, we can assume that the characteristic polynomial $f(T)$ is a distinguished polynomial. There exists an integer $r \geq 0$, such that the distinguished polynomial $g(T) = \omega_r(T)f(T)$ annihilates X'_∞. Now we use a classical computation in Iwasawa theory. For n sufficiently large we have

$$(1+T)^{p^{n-1}} \equiv 1 \bmod (g(T), p).$$

Raising to the p-th power gives

$$(1+T)^{p^n} \equiv 1 \bmod (g(T), p^2).$$

Hence

$$\begin{aligned}\nu_{n+1,n} &= \sum_{i=0}^{p-1} (1+T)^{ip^n} \\ &\equiv p + p^2 h(T) \bmod g(T) \\ &\equiv p(1+ph(T)) \bmod g(T)\end{aligned}$$

where $h(T) \in \Lambda$.

By induction we easily see that for $m \geq n$ there exists an invertible element $u_m(T) \in \Lambda$ such that $\nu_{m,n} \equiv p^{m-n} u_{m,n}(T) \bmod g(T)$. Hence we have $\mathrm{im}(i_{m,n}) = p^{m-n}(X'_\infty)(i)_{\Gamma_n}$. Finally the proposition follows from Schneider's isomorphism. □

We can now conclude:

Proposition 3.3. *Assume that the μ-invariant of X'_∞ is trivial. For all $i \geq 1$ and for all n sufficiently large the exact sequence (3.1):*

$$0 \to \mathrm{Cap}_i(F_n) \to WK_{2i}^{\acute{e}t}(F_n) \to WK_{2i}^{\acute{e}t}(F_\infty)^{\Gamma_n} \to 0,$$

is a split exact sequence of abelian groups.

Proof. First we reduce to the case of a number field containing the roots of unity of order p. Indeed the action of the semi-simple algebra $\mathbb{Z}_p[\Delta]$ on a finite abelian p-group keeps the direct summands. Then it is sufficient to prove that

$$0 \to \mathrm{Cap}_i(E_n) \to WK_{2i}(E_n) \to WK_{2i}^{\acute{e}t}(E_\infty)^{\Gamma_n} \to 0,$$

is a split exact sequence of abelian groups to get the result.

Now choose an integer r sufficiently large. Then for all $h \geq 0$ there is a commutative diagram (with natural maps):

$$\begin{tikzcd} 0 \arrow[r] & \mathrm{Cap}_i(E_{r+h}) \arrow[r] & WK_{2i}^{\acute{e}t}(E_{r+h}) \arrow[r] & WK_{2i}^{\acute{e}t}(E_\infty)^{\Gamma_{r+h}} \arrow[r] & 0 \\ 0 \arrow[r] & \mathrm{Cap}_i(E_r) \arrow[r] \arrow[u] & WK_{2i}^{\acute{e}t}(E_r) \arrow[r] \arrow[u] & WK_{2i}^{\acute{e}t}(E_\infty)^{\Gamma_r} \arrow[r] \arrow[u] & 0 \end{tikzcd}$$

The vertical right arrow is injective. Hence by the snake lemma:

$$\mathrm{coker}\,(\mathrm{Cap}_i(E_r) \to \mathrm{Cap}_i(E_{r+h})) \hookrightarrow \mathrm{coker}\left(WK_{2i}^{\acute{e}t}(E_r) \to WK_{2i}^{\acute{e}t}(E_{r+h})\right).$$

Corollary 3.1 gives a description for the left cokernel and proposition 3.2 (we assume that $\mu = 0$) gives a description for the right cokernel. Let e be an integer such that p^e annihilates $(X'_\infty)^0$ and $n = r + e$. For all h, with $0 \leq h \leq e$ we have:

$$\mathrm{Cap}_i(E_n)/p^h \hookrightarrow WK_{2i}^{\acute{e}t}(E_n)/p^h.$$

Thus by lemma 1.1 the abelian group $\mathrm{Cap}_i(E_n)$ is a direct summand in $WK_{2i}^{\acute{e}t}(E_n)$. □

