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HIROMICHI NAKAYAMA

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TRANSVERSELY AFFINE FOLIATIONS OF SOME SURFACE BUNDLES OVER S^1 OF PSEUDO-ANOSOV TYPE

by Hiromichi NAKAYAMA

Introduction.

E. Ghys and V. Sergiescu classified codimension one foliations without compact leaves of torus bundles over S^1 whose monodromy matrices are hyperbolic automorphism ([2]). They cut the manifold along some fiber transverse to the foliation $\mathcal{F}$ and modified the resulting foliation $\mathcal{F}|(T^2 \times I)$ ($I = [0,1]$) so that $\mathcal{F}|(T^2 \times I)$ is tangent to each $\{*\} \times I (* \in T^2)$. Then $\mathcal{F}|(T^2 \times \{0\})$ is equal to $\mathcal{F}|(T^2 \times \{1\})$. However it is difficult to classify foliations without compact leaves of higher genus surface bundles over S^1 because it is difficult to find a fiber S so that the singular foliation $\mathcal{F}|(S \times \{0\})$ coincides with $\mathcal{F}|(S \times \{1\})$ and to classify the foliation of $\Sigma \times I$. In this paper, we restrict our attention to transversely affine foliations without compact leaves of some higher genus surface bundles over S^1 of pseudo-Anosov type and obtain the following results :

MAIN THEOREM. — *Let Σ be a closed orientable surface with genus greater than 1 and let $\pi : M \to S^1$ be an oriented Σ-bundle over S^1 of pseudo-Anosov type such that the real eigenvalues of its monodromy matrix are λ and $\frac{1}{\lambda}$, and the eigenspace with respect to λ (resp. $\frac{1}{\lambda}$) is one dimensional, where λ (> 1) is the dilatation number of M. Suppose that $\mathcal{F}$ is a transversely oriented and transversely affine codimension one*

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foliation of M without compact leaves satisfying the Euler class equality $\chi(T\mathcal{F}) = \pm\chi(T\pi)$ ($\in H^2(M;\mathbb{Z})$), where $T\mathcal{F}$ and $T\pi$ denote the tangent bundles of the foliation $\mathcal{F}$ and the bundle foliation of π respectively. Then there is a finite covering of $\mathcal{F}$ which is C^0 isotopic to a suspension foliation of a pseudo-Anosov diffeomorphism.

PROPOSITION. — *There is an orientable Σ-bundle over S^1 of pseudo-Anosov type satisfying the conditions of the main theorem. (I.e. the real eigenvalues of its monodromy matrix are λ and $\frac{1}{\lambda}$, and the eigenspace with respect to λ (resp. $\frac{1}{\lambda}$) is one dimensional, where λ is the dilatation number.)*

In Section 1, we give a precise definition of suspension foliations of pseudo-Anosov diffeomorphisms introduced by Meigniez [8], and prove the above proposition. For each bundle structure of pseudo-Anosov type, there exist suspension foliations of the pseudo-Anosov diffeomorphism. The hypothesis of the main theorem on the real eigenvalues of the monodromy and their eigenspaces restricts the bundle structures of M. S. Matsumoto showed the author examples of transversely affine foliations of M which are not isotopic to the suspension foliations of pseudo-Anosov diffeomorphisms and have the same holonomy representation as the suspension foliations have ($\chi(T\mathcal{F}) \neq \pm\chi(T\pi)$), which we also describe. In Section 2, we show the existence of a finite covering $\widehat{p} : \widehat{M} \to M$ and an embedding $\widehat{g} : \Sigma \to \widehat{M}$ isotopic to a fiber of the Σ-bundle $\widehat{M}$ over S^1 such that $\widehat{g}^*\widehat{p}^*\mathcal{F}$ is C^0 isotopic to a stable or unstable foliation of a pseudo-Anosov diffeomorphism which is C^0 isotopic to the monodromy map of $\widehat{M}$ (Theorem 2). We prove the main theorem in Section 3.

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1. Pseudo-Anosov diffeomorphisms and their suspension foliations.

Let Σ be a closed orientable surface with genus greater than 1. A *pseudo-Anosov diffeomorphism* $f : \Sigma \to \Sigma$ ([1]) is a homeomorphism with two measured foliations $(\mathcal{G}^s, \mu^s)$ and $(\mathcal{G}^u, \mu^u)$ such that $\mathcal{G}^s$ and $\mathcal{G}^u$ are mutually transverse with the same saddle singularities, $f(\mathcal{G}^s, \mu^s) =$

$(\mathcal{G}^s, \frac{1}{\lambda}\mu^s)$ $(\lambda > 1)$ and $f(\mathcal{G}^u, \mu^u) = (\mathcal{G}^u, \lambda\mu^u)$, where we adopt the definition of measured foliations written in [1] and f is supposed to be a C^∞ diffeomorphism except at the saddle singularities of $\mathcal{G}^s$. The measured foliation $(\mathcal{G}^s, \mu^s)$ (resp. $(\mathcal{G}^u, \mu^u)$) is called the *stable* (resp. *unstable*) *foliation* of f, and λ is called the *dilatation number* of f.

W. Thurston showed that every diffeomorphism of Σ is C^0 isotopic to a "reducible" diffeomorphism or a periodic map or a pseudo-Anosov diffeomorphism ([1], [16]), and a pseudo-Anosov diffeomorphism is C^0 isotopic to neither a "reducible" diffeomorphism nor a periodic map.

Throughout this paper, we assume that $\mathcal{G}^\sigma$ $(\sigma = s, u)$ is transversely oriented and f preserves the transverse orientation of $\mathcal{G}^\sigma$. In particular, the number of separatrices passing through each saddle singularity is an even number.

A surface bundle M over S^1 is of *pseudo-Anosov type* if its monodromy map is C^0 isotopic to a pseudo-Anosov diffeomorphism. The *dilatation number* λ of M is defined by that of the pseudo-Anosov diffeomorphism. By the arguments of Exposé 12 of [1], λ does not depend on the choice of pseudo-Anosov diffeomorphisms C^0 isotopic to the monodromy map of M. The *monodromy matrix* of M is the linear automorphism of $H_1(\Sigma)$ induced by f. Since we assume that f preserves the transverse orientation of $\mathcal{G}^\sigma$, λ and $\frac{1}{\lambda}$ are eigenvalues of the monodromy matrix.

