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DU BOIS INVARIANTS OF ISOLATED COMPLETE INTERSECTION SINGULARITIES

by Joseph H.M. STEENBRINK (*)

1. Introduction.

In this paper we study invariants of isolated singularities which arise from Du Bois' work on the filtered de Rham complex [1]. Therefore we call them Du Bois invariants. In general, these are hard to compute, and their meaning is not very clear. The reason to consider them here is that one of them occurs in the deformation theory of singular Calabi-Yau threefolds. This note is the result of an attempt to understand them better, and aims to give a complete survey of what is known about them until now. We relate them to the Hodge numbers of the local and vanishing cohomology groups. Our main results are Theorem 4 which expresses the Tjurina number of certain Gorenstein singularities in terms of Du Bois invariants and Hodge numbers of the link, and Theorem 6 which expresses the Hodge numbers of the Milnor fibre of certain three-dimensional complete intersections in similar terms. We also address the question whether the Du Bois invariants are semicontinuous under deformation of the singularity.

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2. Du Bois invariants.

Let (X,x) be an isolated singularity of pure dimension n and let $\pi : (Y,E) \to (X,x)$ be a good resolution of it, i.e. E is a divisor with normal crossings on Y. We fix a representative $\pi : Y \to X$ with X a contractible Stein space; then $E \hookrightarrow Y$ is a homotopy equivalence. We put $U := Y \setminus E \simeq X \setminus \{x\}$.

The Du Bois invariants $b^{p,q}$ of (X,x) are defined by

$$b^{p,q}(X,x) := \dim H^q(Y, \Omega^p_Y(\log E)(-E))$$

for $p \geq 0$ and $q > 0$. Their definition does not depend on the choice of the good resolution, as the Du Bois invariants can be defined in terms of the filtered de Rham complex $(\tilde{\Omega}^{\cdot}_X, F)$ of X (see [1]):

$$b^{p,q} = \dim H^{p+q} Gr^p_F(\tilde{\Omega}^{\cdot}_{X,x}).$$

This follows from [13, Prop. 3.3].

We list some properties of the Du Bois invariants:

THEOREM 1. — *$b^{p,q}(X,x) = 0$ if $p+q > n$ or $q \geq n$.*

See [3, Thm 2] and [5, Thm 6.6.1, Thm 6.7.1] for the first case. The second case follows from the fact that Y is an n-dimensional complex manifold without compact component: by Serre duality, $H^n(Y,F) = 0$ for all locally free sheaves F on Y.

THEOREM 2. — *If (X,x) is a toric isolated singularity, then $b^{p,q} = 0$ for all $p \geq 0$ and $q > 0$.*

See [5, Exp. V Sect. 4].

PROPOSITION 1. — *If (X,x) has depth $\geq k$, then $b^{0,q} = 0$ for $0 < q < k-1$.*

Proof. — Suppose that (X,x) has depth $\geq k$. Then $H^i_{\{x\}}(X, O_X) = 0$ for $i < k$ by [4]. As X is a Stein space, this implies that $H^i(U, O_U) = 0$ for $0 < i < k-1$. By duality of Grauert–Riemenschneider's vanishing theorem we have $H^i_E(Y, O_Y) = 0$ for $i < n$, so $H^i(Y, O_Y) \simeq H^i(U, O_U)$ for $i < n-1$. Hence $H^i(Y, O_Y) = 0$ for $0 < i < k-1$. By [11, Lemma 2.14], the maps $H^i(Y, O_Y(-E)) \to H^i(Y, O_Y)$ are injective for all i, so $H^i(Y, O_Y(-E)) = 0$ for $0 < i < k-1$.

In case (X, x) is Cohen-Macaulay, i.e. has depth n, the only possibly non-zero $b^{0,q}$ is $b^{0,n-1}$. We call (X, x) a *Du Bois singularity* if $b^{0,q} = 0$ for $0 < q < n$. Note that the vanishing of $H^i(Y, O_Y)$ in this range characterizes rational singularities. Du Bois singularities, in particular Gorenstein ones, have been studied by Ishii [6], [7]. She gives an example of a small deformation of a normal isolated Du Bois singularity such that the deformed singularity is no longer Du Bois, and shows that this cannot happen in the Gorenstein case. It follows that the Du Bois invariants are not upper semicontinuous under deformation in general, but might still be semicontinuous for Gorenstein singularities. However, we will give a counterexample to this in Section 4.

If one only considers deformations which can be simultaneously resolved, then the invariants $b^{p,q}$ do depend upper semicontinously on the parameters. This follows from general semicontinuity theorems concerning higher direct images of coherent sheaves under flat proper mappings.