It is well-known (cp.[KM] or [N3]) that co-descent holds for the wild kernels in the cyclotomic $\mathbb{Z}_p$-extension. In other words the norm map induces a canonical isomorphism for all $m \geq n \geq 0$:

$$WK_{2i}^{\acute{e}t}(F_m)_{G_{m,n}} \simeq WK_{2i}^{\acute{e}t}(F_n).$$

Although the extension map does not induce an isomorphism we have a *non canonical* Galois descent in the cyclotomic $\mathbb{Z}_p$-extension:

Proposition 3.4. *Assume that $\mu = 0$. Then for all $m \geq n \gg 0$, the groups $WK_{2i}^{\acute{e}t}(F_m)^{G_{m,n}}$ and $WK_{2i}^{\acute{e}t}(F_n)$ are isomorphic as abelian groups.*

Proof. Choose n large and $m \geq n$. We have $\mathrm{Cap}_i(F_m)^{G_{m,n}} = \mathrm{Cap}_i(F_m)$. Consider the short exact sequence of $\mathbb{Z}_p[G_{m,n}]$-modules:

$$0 \to \mathrm{Cap}_i(F_m) \to WK_{2i}^{\acute{e}t}(F_m) \to WK_{2i}^{\acute{e}t}(F_\infty)^{\Gamma_m} \to 0.$$

The snake lemma yields the long exact sequence:

$$\begin{aligned} 0 \to \mathrm{Cap}_i(F_m) \to WK_{2i}^{\acute{e}t}(F_m)^{G_{m,n}} &\to WK_{2i}^{\acute{e}t}(F_\infty)^{\Gamma_n} \\ &\to \mathrm{Cap}_i(F_m)_{G_{m,n}} \to WK_{2i}^{\acute{e}t}(F_m)_{G_{m,n}} \to \dots \end{aligned}$$

Furthermore we have the commutative diagram:

$$\begin{tikzcd} \mathrm{Cap}_i(F_m)_{G_{m,n}} \arrow[r] \arrow[d, "\simeq"] & WK_{2i}^{\acute{e}t}(F_m)_{G_{m,n}} \arrow[d, "\simeq"] \\ \mathrm{Cap}_i(F_n) \arrow[r, hook] & WK_{2i}^{\acute{e}t}(F_n) \end{tikzcd}$$

where the vertical maps are induced by the norm.

Hence the map $\mathrm{Cap}_i(F_m)_{G_{m,n}} \to WK_{2i}^{\acute{e}t}(F_m)_{G_{m,n}}$ is injective. Thus we have a short exact sequence:

$$0 \to \mathrm{Cap}_i(F_m) \to WK_{2i}^{\acute{e}t}(F_m)^{G_{m,n}} \to WK_{2i}^{\acute{e}t}(F_\infty)^{\Gamma_n} \to 0. \tag{3.2}$$

On the other hand $\mathrm{Cap}_i(F_m)$ is a direct summand in $WK_{2i}^{\acute{e}t}(F_m)$. Hence it is a direct summand in the subgroup $WK_{2i}^{\acute{e}t}(F_m)^{G_{m,n}}$ so (3.2) is a split exact sequence of abelian groups. Thus we have the isomorphisms of abelian groups:

$$\begin{aligned} WK_{2i}^{\acute{e}t}(F_m)^{G_{m,n}} &\simeq \mathrm{Cap}_i(F_m) \oplus WK_{2i}^{\acute{e}t}(F_\infty)^{\Gamma_n} \\ &\simeq \mathrm{Cap}_i(F_n) \oplus WK_{2i}^{\acute{e}t}(F_\infty)^{\Gamma_n} \\ &\simeq WK_{2i}^{\acute{e}t}(F_n). \end{aligned}$$

□

Finally let us recall a descprition for the groups $WK_{2i}^{\acute{e}t}(E_\infty)$ (cp.[N3]).

Proposition 3.5. *Assume that μ is trivial. For all $i \geq 1$ we have:*

$$WK_{2i}^{\acute{e}t}(E_\infty) \simeq (\mathbb{Q}_p/\mathbb{Z}_p)^\lambda,$$

as abelian group.