Next we define suspension foliations of pseudo-Anosov diffeomorphisms. Let M be an oriented Σ-bundle over S^1 of pseudo-Anosov type and let f be a pseudo-Anosov diffeomorphism C^0 isotopic to the monodromy map of M. Denote by $(\mathcal{G}^s, \mu^s)$ and $(\mathcal{G}^u, \mu^u)$ the stable and unstable foliations of f respectively, and denote by K the set of saddle singularities of $\mathcal{G}^s$. Since $\mathcal{G}^\sigma$ $(\sigma = s, u)$ is transversely oriented, there exists a non-singular closed 1-form ω^σ of $\Sigma - K$ *defining* the measured foliation $(\mathcal{G}^\sigma, \mu^\sigma)$. (I.e. the kernel of ω^σ coincides with the tangent bundle of $\mathcal{G}^\sigma$ and $\int_\gamma \omega^\sigma = \mu^\sigma(\gamma)$, where γ is a transverse arc of $\mathcal{G}^\sigma$ oriented by the transverse orientation of $\mathcal{G}^\sigma$). Let $\mathcal{H}(\sigma, \alpha, \mathcal{G}^\sigma, \mu^\sigma)$ $(\sigma = s, u, \alpha \neq 0)$ denote the foliation of $(\Sigma - K) \times \mathbf{R}$ *defined* by the non-singular 1-form $\lambda^{\varepsilon(\sigma)t}\omega^\sigma + \alpha dt$ $(t \in \mathbf{R})$, where $\varepsilon(s) = 1$ and $\varepsilon(u) = -1$. (I.e. $T\mathcal{H}(\sigma, \alpha, \mathcal{G}^\sigma, \mu^\sigma) = \mathrm{Ker}(\lambda^{\varepsilon(\sigma)t}\omega^\sigma + \alpha dt)$.) The completion of $\mathcal{H}(\sigma, \alpha, \mathcal{G}^\sigma, \mu^\sigma)$ in $\Sigma \times \mathbf{R}$ is denoted by $\widehat{\mathcal{H}}(\sigma, \alpha, \mathcal{G}^\sigma, \mu^\sigma)$. For the $\mathbf{Z}$-action θ of $\Sigma \times \mathbf{R}$ given by $\theta_n(x, t) = (f^{-n}(x), t + n)$ $(n \in \mathbf{Z})$, the quotient space of $\Sigma \times \mathbf{R}$ by

θ is C^0 isotopic to M. Since $\theta_n^*(\lambda^{\varepsilon(\sigma)t}\omega^\sigma + \alpha dt) = \lambda^{\varepsilon(\sigma)t}\omega^\sigma + \alpha dt$ (here $f^*\omega^\sigma = \lambda^{\varepsilon(\sigma)}\omega^\sigma$), $\widehat{\mathcal{H}}(\sigma, \alpha, \mathcal{G}^\sigma, \mu^\sigma)/\theta$ is a transversely orientable minimal C^0 foliation of M with holonomy (having a locally dense resilient leaf [4]), denoted by $\mathcal{F}(\sigma, \alpha, \mathcal{G}^\sigma, \mu^\sigma, f)$.

PROPOSITION. — *Let f and $\overline{f}$ be pseudo-Anosov diffeomorphisms C^0 isotopic to the monodromy map of M, and let $(\mathcal{G}^\sigma, \mu^\sigma)$ and $(\overline{\mathcal{G}}^\sigma, \overline{\mu}^\sigma)$ be the (un-)stable foliations of f and $\overline{f}$ respectively ($\sigma = s, u$). Then $\mathcal{F}(\sigma, \alpha, \overline{\mathcal{G}}^\sigma, \overline{\mu}^\sigma, \overline{f})$ is C^0 isotopic to $\mathcal{F}(\sigma, \pm 1, \mathcal{G}^\sigma, \mu^\sigma, f)$ for any non-zero number α.*

Proof. — Since f and $\overline{f}$ are C^0 isotopic pseudo-Anosov diffeomorphisms, there is a diffeomorphism g of Σ isotopic to the identity map satisfying $gf = \overline{f}g$ and $g(\mathcal{G}^\sigma, \mu^\sigma) = (\overline{\mathcal{G}}^\sigma, k\overline{\mu}^\sigma)$ ($\sigma = s, u$) for some $k > 0$ ([1], Exposé 12). Denote by ω^σ (resp. $\overline{\omega}^\sigma$) the closed 1-form defining $(\mathcal{G}^\sigma, \mu^\sigma)$ (resp. $(\overline{\mathcal{G}}^\sigma, \overline{\mu}^\sigma)$), which is defined except at the saddle singularities of $\mathcal{G}^\sigma$ (resp. $\overline{\mathcal{G}}^\sigma$). Then $g^*\overline{\omega}^\sigma = \pm\frac{1}{k}\omega^\sigma$. We define the diffeomorphism $h : \Sigma \times \mathbf{R} \to \Sigma \times \mathbf{R}$ by $h(x,t) = \left(g(x), t + \frac{\varepsilon(\sigma)\log(k|\alpha|)}{\log\lambda}\right)$ ($(x,t) \in \Sigma \times \mathbf{R}$). Then h satisfies that

$$h^*(\lambda^{\varepsilon(\sigma)t}\overline{\omega}^\sigma + \alpha dt) = \pm|\alpha|\left(\lambda^{\varepsilon(\sigma)t}\omega^\sigma \pm (\alpha/|\alpha|)dt\right) \quad \text{and}$$
$$h\theta_n = \overline{\theta}_n h,$$

where $\theta_n(x,t) = (f^{-n}(x), t+n)$ and $\overline{\theta}_n(x,t) = \left(\overline{f}^{-n}(x), t+n\right)$ ($n \in \mathbf{Z}$). This implies that $\mathcal{F}(\sigma, \alpha, \overline{\mathcal{G}}^\sigma, \overline{\mu}^\sigma, \overline{f})$ is C^0 isotopic to $\mathcal{F}(\sigma, \pm 1, \mathcal{G}^\sigma, \mu^\sigma, f)$. □

We call $\mathcal{F}(\sigma, \pm 1, \mathcal{G}^\sigma, \mu^\sigma, f)$ ($\sigma = s, u$) the *suspension foliations of the pseudo-Anosov diffeomorphism* of M, denoted by $\mathcal{F}^\sigma_\pm$. By the above proposition, the definition of the suspension foliations of the pseudo-Anosov diffeomorphism of M does not depend on the choice of pseudo-Anosov diffeomorphisms C^0 isotopic to the monodromy map of M.

Next we construct a smooth model of $\mathcal{F}^\sigma_\pm$, where $\mathcal{F}^\sigma_\pm$ is a C^∞ foliation except at $(K \times \mathbf{R})/\theta$, denoted by K'. First we choose a small closed tubular neighborhood V of K' in M such that $\mathcal{F}^\sigma_\pm|\partial V$ is the union of C^∞ product foliations of tori whose leaves are isotopic to $\partial V \cap \left((\Sigma \times \{t\})/\theta\right)$ ($t \in \mathbf{R}$). By attaching the copies of the product foliation $\{D^2 \times \{*\}; * \in S^1\}$ of $D^2 \times S^1$

to $\mathcal{F}^\sigma_\pm|(M - \operatorname{int} V)$ along the leaves of $\partial D^2 \times S^1$ and ∂V, we obtain a C^∞ foliation of M, denoted by $\widetilde{\mathcal{F}}^\sigma_\pm$. The foliation $\widetilde{\mathcal{F}}^\sigma_\pm$ is C^0 isotopic to $\mathcal{F}^\sigma_\pm$.