The restriction of the complex $\Omega^\cdot_Y(\log E)(-E))$ to E is acyclic, hence

$$\mathbf{H}^i(Y, \Omega^\cdot_Y(\log E)(-E))) \simeq H^i(Y, E, \mathbf{C}) = 0.$$

This implies that the first spectral sequence of hypercohomology

$$(1) \qquad E_1^{p,q} = H^q(Y, \Omega^p_Y(\log E)(-E)) \Rightarrow \mathbf{H}^{p+q}(Y, \Omega^\cdot_Y(\log E)(-E)))$$

converges to 0. It follows, that also $b^{1,n-1} = 0$ for a Du Bois singularity.

The Du Bois invariant $b^{1,1}$ has a particular importance for rational singularities:

THEOREM 3. — *Let (X, x) be a rational isolated singularity and let $\pi : Y \to X$ be a good resolution. Then the subspace generated by the image of the map*

$$d\log : H^1(Y, O^*_Y) \to H^1(Y, \Omega^1_Y)$$

has codimension $b^{1,1}(X, x)$.

See [10, Prop. 2.1].

3. Invariants from local cohomology.

Let X, Y be as in the previous section. Let $L = \partial X$ be the link of (X, x). The cohomology groups $H^i(L)$ carry a mixed Hodge structure. (See

[11, Sect. 1] for the proofs of the following statements.) We will focus on its Hodge filtration, and define invariants $l^{p,q}$ of (X,x) by

$$l^{p,q} := \dim Gr_F^p H^{p+q}(L,\mathbf{C}) = \dim H^q(E,\Omega_Y^p(\log E)\otimes O_E).$$

Poincaré duality for L induces a duality of mixed Hodge structures

$$H^{2n-1-i}(L) \simeq \mathrm{Hom}(H^i(L),\mathbf{Q}(-n))$$

and we obtain the equalities

$$l^{p,q} = l^{n-p,n-q-1}.$$

If we put $h_i^{p,q} := \dim Gr_F^p Gr_{p+q}^W H^i(L,\mathbf{C})$, then one has $h_i^{p,q} = h_i^{q,p}$ and

$$l^{p,i-p} = \sum_q h_i^{p,q}.$$

By the semipurity theorem [11, Cor. 1.12], $h_i^{p,q} = 0$ in the following cases: if $i < n, p+q > i$ and if $i \geq n, p+q \leq i$.

LEMMA 1. — *If $p < n$ then $\sum_{i=0}^{p} l^{i,p-i} \geq \sum_{i=0}^{p} l^{p-i,i}$.*

The proof is an easy consequence of semipurity.

LEMMA 2. — *Let (X,x) be a rational singularity. Then $l^{0,i} = l^{i,0} = 0$ for all i.*

Proof. — We have $l^{0,i} = \dim H^i(E,O_E) = 0$ because $H^i(Y,O_Y) = 0$ (as (X,x) is rational) and $H^i(Y,O_Y) \to H^i(E,O_E)$ is surjective by [11, Lemma 2.14]. Moreover, $l^{i,0} \leq l^{0,i}$.

We recall the following invariants:

$\tau(X,x) = \dim T^1_{X,x}$ (the Tjurina number);

$q'(X,x) = \dim H^0(U,\Omega_U^{n-1})/H^0(Y,\Omega_Y^{n-1}(\log E)(-E))$;

$\alpha(X,x) := \dim H^0(Y,\Omega_Y^n)/dH^0(Y,\Omega_Y^{n-1}(\log E)(-E))$ (compare [15, 1.8] for the case $n=2$).

THEOREM 4. — *Let (X,x) be an isolated Gorenstein singularity of dimension at least three for which $l^{1,n-3} = 0$. Then*

$$\tau = b^{n-1,1} + b^{1,n-2} - b^{1,n-1} + l^{1,n-2} - l^{n-1,0} + q'.$$

If moreover (X,x) is Du Bois and $l^{n-1,0} = 0$, then

$$\tau = b^{n-1,1} + b^{1,n-2} + l^{1,n-2}.$$

Proof. — We have $T^1_{X,x} \simeq H^1(U,\Theta_U) \simeq H^1(U,\Omega^{n-1}_U)$ as (X,x) is Gorenstein. We have the exact sequence of local cohomology for $\Omega^{n-1}_Y(\log E)(-E)$ which reduces to

$$0 \to \mathbf{C}^{q'} \to H^1_E(Y,\Omega^{n-1}_Y(\log E)(-E)) \to H^1(Y,\Omega^{n-1}_Y(\log E)(-E)) \to H^1(U,\Omega^{n-1}_U) \to H^2_E(Y,\Omega^{n-1}_Y(\log E)(-E)) \to 0$$

as $b^{n-1,2}=0$. Hence

$$\tau = q' + b^{n-1,1} + h^2_E - h^1_E$$

where we write

$$h^i_E = \dim H^i_E(Y,\Omega^{n-1}_Y(\log E)(-E)) = \dim H^{n-i}(Y,\Omega^1_Y(\log E))$$

for $i=1,2.$.