Proof. Since the groups $WK_{2i}^{\acute{e}t}(E_n)$ are finite ($i \geq 1$) the sequence $\{\omega_n\}_{n\geq 1}$ is an admissible sequence for the Λ-torsion module $X'_\infty(i)$. Hence we have

$$\beta\left(X'_\infty(i)\right) \simeq \varinjlim \left(X'_\infty(i)\right)_{\Gamma_n} = WK_{2i}^{\acute{e}t}(E_\infty),$$

where $\beta\left(X'_\infty(i)\right)$ denotes the co-adjoint of $X'_\infty(i)$.
Since we suppose $\mu = 0$ the sequence $\{p^n\}_{n\geq 1}$ is also an admissible sequence for $X'_\infty(i)$, whence

$$\beta\left(X'_\infty(i)\right) \simeq \varinjlim \left(X'_\infty(i)\right)/p^n = (\mathbb{Q}_p/\mathbb{Z}_p)^\lambda.$$

□

Remark. The results of this section are true for any finitely generated torsion Λ-module X: assume that X has a trivial μ-invariant and that X_{Γ_n} is finite for all $n \gg 0$, then the sequence

$$0 \to X^0 \to X_{\Gamma_n} \to \left(\varinjlim X_{\Gamma_n}\right)^{\Gamma_n} \to 0,$$

is a split exact sequence of abelian groups for all $n \gg 0$ (cp. [V, Théorème 3.1.8]).

4. Capitulation for odd K-groups

In the previous section we have shown that for n sufficiently large the capitulation kernel is a direct summand in a subgroup of $K_{2i}^{\acute{e}t}(\mathcal{O}_{F_n}^S)$. In this final section we prove that the capitulation kernel is still a direct summand in the entire group $K_{2i}^{\acute{e}t}(\mathcal{O}_{F_n}^S)$.

Since Tate's works on K_2, many relations between the wild kernels and the class group have been highlighted. The following proposition shows that the deviation between the p-quotients of the wild kernels and the p-class group is asymptotically trivial in the cyclotomic $\mathbb{Z}_p$-extension.

Assume that E contains the roots of unity of order p^n. At the end of section 2 we contructed the canonical map:

$$f_{(i,E,S)} := WK_{2i}^{\acute{e}t}(E)/p^n \to Cl(\mathcal{O}_E^S)/p^n(i), \text{ for all } i \geq 1.$$

This morphism could be surjective and not bijective for any set of primes containing $S_p \cup S_\infty$: for example let E be the Hilbert class field of $\mathbb{Q}(\mu_{37})$. For the irregular prime number $p = 37$ and for all $i \geq 1$, the wild kernels $WK_{2i}^{\acute{e}t}(E)$ are cyclic while the class group $Cl(\mathcal{O}'_E)$ is trivial.

However the map $f_{(i,E,S)}$ is asymptotically bijective.

Proposition 4.1. *Let E be a number field containing μ_p and assume that the μ invariant of X'_∞ is trivial. Let S be a set of primes containing the primes above p and the infinite primes.*

For all $h \geq 1$ there exists an integer N such that for all $n \geq N$ and for all $i \geq 1$ the map $f_{(i,E_n,S)}$ yields the isomorphism:

$$WK_{2i}^{\acute{e}t}(E_n)/p^h \simeq Cl(\mathcal{O}_{E_n}^S)/p^h(i).$$

Proof. We fix a positive integer h and a set of primes S as above. By Proposition 2.4 for all n larger than a fixed integer N the map

$$WK_{2i}^{\acute{e}t}(E_n)/p^h \to Cl(\mathcal{O}_{E_n}^S)/p^h(i) \text{ is onto.}$$

Let us compute the order of both groups for $n \gg 0$. Since $WK_{2i}^{\acute{e}t}(E_\infty)^{\Gamma_n}$ is finite we have

$$WK_{2i}^{\acute{e}t}(E_\infty)^{\Gamma_n}/p^h \simeq {}_{p^h}WK_{2i}^{\acute{e}t}(E_\infty)^{\Gamma_n}, \text{ as abelian groups.}$$