The transverse orientation of $\widetilde{\mathcal{F}}^\sigma_+$ (resp. $\widetilde{\mathcal{F}}^\sigma_-$) is given by the positive orientation of $\lambda^{\varepsilon(\sigma)t}\omega^\sigma + dt$ (resp. $\lambda^{\varepsilon(\sigma)t}\omega^\sigma - dt$). Then the Euler class $\chi(T\widetilde{\mathcal{F}}^\sigma_+)$ (resp. $\chi(T\widetilde{\mathcal{F}}^\sigma_-)$) is equal to $\chi(T\pi)$ (resp. $-\chi(T\pi)$). By using this fact and Seke's theorem ([12]), Meigniez ([8]) showed that $\widetilde{\mathcal{F}}^\sigma_+$ is not isotopic to $\widetilde{\mathcal{F}}^\sigma_-$.

We say that a transversely orientable codimension one foliation $\mathcal{F}$ is *transversely affine* if there exists a system of transition functions consisting of elements of $\mathrm{Aff}^+\mathbf{R} = \{x \mapsto ax + b; a > 0\}$. By Seke's theorem ([12]), transversely affine structures are characterized by the pairs (ω, ω_1) of 1-forms of M such that

1) ω defines the foliation $\mathcal{F}$,

(i.e. the tangent bundle of $\mathcal{F}$ coincides with $\ker \omega$.)

2) $d\omega = \omega \wedge \omega_1$,

3) $d\omega_1 = 0$,

modulo the identifications $(\omega, \omega_1) \sim (g\omega, \omega_1 - \dfrac{dg}{g})$ where g is a non-zero function of M.

For example, $\widetilde{\mathcal{F}}^\sigma_\pm$ is a transversely affine foliation. In fact, $\widetilde{\mathcal{F}}^\sigma_\pm|(M - \operatorname{int} V)$ has the transversely affine structure $(\lambda^{\varepsilon(\sigma)t}\omega^\sigma \pm dt, -\varepsilon(\sigma)\log\lambda \cdot dt)$, and this transversely affine structure extends to M.

Next we define the holonomy representation of a transversely affine foliation $\mathcal{F}$. Let x_0 denote the base point of M and let $p : (\widetilde{M}, \tilde{x}_0) \to (M, x_0)$ be a universal covering of M with the base point $\tilde{x}_0$ ($p(\tilde{x}_0) = x_0$). Then there exist two functions $k : (\widetilde{M}, \tilde{x}_0) \to (\mathbf{R}, 0)$ and $h : (\widetilde{M}, \tilde{x}_0) \to (\mathbf{R}^*_+, 1)$ ($\mathbf{R}^*_+ = \{t > 0\}$) satisfying $p^*(\omega, \omega_1) = \left(\dfrac{dk}{h}, \dfrac{dh}{h}\right)$ ([12]). For each element $\gamma \in \pi_1(M, x_0)$, there is an element $(a, b) \in \mathbf{R}^*_+ \times \mathbf{R}$ such that $k \cdot \gamma = ak + b$ and $h \cdot \gamma = ah$. We define the *holonomy representation* $\mathrm{hol}_{\mathcal{F}} : \pi_1(M, x_0) \to \mathrm{Aff}^+\mathbf{R}$ of $\mathcal{F}$ by $\mathrm{hol}_{\mathcal{F}}(\gamma) = (x \mapsto ax + b)$. The holonomy representation is uniquely determined up to an inner automorphism of $\mathrm{Aff}\,\mathbf{R}(= \{x \mapsto ax + b; a \neq 0\})$.

For example, the holonomy representation of $\widetilde{\mathcal{F}}^\sigma_\pm$ is as follows (up to an inner automorphism of $\mathrm{Aff}\,\mathbf{R}$). Let β be a section of $\pi : M \to S^1$ passing through the base point x_0 and oriented by the positive orientation of S^1.

Then $\mathrm{hol}_{\tilde{\mathcal{F}}^\sigma_\pm}([\beta])$ is equal to $(x \mapsto \lambda^{-\varepsilon(\sigma)}x)$. Let $\iota : \Sigma \to M$ denote the inclusion map of the fiber passing through x_0 and let $y_0 = \iota^{-1}(x_0)$. Then $\mathrm{hol}_{\tilde{\mathcal{F}}^\sigma_\pm}(\iota_*\pi_1(\Sigma, y_0))$ is contained in the group of translations $\{x \mapsto x + b\}$, identified with $\mathbf{R}$, and $[\mathrm{hol}_{\tilde{\mathcal{F}}^\sigma_\pm} \cdot \iota_*]$ $(\in H^1(\Sigma;\mathbf{R}))$ is cohomologous to $[\mathrm{Per}_\mu\sigma]$, where $\mathrm{Per}_\mu\sigma : \pi_1(\Sigma, y_0) \to \mathbf{R}$ is defined by $\mathrm{Per}_\mu\sigma(\gamma) = \int_\gamma \omega^\sigma$.

S. Matsumoto constructed examples of transversely affine foliations of M which are not isotopic to the suspension foliations of the pseudo-Anosov diffeomorphisms.

THEOREM (S. Matsumoto). — *Let Σ be a closed orientable surface with genus greater than 1 and let $\pi : M \to S^1$ be an orientable Σ-bundle over S^1 of pseudo-Anosov type such that the saddle singularities of the (un-)stable foliation $\mathcal{G}^\sigma$ $(\sigma = s, u)$ of the pseudo-Anosov diffeomorphism f isotopic to the monodromy map of M are the fixed points of f and have 4 separatrices (4-saddle singularities). Then, for each $k \in \mathbf{Z}$ satisfying $|k| \leq -\chi(\Sigma)/2$, there exists a transversely affine foliation $\mathcal{F}^\sigma_k$ of M satisfying the following conditions :*

1) $\langle \chi(T\mathcal{F}^\sigma_k), [\Sigma] \rangle = 2k$ where $[\Sigma] \in H_2(M;\mathbf{Z})$ denotes the homology class represented by the fiber of π.

2) $\mathrm{hol}_{\mathcal{F}^\sigma_k}$ is equal to $\mathrm{hol}_{\tilde{\mathcal{F}}^\sigma_\pm}$ up to an inner automorphism of $\mathrm{Aff}\,\mathbf{R}$.