Because $l^{1,n-3}=0$, we have the exact sequence

$$0 \to H^{n-2}(Y,\Omega^1_Y(\log E)(-E)) \to H^{n-2}(Y,\Omega^1_Y(\log E)) \to Gr^1_F H^{n-1}(L) \to H^{n-1}(Y,\Omega^1_Y(\log E)(-E)) \to H^{n-1}(Y,\Omega^1_Y(\log E)) \to Gr^1_F H^n(L) \to 0$$

which implies that

$$h^2_E - h^1_E = b^{1,n-2} - b^{1,n-1} + l^{1,n-2} - l^{1,n-1}.$$

Finally use that $l^{1,n-1} = l^{n-1,0}$.

Suppose that (X,x) is moreover Du Bois. Consider the exact sequence

$$H^0(E,\Omega^{n-1}_Y(\log E)\otimes O_E) \to H^1_E(Y,\Omega^{n-1}_Y(\log E)(-E)) \to H^1_E(Y,\Omega^{n-1}_Y(\log E)).$$

By duality, the third space has dimension $b^{1,n-1}=0$ as (X,x) is Du Bois, and the first one has dimension $l^{n-1,0}$. Hence $l^{n-1,0}=0$ implies $h^1_E=0$, so $q'=0$.

COROLLARY. — *Let (X,x) be an isolated toric Gorenstein singularity of dimension at least three. Then $\tau = l^{1,n-2}$. Indeed, toric singularities are rational. As $b^{1,n-2}=0$, the formula for τ holds even without the hypothesis $l^{1,n-3}=0$.*

The condition $l^{1,n-3}=0$ is fulfilled for isolated complete intersection singularities, see the next section, and for rational singularities in dimension three. More generally we have

PROPOSITION 2. — *Let (X,x) be a three–dimensional Gorenstein Du Bois singularity with $l^{1,0}=l^{2,0}=0$ (e.g. a rational singularity). Then*

$$\tau = 2b^{1,1} + l^{1,1} - \alpha.$$

Proof. — As (X,x) is Du Bois, $b^{1,2}=0$. The convergence to zero of the spectral sequence (1) implies that in this case, $\alpha = b^{1,1}-b^{2,1}$.

This improves [9, Theorem 1]. If moreover (X,x) is a toric singularity, we get $\tau = l^{1,1}$. The rigidity of three–dimensional cyclic quotient singularities (for which $l^{1,1}=0$) is a direct consequence.

4. The case of isolated complete intersection singularities.

The link L of an n–dimensional isolated complete intersection singularity is $(n-2)$–connected, so $H^i(L)\neq 0$ can only occur for $i\in\{0,n-1,n,2n-1\}$. Hence $l^{p,q}=0$ unless $p+q\in\{0,n-1,n,2n-1\}$.

Recall that for an n–dimensional isolated complete intersection singularity (X,x) and $p=0,\dots,n$, an invariant $s_p(X,x)$ can be defined in the following way. Choose any one parameter smoothing $f:(X',x)\to(\mathbf{C},0)$ of (X,x) and define

$$s_p(X,x) := \dim Gr_F^p\phi_f\mathbf{C}_{X'}$$

as a Hodge number of the Milnor fibre of (X,x). It does not depend on the choice of the smoothing (see [12]). Moreover it is an upper semicontinuous invariant under deformations of (X,x). Of course, $\sum_{p=0}^{n} s_p(X,x)=\mu(X,x)$, the Milnor number of (X,x).

For each one-parameter smoothing with Milnor fibre F one has the exact sequence of mixed Hodge structures

$$0\to H^{n-1}(L)\to H^n(F,L)\to H^n(F)\to H^n(L)\to 0.$$

Taking into account the duality of $H^n(F,L)$ and $H^n(F)$ (w.r.t. $\mathbf{Q}(-n)$) we obtain

$$s_p-s_{n-p}=l^{p,n-p}-l^{p,n-p-1}=l^{n-p,p-1}-l^{p,n-p-1}.$$

We conclude that $\sum_{i=0}^{p} s_i \le \sum_{i=0}^{p} s_{n-i}$ for $0\le p\le n$.