By Proposition 3.5 we have for $n \gg 0$:

$${}_{p^h}WK_{2i}^{\acute{e}t}(E_\infty)^{\Gamma_n} \simeq \bigoplus_{k=1}^{\lambda} \mathbb{Z}/p^h, \text{ as abelian groups.}$$

Hence by Proposition 3.3 we see that

$$WK_{2i}^{\acute{e}t}(E_n)/p^h \simeq (X'_\infty)^0/p^h \oplus \bigoplus_{k=1}^{\lambda} \mathbb{Z}/p^h, \text{ as abelian groups.}$$

On the other hand, since S contains the primes above p, it is well known that $X_\infty^S := \varprojlim Cl(\mathcal{O}_{E_m}^S)\{p\} = X'_\infty$, independent of S.

Theorem 0.1 is still true for S-class groups (cp. [GJ]), so for $n \gg 0$ we have

$$Cl(\mathcal{O}_{E_n}^S)/p^h \simeq (X'_\infty)^0/p^h \oplus \bigoplus_{k=1}^{\lambda} \mathbb{Z}/p^h, \text{ as abelian groups.}$$

Hence $WK_{2i}^{\acute{e}t}(E_n)/p^h$ and $Cl(\mathcal{O}_{E_n}^S)/p^h$ have the same order for n sufficiently large and the canonical surjection

$$WK_{2i}^{\acute{e}t}(E_n)/p^h \to Cl(\mathcal{O}_{E_n}^S)/p^h(i)$$

is a bijection. □

We may deduce an asymptotic rank formula for the wild kernels:

Corollary 4.1. *Under the assumptions of Proposition 4.1 we have :*

$$\dim_{\mathbb{F}_p}(WK_{2i}^{\acute{e}t}(E_n)/p) = \dim_{\mathbb{F}_p}(Cl(\mathcal{O}_{E_n}^S)/p), \text{ for } n \text{ large.}$$

In [N1, Corollaire 5.7] (see also [KC, Corollary 3.3]), the author (assuming Leopoldt's conjecture) gives the rank formula:

$$\dim_{\mathbb{F}_p}(WK_2^{\acute{e}t}(E)/p) = \dim_{\mathbb{F}_p}(Cl(\mathcal{O}'_E)/p) + \dim_{\mathbb{F}_p}(W_E \cap p\mathcal{T}_E/pW_E),$$

where

- $\mathcal{T}_E$ denotes the $\mathbb{Z}_p$-torsion of $\left(G_E^{S_p}\right)^{ab}$, and
- $W_E \simeq \Pi_{\mathfrak{p}|p}\mu(E_{\mathfrak{p}})/\mu(E)$.

By Corollary 4.1 the p-rank of the wild kernel and the p-rank of the p-class group are the same for $n \gg 0$. Thus the group $W_{E_n} \cap p\mathcal{T}_{E_n}/pW_{E_n}$ is trivial (i.e. there is an injection $W_{E_n}/p \hookrightarrow \mathcal{T}_{E_n}/p$).

We can now prove the analogue of Theorem 0.1 for even étale K-groups.

Theorem 4.1. *Let S be a finite set of primes containing the primes above p and the infinite primes. Assume that the μ-invariant of the Λ-module X'_∞ is trivial.*

Then for all $i \geq 1$ and for all n sufficiently large the exact sequence (1.1):

$$0 \to \mathrm{Cap}_i(F_n) \to K_{2i}^{\acute{e}t}(\mathcal{O}_{F_n}^S) \to K_{2i}^{\acute{e}t}(\mathcal{O}_{F_\infty}^S)^{\Gamma_n} \to 0,$$

is a split exact sequence of abelian groups.

Proof. As in the previous section it is sufficient to prove the result for the number field $E = F(\mu_p)$.

Let p^e be the order of $(X'_\infty)^0$. Let h be a positive integer, with $0 \leq h \leq e$. Then for n large:

(1) the field E_n contains the roots of unity of order p^h.
(2) the group $\mathrm{Cap}_i(E_n)$ is a direct summand in $WK_{2i}^{\acute{e}t}(E_n)$.
(3) the canonical map $WK_{2i}^{\acute{e}t}(E_n)/p^h \simeq Cl(\mathcal{O}_{E_n}^S)/p^h(i)$ is an isomorphism.