3) $\mathcal{F}^\sigma_k$ has no compact leaves.

Proof. — Let $K = \{s_1, s_2, s_3, \ldots, s_n\}$ denote the set of the saddle singularities of the (un-)stable foliation $\mathcal{G}^\sigma(\sigma = s, u)$ of f. The foliation of $(\Sigma - K) \times \mathbf{R}$ defined by the non-singular 1-form $\lambda^{\varepsilon(\sigma)t}\omega^\sigma$ is denoted by $\mathcal{H}^\sigma_v$. Since $\mathcal{H}^\sigma_v$ is invariant under the $\mathbf{Z}$-action θ $(\theta_n(x,t) = (f^{-n}(x), t+n)$, $n \in \mathbf{Z})$, $\mathcal{H}^\sigma_v/\theta$ is the foliation of $M - K'$ $(K' = (K \times \mathbf{R})/\theta)$, denoted by $\mathcal{F}^\sigma_v$. The transverse orientation of $\mathcal{F}^\sigma_v$ is given by the positive orientation of $\lambda^{\varepsilon(\sigma)t}\omega^\sigma$.

Denote by σ^i_j $(j = 1, 2, 3, 4)$ the separatrices of $\mathcal{G}^\sigma$ passing through the saddle singularity $s_i (1 \leq i \leq n)$. To simplify the explanation, we assume that $f(\sigma^i_j) = \sigma^i_j$ $(1 \leq j \leq n,\ 1 \leq i \leq 4)$.

The leaf $(\sigma^i_j \times \mathbf{R})/\theta$ of $\mathcal{F}^\sigma_v$ is diffeomorphic to $S^1 \times \mathbf{R}$ and has holonomy. Hence there exists a small closed tubular neighborhood V_i of $(\{s_i\} \times \mathbf{R})/\theta$ in M such that ∂V_i is transverse to $\mathcal{F}^\sigma_v$ and $\mathcal{F}^\sigma_v|\partial V_i$ consists of four 2-dimensional Reeb components (Fig. 1).

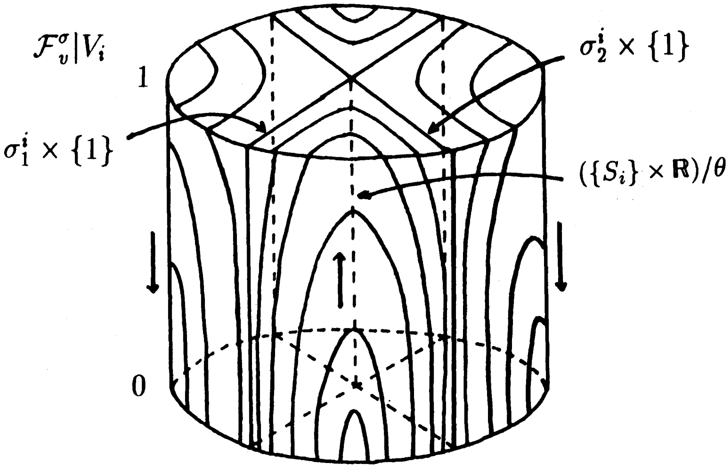

Figure 1

Next we construct two transversely oriented foliations $\mathcal{K}_+$ and $\mathcal{K}_-$ of $S^1 \times D^2$ satisfying the following conditions (Fig. 2) :

1) $\mathcal{K}_\pm|(S^1 \times \partial D^2)$ is isotopic to $\mathcal{F}_v^\sigma|\partial V_i$ with the same transverse orientation.

2) $\mathcal{K}_\pm$ has two annular leaves tangent to $S^1 \times \{*\}$ $(* \in D^2)$, and the other leaves of $\mathcal{K}_\pm$ are transverse to $S^1 \times \{*\}$ (any $* \in D^2$).

3) The transverse orientation of $S^1 \times \{0\}$ $(0 \in D^2)$ induced by the transverse orientation of $\mathcal{K}_+$ (resp. $\mathcal{K}_-$) coincides with the positive (resp. negative) orientation of S^1.

($\mathcal{K}_\pm$ consists of two plus half Reeb components [14] and one dead-end component of $D^1 \times S^1 \times S^1$.)

By attaching $\mathcal{F}_v^\sigma|(M - \bigcup_{i=1}^n \operatorname{int} V_i)$ with $k - \dfrac{\chi(\Sigma)}{2}$ copies of $\mathcal{K}_+$ and $-k - \dfrac{\chi(\Sigma)}{2}$ copies of $\mathcal{K}_-$ along the leaves of $\mathcal{F}_v^\sigma|(\bigcup_{i=1}^n \partial V_i)$, $\partial\mathcal{K}_+$ and $\partial\mathcal{K}_-$, we obtain a transversely orientable C^∞ foliation of M, denoted by $\mathcal{F}_k^\sigma$. By Thurston's proposition of [15], $\langle \chi(T\mathcal{F}_k^\sigma), [\Sigma]\rangle = 2k$. Furthermore, $\mathcal{F}_k^\sigma$ has no compact leaves, because all the leaves of $\mathcal{F}_v^\sigma|(M - \bigcup_{i=1}^n \operatorname{int} V_i)$ are non-compact.

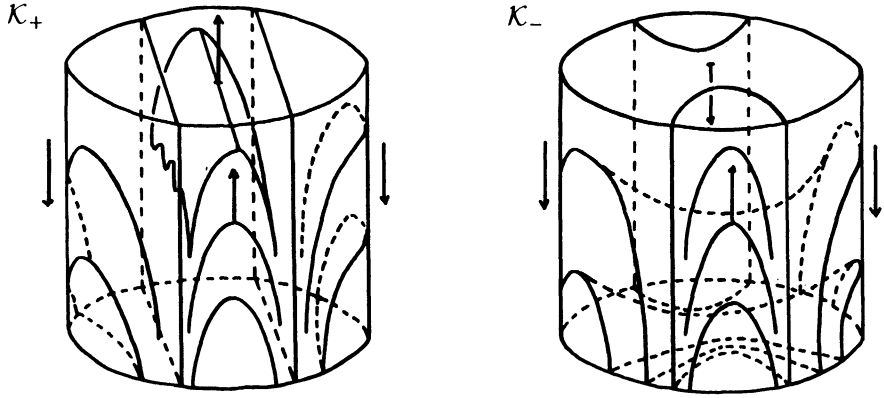

Figure 2

The transversely affine structure of $\mathcal{F}_k^\sigma$ is given as follows. First we define the transversely affine structure of $\mathcal{F}_v^\sigma|(M - \bigcup_{i=1}^n \operatorname{int} V_i)$ by $(\lambda^{\varepsilon(\sigma)t}\omega^\sigma, -\varepsilon(\sigma)\log\lambda\cdot dt)$. The foliation $\mathcal{K}_\pm$ also has a transversely affine structure. By Seke's theorem ([12]), which shows the uniqueness of the transversely affine structure of a foliation with holonomy, the transversely affine structures of $\mathcal{F}_v^\sigma|(\bigcup_{i=1}^n \partial V_i)$ and $\partial\mathcal{K}_\pm$ are unique. Therefore the transversely affine structure of $\mathcal{K}_\pm$ can be attached to that of $\mathcal{F}_v^\sigma|(M - \bigcup_{i=1}^n \operatorname{int} V_i)$. For this transversely affine structure of $\mathcal{F}_k^\sigma$, the holonomy representation is equal to $\mathrm{hol}_{\widetilde{\mathcal{F}}_\pm^\sigma}$ up to an inner automorphism of $\operatorname{Aff}\mathbf{R}$. □