Note that $s_n(X,x)$ is the geometric genus of (X,x) [11, Prop. 2.13] and that $s_0(X,x)=s_n(X,x)-l^{0,n-1}=\dim H^{n-1}(Y,O_Y)-\dim H^{n-1}(E,O_E)$ $=b^{0,n-1}$ by [11, Lemma 2.14]. In particular $s_0=0$ characterizes Du Bois singularities among isolated complete intersection singularities. In general, there seem no other connections between Du Bois invariants and Hodge numbers of the Milnor fiber.

THEOREM 5. — *Let (X,x) be an isolated complete intersection singularity of dimension n. Then $b^{p,q}(X,x)=0$ unless $p+q\in\{n-1,n\}$.*

Proof. — If $n\le 2$ the theorem reduces to Theorem 1. So suppose that $n\ge 3$. Then $b^{0,1}=0$ as (X,x) is Cohen-Macaulay. So we still have to deal with the case $2\le p+q\le n-2$. We have the exact sheaf sequences

$$0\to\Omega_Y^p(\log E)(-E)\to\Omega_Y^p(\log E)\to\Omega_Y^p(\log E)\otimes O_E\to 0.$$

Note that $H^q(E,\Omega_Y^p(\log E)\otimes O_E)=0$ for $n<p+q<2n-1$ (as $H^{p+q}(L)=0$ in that range). As also $H^q(Y,\Omega_Y^p(\log E)(-E))=0$ for these p,q (by Theorem 1), we obtain

$$H^q(Y,\Omega_Y^p(\log E))=0 \text{ for } n<p+q<2n-1.$$

By duality we get

$$H_E^q(Y,\Omega_Y^p(\log E)(-E))=0 \text{ for } 1\le p+q\le n-1.$$

Suppose $1\le p+q\le n-1$. Then $H^q_{\{x\}}(\Omega_X^p)=0$ by [2, Prop. 2.3]. Hence the natural map

$$H_E^q(Y,\Omega_Y^p(\log E)(-E))\to H^q(Y,\Omega_Y^p(\log E)(-E))$$

is an isomorphism for $2\le p+q<n-1$. Therefore $H^q(Y,\Omega_Y^p(\log E)(-E))=0$ in this range.

The convergence of the spectral sequence (1) to zero implies

PROPOSITION 3. — *For (X,x) an isolated complete intersection singularity of dimension n, we have*

$$\alpha=\sum_{p=1}^{n-1}(b^{p-1,n-p}-b^{p,n-p}).$$

LEMMA 3. — *Let (X,x) be a Du Bois isolated complete intersection singularity of dimension at least three with $l^{n-1,0}=0$. Then*

$$\mu=b^{n-1,1}+b^{1,n-2}+l^{1,n-2}+l^{0,n-1}+\alpha.$$

Proof. — By [8], for any isolated complete intersection singularity of dimension at least two,

$$\mu-\tau=l^{0,n-1}-l^{n-1,0}+a_1+a_2+a_3$$

for certain invariants a_1,a_2,a_3. For a Du Bois singularity, $a_1=0$, and $a_3\le l^{n-1,0}$ by its definition. Moreover, $l^{n-1,0}=0$ implies that $a_2=\alpha$. So we have $\mu-\tau=l^{0,n-1}+\alpha$. We conclude by the second case of Theorem 4.

THEOREM 6. — *Let (X, x) be a three-dimensional Du Bois isolated complete intersection singularity with $l^{2,0} = 0$. Then the Hodge numbers of the Milnor fibre are given by*

$$s_0 = 0, s_1 = b^{1,1}, s_2 = b^{1,1} + l^{1,1}, s_3 = l^{0,2}.$$

Proof. — The statements about s_0 and s_3 have been proved previously, as well as the identity $s_2 - s_1 = l^{1,1} - l^{2,0} = l^{1,1}$. Finally $s_1 + s_2 + s_3 = \mu = 2b^{1,1} + l^{1,1} + l^{0,2}$.

COROLLARY. — *For rational isolated complete intersection singularities of dimension three, the invariant $b^{1,1}$ is upper semicontinuous under deformation.*

Indeed, this follows from the semicontinuity of s_1. As a consequence, we get a generalization of [10, Theorem 2.2]: if (X, x) is a rational isolated complete intersection singularity of dimension three, which is not an ordinary double point, then $b^{1,1}(X, x) > 0$. Indeed, if (X, x) is not an ordinary double point, it deforms into a singularity of type A_2 or to the intersection of two quadrics (type $\tilde{D}_6$) which have $s_1 = 1$ and $s_1 = 2$ respectively.