Points (2) and (3) follow from the assumption $\mu = 0$.

Using points (1) and (3) and Proposition 2.2 we can write the commutative diagram

$$\begin{tikzcd}
WK_{2i}^{\acute{e}t}(E_n)/p^h \arrow[r] \arrow[d, "\simeq"] & K_{2i}^{\acute{e}t}(\mathcal{O}_{E_n}^S)/p^h \arrow[d, "="] \\
Cl(\mathcal{O}_{E_n}^S)/p^h(i) \arrow[r, hook] & K_{2i}^{\acute{e}t}(\mathcal{O}_{E_n}^S)/p^h
\end{tikzcd}$$

The left vertical map is bijective : it follows from Proposition 4.1. Thus the top horizontal arrow, induced by the inclusion, is injective.

On the other hand point (2) implies that $\mathrm{Cap}_i(E_n)/p^h \hookrightarrow WK_{2i}^{ét}(E_n)/p^h$. Hence we have:

$$\mathrm{Cap}_i(E_n)/p^h \hookrightarrow K_{2i}^{ét}(\mathcal{O}_{E_n}^S)/p^h.$$

We finally use Lemma 1.1 to conclude that $\mathrm{Cap}_i(E_n)$ is a direct summand in the abelian group $K_{2i}^{ét}(\mathcal{O}_{E_n}^S)$. $\square$

The Galois co-descent holds for the étale K-groups in a p-ramified extension. Hence, as for the wild kernels, we have

Corollary 4.2. *Under the assumptions of the previous theorem, for n sufficiently large, and for all $m \geq n$, the groups $K_{2i}^{ét}(\mathcal{O}_{E_m}^S)^{G_{m,n}}$ and $K_{2i}^{ét}(\mathcal{O}_{E_n}^S)$ are isomorphic as abelian groups.*

Finally, to get the result for the *algebraic* K-groups, we may use the Quillen-Lichtenbaum conjecture to identify algebraic and étale K-theory. This conjecture predicts that the Chern character yields the canonical isomorphism (see [Ko] and [W, Theorem 70]):

$$K_{2i}(\mathcal{O}_F^T) \otimes \mathbb{Z}_p \simeq K_{2i}^{ét}(\mathcal{O}_F^T[1/p]),$$

for all $i \geq 1$ and all finite sets of primes T.
Unpublished Voevodsky 's results on the Bloch-Kato conjecture for number fields seem prove this conjecture.

Theorem 4.2. *Let p be an prime number and T be a finite set of primes of a number field F containing $\sqrt{-1}$ if $p = 2$. Let F_∞ be the cyclotomic $\mathbb{Z}_p$-extension of F with finite layers F_n and assume that the μ-invariant of the Λ-module $\varprojlim Cl(\mathcal{O}'_{F(\mu_{p^n})})\{p\}$ is trivial. Let i be a non-negative integer. Then, assuming the Quillen-Lichtenbaum conjecture, for n large the capitulation kernel $\mathrm{Cap}_i(F_n)$ in F_∞ is a direct summand in the abelian group $K_{2i}(\mathcal{O}_{F_n}^T)\{p\}$.*

Remark. We can wonder if the result still holds for the even K-groups of the fields F_n (instead of its ring of integers). For $i = 0$ the answer is trivial. For $i \geq 1$ it is well known that the étale wild kernels are isomorphic to the divisible part of $K_{2i}(F_n)\{p\}$ (we recall that p is odd and $\sqrt{-1} \in F$, if $p = 2$). Thus the capitulation kernel is contained in the divisible part of $K_{2i}(F_n)\{p\}$ and it can not be a direct summand (except if it is trivial).

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Romain VALIDIRE
XLIM DMI - UMR CNRS 6172
123, avenue Albert Thomas
87060 LIMOGES CEDEX (France).
E-mail: romain.validire@gmail.com
URL: http://www.math.u-bordeaux.fr/A2X/