Remark. — If $2k \neq \pm\chi(\Sigma)$, then $\mathcal{F}_k^\sigma$ is not homotopic to $\widetilde{\mathcal{F}}_\pm^\sigma$. Therefore $\mathcal{F}_k^\sigma$ is not isotopic to $\widetilde{\mathcal{F}}_\pm^\sigma$.

In the end of this section, we prove the proposition in the introduction.

Proof of Proposition. — Let f denote the hyperbolic automorphism of the torus T^2 given by the 2×2 matrix $\begin{pmatrix} 5 & 3 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}^2$. Then the fixed points of f are $[(0,0)]$, $\left[\left(\frac{1}{5},\frac{2}{5}\right)\right]$, $\left[\left(\frac{2}{5},\frac{4}{5}\right)\right]$, $\left[\left(\frac{3}{5},\frac{1}{5}\right)\right]$ and $\left[\left(\frac{4}{5},\frac{3}{5}\right)\right]$, where T^2 is identified with the quotient of $\mathbf{R}^2$ by the integer lattice and the element of T^2 represented by $z \in \mathbb{R}^2$ is denoted by $[z]$. Let K denote the set $\left\{\left[\left(\frac{1}{5},\frac{2}{5}\right)\right],\left[\left(\frac{4}{5},\frac{3}{5}\right)\right]\right\}$ and let α, β and ε denote the generators of $\pi_1(T^2-K)$ where α, β and ε are represented by $([0,1]\times\{0\})/\sim, (\{0\}\times[0,1])/\sim$ and a loop winding around $\left[\left(\frac{1}{5},\frac{2}{5}\right)\right]$, respectively.

Let S_1 and S_2 denote two copies of $T^2-\left\{[(t,2t)]; -\frac{1}{5}\le t\le\frac{1}{5}\right\}$. By attaching S_1 to S_2 along $\left\{[(t,2t)]; -\frac{1}{5}< t<\frac{1}{5}\right\}$ alternatively, we obtain a double covering $p:\mathring{\Sigma}_2 \to T^2-K$, where $\mathring{\Sigma}_2$ is a 2-punctured surface with genus 2. Let $\eta:\pi_1(T^2-K)\to\mathbb{Z}/2\mathbb{Z}$ denote the homomorphism satisfying $\eta(\alpha)=\eta(\beta)=\eta(\varepsilon)=1$. Then $p_*\pi_1(\mathring{\Sigma}_2)=\operatorname{Ker}\eta$. Since $\eta f_*([\alpha])=\eta f_*([\beta])=\eta f_*([\varepsilon])=1$, there is a lift f' of f.

By collapsing two holes of $\mathring{\Sigma}_2$, f' extends to a homeomorphism f'' of the closed orientable surface Σ_2 with genus 2, which is a pseudo-Anosov diffeomorphism ([1], Exposé 13). We take two lifts of $\left\{\left[\left(t,\frac{1}{2}\right)\right];0\le t\le 1\right\}$ and $\left\{\left[\left(\frac{1}{2},t\right)\right];0\le t\le 1\right\}$ as the generators of $H_1(\Sigma_2)$. Since f maps $\left\{\left[\left(t,\frac{1}{2}\right)\right];0\le t\le 1\right\}$ (resp. $\left\{\left[\left(\frac{1}{2},t\right)\right];0\le t\le 1\right\}$) on $\left\{\left[\left(5t+\frac{3}{2},3t+1\right)\right];\ 0\le t\le 1\right\}$ (resp. $\left\{\left[\left(3t+\frac{5}{2},2t+\frac{3}{2}\right)\right];0\le t\le 1\right\}$) which intersects $\left\{[(t,2t)]; -\frac{1}{5}<t<\frac{1}{5}\right\}$ two times, the isomorphism of $H_1(\Sigma_2;\mathbb{Z})$ induced by f" is represented by the 4×4 matrix $\begin{pmatrix}2&2&3&1\\1&1&2&1\\3&1&2&2\\2&1&1&1\end{pmatrix}$, whose eigenvalues are $\frac{7\pm3\sqrt{5}}{2}$ and $\frac{-1\pm\sqrt{-3}}{2}$. Therefore the Σ_2-bundle over S^1 whose monodromy map is C^0 isotopic to f'' satisfies the conditions of the main theorem. □

2. An embedded surface with the (un-)stable foliation.

The purpose of this section is to prove the existence of a finite covering of $\mathcal{F}$ whose restriction to a fiber is C^0 isotopic to an (un-)stable foliation of a pseudo-Anosov diffeomorphism (Theorem 2). First we show the following theorem.

THEOREM 1. — *Let $\pi : M \to S^1$ be as in the main theorem. If $\mathcal{F}$ is a transversely oriented and transversely affine foliation of M without compact leaves, then the holonomy representation of $\mathcal{F}$ is equal to $\mathrm{hol}_{\widetilde{\mathcal{F}}^s_\pm}$ or $\mathrm{hol}_{\widetilde{\mathcal{F}}^u_\pm}$ up to an inner automorphism of* $\mathrm{Aff}\,\mathbf{R}$, *where $\mathrm{hol}_{\widetilde{\mathcal{F}}^\sigma_\pm}$ $(\sigma = s, u)$ is the holonomy representation of the suspension foliation of the pseudo-Anosov diffeomorphism defined in Section 1.*

Proof. — We define homomorphisms $u : \mathbf{R} \to \mathrm{Aff}^+\mathbf{R}$ by $u(b) = (x \mapsto x + b)$ and $v : \mathrm{Aff}^+\mathbf{R} \to \mathbf{R}^*_+$ by $v(x \mapsto ax + b) = a$. Then the sequence $0 \to \mathbf{R} \xrightarrow{u} \mathrm{Aff}^+\mathbf{R} \xrightarrow{v} \mathbf{R}^*_+ \to 1$ is an exact sequence ([8]).