We now give our example that the invariant $b^{1,1}$ is not upper semicontinuous under deformations in general, even for two–dimensional hypersurfaces.

Example. — Let $f_t(x, y, z) = tx^6 + x^7 + x^4y^4 + y^7 + z^2$ and let X_t be the surface germ at $0 \in \mathbf{C}^3$ given by $f_t(x, y, z) = 0$. Then $b^{0,1}(X_t) = 3$ if $t \neq 0$ whereas $b^{0,1}(X_0) = 2$. This shows that the invariant $b^{0,1}$ does not depend upper semicontinuously on deformation parameters in general.

To compute these numbers, we use the formula $p_g + \alpha = \dim Q^f/\overline{K} \cap V_{>0}$ from [14, Cor. (6.3)] which is valid for a two–dimensional isolated hypersurface singularity. Here $Q^f := \Omega^3/df \wedge \Omega^2$, V is its Kashiwara-Malgrange filtration, $p_g = \dim Q^f/V_{>0}$ is the geometric genus and $\overline{K}$ is the kernel of multiplication by f in Q^f. In the example at hand, f_t is semi-quasi-homogeneous for all t, with weights $\left(\frac{1}{6}, \frac{1}{7}, \frac{1}{2}\right)$ for $t \neq 0$ and with weights $\left(\frac{1}{7}, \frac{1}{7}, \frac{1}{2}\right)$ for $t = 0$. For all t we have the equalities

$$V_0 Q^f = V_{>0} Q^f = (x^2, xy, y^2) Q^f$$

which imply that $b^{0,1} = p_g = 3$ so $b^{1,1} = 3 - \alpha$.

Comparing the difference between the maximal and minimal weights of Q^f with the weights of x and y, one checks that $x^{10}Q^f = y^{10}Q^f = 0$. A simple Gröbner basis calculation shows that for $t \neq 0$ one has $\overline{K} = (x, y^2)Q^f$ hence $\alpha = 0$ and $b^{1,1} = 3$. On the other hand, if $t = 0$ then $\overline{K} = (x^2, y^2)Q^f \subset V_{>0}$ and $\alpha = 1$ so $b^{1,1} = 2$.

5. Some consequences and remarks.

PROPOSITION 4. — *Let (X, x) be an isolated toric complete intersection singularity. Then either (X, x) is a surface singularity of type A_k with $k \geq 1$ or an ordinary double point in dimension three.*

Proof. — It is well-known that the only toric surface singularities are the cyclic quotient singularities; these can only be Gorenstein if they are of type A_k.

A toric isolated complete intersection singularity of dimension three has $s_1 = b^{1,1} = 0$ so must be the ordinary double point.

If (X, x) is a complete intersection of dimension at least four, then $H^2(L) = 0$. For a toric isolated singularity this group has rank equal to $k - \dim(X)$ where k is the number of one-dimensional faces of the corresponding cone. One has $k = \dim(X)$ if and only if the cone is simplicial, i.e. X is a quotient singularity. However, isolated quotient singularities in dimension at least three are rigid, so cannot be complete intersections.

We end with the following question. Let (X, x) be a three-dimensional isolated rational complete intersection singularity (more generally: a hypersurface section of an isolated Gorenstein singularity X' with $T^2_{X'} = 0$) and let $(Y, E) \to (X, x)$ be a good resolution. One has the exact sequence

$$0 \to H^1(Y, \Omega^2_Y(\log E)(-E)) \to T^1_{X,x} \to H^2_E(Y, \Omega^2_Y(\log E)(-E)) \to 0$$

as $T^1_{X,x} \simeq H^1(Y \setminus E, \Omega^2_Y)$. Is it true that the tangent cone to the discriminant of the versal deformation of (X, x) (which is in a natural way a cone inside $T^1_{X,x}$) is invariant under translations by the subspace $H^1(Y, \Omega^2_Y(\log E)(-E))$?

In [3] it is shown that the tangent cone to the discriminant is invariant under translations by $\mathbf{m}T^1_X$ where $\mathbf{m}$ is the maximal ideal of $O_{X,x}$. So one may as well ask whether $H^1(Y, \Omega^2_Y(\log E)(-E)) \subset \mathbf{m}T^1_X$. This has been proven in the hypersurface case in [10].

If the answer to the question is positive, then the first proof of smoothability of $\mathbf{Q}$-factorial Calabi-Yau threefolds from [10] goes through if these have isolated complete intersection singularities.

In [9], the second proof of [10] is generalized even further: Namikawa there admits smoothable isolated singularities which are either toric or have smooth semiuniversal base space.

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