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Appendix to the paper “On the Billingsley dimension of Birkhoff average in the countable symbolic space”


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Appendix to the paper “On the Billingsley dimension of Birkhoff average in the countable symbolic space”

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Abstract. This appendix gives a lower bound of the Billingsley-Hausdorff dimension of a level set related to Birkhoff average in the "non-compact" symbolic space $\mathbb{N}^\mathbb{N}$, defined by Gibbs measure.

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The authors in [1] estimate the upper bound of the Billingsley dimension of the levels sets $\hat{E}_f(\alpha)$, defined by Gibbs measures. In the following, we give the lower bound.

**Theorem 1.** Let $\varphi$ be a potential function of summable variations. We assume that $\varphi$ admits a unique Gibbs measure $\nu$, then

\[ \dim_{\nu} \hat{E}_f(\alpha) = \sup \left\{ \gamma(\nu, \mu); \int_{\mathcal{X}} f \, d\mu = \alpha \right\}. \]

**Proof.** For any $\mu \in \mathcal{P}_f(\mathcal{X})$, define the set of $\mu$-generic points by

\[ G_{\mu} = \left\{ x \in \mathcal{X}; \lim_{n \to +\infty} \frac{1}{n} S_n f(x) = \int_{\mathcal{X}} f \, d\mu \quad \text{for all} \quad C_b(\mathcal{X}) \right\}. \]

Remark that the sets $G_{\mu}$ for which $\int_{\mathcal{X}} f \, d\mu = \alpha$ are all included in the set $\hat{E}_f(\alpha)$. Thus by using Theorem 1.1 in [2], we obtain

\[ \sup \left\{ \gamma(\nu, \mu); \int_{\mathcal{X}} f \, d\mu = \alpha \right\} \leq \dim_{\nu} \hat{E}_f(\alpha). \]

References