Let $\iota : \Sigma \to M$ be the inclusion map of a fiber, and let $f : \Sigma \to \Sigma$ be a monodromy map of M according to ι. (I.e. there is a diffeomorphism $\phi : (\Sigma \times I)/((x, 1) \sim (f(x), 0)) \to M$ $(I = [0, 1])$ such that $\phi|(\Sigma \times \{0\}) = \iota$). Choose a fixed point y_0 of f, and the base point of M is given by $\iota(y_0)$. Let ℓ denote the loop $\phi(\{y_0\} \times I)$ of M oriented by the positive orientation of $\{y_0\} \times I$, let β denote the element of $\pi_1(M, \iota(y_0))$ represented by ℓ. Then $\iota_* f_* \gamma = \beta^{-1}(\iota_*\gamma)\beta$ for any $\gamma \in \pi_1(\Sigma, y_0)$.

For the homomorphism $\log\cdot v \cdot \mathrm{hol}_{\mathcal{F}} \cdot \iota_* : \pi_1(\Sigma, y_0) \to \mathbf{R}$, the following equation holds for any $\gamma \in \pi_1(\Sigma, y_0)$:

$$\begin{aligned}
&\log\cdot v \cdot \mathrm{hol}_{\mathcal{F}} \cdot \iota_*(f_*\gamma) \\
&= \log\cdot v \cdot \mathrm{hol}_{\mathcal{F}}(\beta^{-1}(\iota_*\gamma)\beta) \\
&= \log\cdot v \cdot \mathrm{hol}_{\mathcal{F}}(\beta) + \log\cdot v \cdot \mathrm{hol}_{\mathcal{F}}(\iota_*\gamma) + \log\cdot v \cdot \mathrm{hol}_{\mathcal{F}}(\beta^{-1}) \\
&= \log\cdot v \cdot \mathrm{hol}_{\mathcal{F}} \cdot \iota_*(\gamma).
\end{aligned}$$

This shows that the cohomology class $[\log\cdot v \cdot \mathrm{hol}_{\mathcal{F}} \cdot \iota_*]$ $(\in H^1(\Sigma; \mathbf{R}))$ is a fixed point of $f^\# : H^1(\Sigma; \mathbf{R}) \to H^1(\Sigma; \mathbf{R})$. Since $f_\# : H_1(\Sigma; \mathbf{Z}) \to H_1(\Sigma; \mathbf{Z})$ has no eigenvalue equal to 1, $[\log \cdot v \cdot \mathrm{hol}_{\mathcal{F}} \cdot \iota_*] = 0$ in $H^1(\Sigma; \mathbf{R})$, and $v \cdot \mathrm{hol}_{\mathcal{F}} \cdot \iota_*(\pi_1(\Sigma, y_0)) = \{1\}$. Thus the following commutative diagram exists :

$$\begin{array}{ccccccccc} 1 & \longrightarrow & \pi_1(\Sigma, y_0) & \xrightarrow[\iota_*]{} & \pi_1(M, \iota(y_0)) & \xrightarrow[\pi_*]{} & \pi_1(S^1) & \longrightarrow & 1 \\ & & \downarrow H_N & & \downarrow \mathrm{hol}_{\mathcal{F}} & & \downarrow H_L & & \\ 1 & \longrightarrow & \mathbf{R} & \xrightarrow[u]{} & \mathrm{Aff}^+\mathbf{R} & \xrightarrow[v]{} & \mathbf{R}_+^* & \longrightarrow & 1 \end{array}$$

where the upper sequence is the homotopy exact sequence of the fibration π. For the cohomology class $[H_N]$ represented by H_N, the following equation holds for any element $\gamma \in \pi_1(\Sigma, y_0)$:

$$\begin{aligned} & f^{\#}[H_N](\gamma) \\ & \quad = u^{-1}\mathrm{hol}_{\mathcal{F}}\ \iota_*(f_*\gamma) \\ & \quad = u^{-1}\mathrm{hol}_{\mathcal{F}}(\beta^{-1}(\iota_*\gamma)\beta) \\ & \quad = u^{-1}(x \mapsto x + ce) \end{aligned}$$

where $\mathrm{hol}_{\mathcal{F}}(\beta) = \left(x \mapsto \frac{1}{c}x + d\right)$ and $\mathrm{hol}_{\mathcal{F}}(\iota_*\gamma) = (x \mapsto x + e)$

$$\begin{aligned} & = cu^{-1}(\mathrm{hol}_{\mathcal{F}}(\iota_*\gamma)) \\ & = c[H_N](\gamma). \end{aligned}$$

First assume that $[H_N] \neq 0$ in $H^1(\Sigma; \mathbf{R})$. Then c is an eigenvalue of $f^{\#}$ and $[H_N]$ is an eigenvector with respect to c. By the conditions of the monodromy matrix, c is equal to λ or $\frac{1}{\lambda}$. Since the cohomology class $[\mathrm{Per}_\mu s]$ (resp. $[\mathrm{Per}_\mu u]$) is also an eigenvector of $f^{\#}$ with respect to λ (resp. $\frac{1}{\lambda}$), there is a non-zero number c' such that $[H_N] = c'[\mathrm{Per}_\mu s]$ (resp. $[H_N] = c'[\mathrm{Per}_\mu u]$) if $c = \lambda$ (resp. $c = \frac{1}{\lambda}$). Therefore $\mathrm{hol}_{\mathcal{F}}$ is equal to $\mathrm{hol}_{\tilde{\mathcal{F}}^s_\pm}$ or $\mathrm{hol}_{\tilde{\mathcal{F}}^u_\pm}$ up to an inner automorphism of $\mathrm{Aff}\,\mathbf{R}$.

If $[H_N] = 0$, then $\mathrm{hol}_{\mathcal{F}}\pi_1(M, \iota(y_0))$ is an abelian subgroup. Such transversely affine foliations were studied in [12], [17]. Since $\mathcal{F}$ has no compact leaves, $\mathcal{F}$ has no holonomy and $\mathcal{F}$ is defined by a non-singular closed 1-form ([12], Theorem 7, 8). The cohomology class of this closed 1-form is $\pi^*(c''[dt])$ for some non-zero number c'' where $[dt]$ is the generator of $H^1(S^1; \mathbf{Z})$. By the theorem ([6]) of Laudenbach-Blank in a weak form, $\mathcal{F}$ is isotopic to a bundle foliation (the referee showed the author the existence of direct proofs). This contradicts the non-existence of compact leaves of $\mathcal{F}$. □

THEOREM 2. — *Let $\pi : M \to S^1$ be an oriented Σ-bundle over S^1 of pseudo-Anosov type. If $\mathcal{F}$ is a transversely oriented and transversely affine foliation of M without compact leaves such that $\chi(T\mathcal{F}) = \pm\chi(T\pi)$ and the holonomy representation of $\mathcal{F}$ is equal to $\mathrm{hol}_{\widetilde{\mathcal{F}}^s_{\pm}}$ (resp. $\mathrm{hol}_{\widetilde{\mathcal{F}}^u_{\pm}}$) up to an inner automorphism of $\mathrm{Aff}\,\mathbf{R}$, then there exists a finite covering $\widehat{p} : \widehat{M} \to M$ and an embedding $\widehat{g} : \Sigma \to \widehat{M}$ isotopic to a fiber of the Σ-bundle $\widehat{M}$ over S^1 such that $\widehat{g}^*\widehat{p}^*\mathcal{F}$ is C^0 isotopic to the stable (resp. unstable) foliation of a pseudo-Anosov diffeomorphism which is C^0 isotopic to the monodromy map of $\widehat{M}$.*

The holonomy representation $\mathrm{hol}_{\mathcal{F}}$ satisfies that either $v\cdot\mathrm{hol}_{\mathcal{F}}(\beta) = \dfrac{1}{\lambda}$ and $[H_N] = c[\mathrm{Per}_\mu s]$ $(c \neq 0)$ or $v\cdot\mathrm{hol}_{\mathcal{F}}(\beta) = \lambda$ and $[H_N] = c[\mathrm{Per}_\mu u]$ $(c \neq 0)$. To simplify the following proof of Theorem 2, we assume that $v\cdot\mathrm{hol}_{\mathcal{F}}(\beta) = \lambda$ and $[H_N] = c[\mathrm{Per}_\mu u]$.

By the Roussarie's lemma ([11], [9]), there exists an embedding $g : \Sigma \to M$ isotopic to a fiber of M such that $g^*\mathcal{F}$ is a singular foliation with 4-saddle singularities, which are saddle singularities with four separatrices.

Let $f : \Sigma \to \Sigma$ be a monodromy map of M with respect to $g(\Sigma)$. (I.e. there exists a diffeomorphism $\phi : (\Sigma \times I)/((x,1) \sim (f(x),0)) \to M$ satisfying $\phi|(\Sigma\times\{0\}) = g$.) We define the infinite cyclic covering $q : N \to M$ $(N = \Sigma\times\mathbf{R})$ by $q(x,t) = \phi(f^i(x), t-i)$ $(i \leq t \leq i+1, i \in \mathbf{Z})$. In the following, we give the base point $\overline{x}_0$ of N by $(y_0, 0)$ where y_0 is a fixed point of f, and the base point x_0 of M by $g(y_0)$. The holonomy representation does not depend on the choice of the base points up to inner automorphisms.

Let $r : (\widetilde{M}, \widetilde{x}_0) \to (N, \overline{x}_0)$ be a universal covering of N with the base point and let $p = q \cdot r$. For the transversely affine structure (ω, ω_1) of $\mathcal{F}$, there are two functions $h : (\widetilde{M}, \widetilde{x}_0) \to (\mathbf{R}^*_+, 1)$ and $k : (\widetilde{M}, \widetilde{x}_0) \to (\mathbf{R}, 0)$ such that $p^*(\omega, \omega_1) = \left(\dfrac{dk}{h}, \dfrac{dh}{h}\right)$.

In order to prove Theorem 2, we need the following lemmas.

LEMMA 1. — *$q^*\mathcal{F}$ is defined by a non-singular closed 1-form. Especially $g^*\mathcal{F}\Big(= (q|\Sigma \times \{0\})^*\mathcal{F}\Big)$ is defined by a closed 1-form.*

Proof. — For each element $\gamma \in \pi_1(N, \overline{x}_0)$, $q_*\gamma \in \pi_1(M, x_0)$ is homotopic to an element of $g_*\pi_1(\Sigma, y_0)$. Hence $\mathrm{hol}_{\mathcal{F}}(q_*\gamma)$ is a translation, and $h \cdot q_*\gamma(x) = h(x)$ $(x \in \widetilde{M})$ by the definition of the holonomy representation. For any elements z_1 and z_2 ($\in \widetilde{M}$), $h(z_1) = h(z_2)$ if $r(z_1) = r(z_2)$.

We define $s : (N, \overline{x}_0) \to (\mathbb{R}_+^*, 1)$ by $s = h \cdot r^{-1}$. Since $r^*(q^*\omega_1 - \frac{ds}{s}) = p^*\omega_1 - \frac{d(s \cdot r)}{s \cdot r} = 0$, $q^*\omega_1$ is equal to $\frac{ds}{s}$. Hence $d(sq^*\omega) = ds \wedge q^*\omega + sdq^*\omega = 0$. Therefore $q^*\mathcal{F}$ is defined by the non-singular closed 1-form $sq^*\omega$. □

In the following, the non-singular closed 1-form $sq^*\omega$ is denoted by Ω, which defines $q^*\mathcal{F}$.

LEMMA 2. — *There exists a non-singular vector field X of M transverse to both $\mathcal{F}$ and $g(\Sigma)$.*

Proof. — Let s_i $(1 \le i \le n)$ denote the saddle singularities of $\mathcal{F}|g(\Sigma)$. Then there exists a non-singular vector field X of M and pairwise disjoint small neighborhoods U_i of s_i contained in $g(\Sigma)$ such that X is transverse to $\mathcal{F}$ and tangent to $g(\Sigma) - \bigcup_{i=1}^{n} U_i$.

The saddle singularity s_i is called *positive* (resp. *negative*) if the orientation of X at s_i is equal to the positive (resp. negative) orientation of the base space S^1. Let I_+ (resp. I_-) denote the number of positive (resp. negative) saddle singularities. By Thurston's lemma ([15]), the following equations hold :

1) $-I_+ + I_- = \langle \chi(T\mathcal{F}), [g(\Sigma)] \rangle$,

2) $-I_+ - I_- = \chi(\Sigma)$,

where $\chi(T\mathcal{F}) \in H^2(M;\mathbb{Z})$ denotes the Euler class of the tangent bundle of $\mathcal{F}$, and $[g(\Sigma)]$ denotes the element of $H_2(M;\mathbb{Z})$ represented by $g(\Sigma)$. Since $\chi(T\mathcal{F}) = \pm\chi(T\pi)$, either I_+ or I_- is equal to 0. Hence the saddle singularities of $\mathcal{F}|g(\Sigma)$ are all negative or all positive. If all the saddle singularities of $\mathcal{F}|g(\Sigma)$ are positive (resp. negative), then we can perturb X toward the positive (resp. negative) direction of the base space S^1 in a neighborhood of $g(\Sigma)$ so that X is transverse to both $\mathcal{F}$ and $g(\Sigma)$. □

LEMMA 3. — *There exists an embedding $\Gamma : \Sigma \times \mathbb{R}_+ \to N$ such that $\Gamma(\Sigma \times \{0\}) = \Sigma \times \{0\}$, $\Gamma(\Sigma \times \mathbb{R}_+) \subset \Sigma \times \mathbb{R}_+$ and $\Gamma^*\Omega = \iota_0^*\Omega \pm dt$, where the inclusion map $\iota_t : \Sigma \to N$ $(t \in \mathbb{R})$ is defined by $\iota_t(x) = (x,t)$.*

Proof. — Let $\widetilde{X}$ denote the lift of X with respect to q. Then there is a non-singular vector field Y of N such that $\Omega(Y) = \pm 1$, $Y = u\widetilde{X}$ for some non-zero function u of N, and the orientation of Y at $\Sigma \times \{0\}$ coincides with the positive orientation of $\{*\} \times \mathbf{R}$ ($* \in \Sigma$). The integral manifolds of Y are called the *leaves* of Y, which are to be oriented by Y.

Let z be an element of N. Denote by L the leaf of Y passing through z. The point w of L satisfying $\int_z^w \Omega|L = \Omega(Y)t$ ($t \in \mathbf{R}$) is denoted by $\psi(z,t)$. Then ψ is the flow of Y because $\Omega\left(\frac{\partial\psi}{\partial t}\right) = \frac{d}{dt}\left(\int_0^t \Omega\left(\frac{\partial\psi}{\partial t}\right)dt\right) = \frac{d}{dt}(\Omega(Y)t) = \Omega(Y)$. Note that ψ is not always defined in the whole $N \times \mathbf{R}$. However ψ is defined on $(\Sigma \times \{0\}) \times \mathbf{R}_+$, which will be shown in the following.

Let $L(x)$ denote the leaf of Y passing through $(x,0) \in \Sigma \times \{0\} \subset N$, and let $L_i(x) = L(x) \cap (\Sigma \times [i, i+1])$ and $L_+(x) = L(x) \cap (\Sigma \times [0,\infty))$.

When $L_+(x)$ is contained in $\Sigma \times [0, n_0)$ for some integer n_0 (> 0), ψ is defined on $(x,0) \times \mathbf{R}_+$ because $\psi|(\Sigma \times [0,n_0])$ is the flow of the compact manifold $\Sigma \times [0, n_0]$ transverse to the boundary.

Suppose that $L_+(x)$ is not contained in a compact region. Then $L_i(x)$ is not empty for every $i \geq 0$ ($i \in \mathbb{Z}$). Let ℓ denote $\min_{y\in\Sigma} \Omega(Y)\left(\int_{L_0(y)} \Omega\right) > 0$. ℓ is the shortest time to reach $\Sigma \times \{1\}$ from $\Sigma \times \{0\}$ by the flow ψ. We define the covering transformation $\theta : \Sigma \times \mathbf{R} \to \Sigma \times \mathbf{R}$ of q by $\theta(x,t) = (f^{-1}(x), t+1)$. Since $\theta^*\Omega = \theta^*(sq^*\omega) = (s \cdot \theta)(q\theta)^*\omega = \lambda sq^*\omega = \lambda\Omega$, $\theta^*\Omega = \lambda\Omega$. Thus the following inequality holds :

$$\Omega(Y)\int_{L_i(x)} \Omega = \Omega(Y)\int_{\theta^{-i}L_i(x)} (\theta^i)^*\Omega = \Omega(Y)\int_{L_0(\theta^{-i}(x_i,i))} \lambda^i\Omega \geq \lambda^i\ell,$$

where $\{x_i\} = L(x) \cap (\Sigma \times \{i\})$. Hence $\Omega(Y)\int_{L_+(x)} \Omega = \infty$ and ψ is defined on $(x,0) \times \mathbf{R}_+$. Therefore ψ is defined on $(\Sigma \times \{0\}) \times \mathbf{R}_+$.

We define an embedding $\Gamma : \Sigma \times \mathbf{R}_+ \to N$ by $\Gamma(x,t) = \psi((x,0),t)$. Then

$$\begin{aligned}
&\Gamma^*\Omega(v,a) \quad (v \in T_x\Sigma, a \in T_t\mathbf{R}_+ = \mathbf{R}) \\
&= \Gamma^*\Omega\left((\iota_t)_*v + a\left(\frac{\partial}{\partial t}\right)\right) \\
&= \iota_t^*\Gamma^*\Omega(v) + a\Omega\Gamma_*\left(\frac{\partial}{\partial t}\right)
\end{aligned}$$

$$
\begin{aligned}
&= (\Gamma \cdot \iota_t)^* \Omega(v) + a\Omega(Y) \\
&= (\psi_t \cdot \iota_0)^* \Omega(v) \pm a \quad (\psi_t(z) = \psi(z,t),\ z \in N,\ t \in \mathbf{R}) \\
&= \iota_0^* \psi_t^* \Omega(v) \pm a \\
&= \iota_0^* \Omega(v) \pm a \quad \text{(See [3], Chapter VIII, Lemma 1.1.2)} \\
&= (p_1^* \iota_0^* \Omega \pm dt)\Big((\iota_t)_* v + a\Big(\frac{\partial}{\partial t}\Big)\Big) \quad (p_1(x,t) = x) \\
&= (\iota_0^* \Omega \pm dt)(v, a).
\end{aligned}
$$

Therefore $\Gamma^*\Omega = \iota_0^*\Omega \pm dt$. □

LEMMA 4. — *There exists a non-zero number c such that $\int_\gamma \iota_0^*\Omega = c[\mathrm{Per}_\mu u](\gamma)$ for any $\gamma \in \pi_1(\Sigma, y_0)$.*

Proof. — For any $\gamma \in \pi_1(\Sigma, y_0)$, $\mathrm{hol}_{\mathcal{F}}(g_*\gamma) = (x \mapsto x + \int_{(\iota_0)_*\gamma} \Omega)$. In fact,

$$
\begin{aligned}
&k \cdot g_*\gamma(\widetilde{x}_0) - k(\widetilde{x}_0) \\
&= \int_{\overline{g_*\gamma}} dk \quad \text{where } \overline{g_*\gamma} \text{ is the lift of } g_*\gamma \text{ with respect to } p \\
&\qquad\qquad \text{whose starting point is } \widetilde{x}_0, \\
&= \int_{\overline{g_*\gamma}} hp^*\omega \\
&= \int_{\overline{g_*\gamma}} r^*(sq^*\omega) \\
&= \int_{r_*\overline{g_*\gamma}} \Omega \\
&= \int_{(\iota_0)_*\gamma} \Omega.
\end{aligned}
$$

Since $\mathrm{hol}_{\mathcal{F}}(g_*\gamma)$ is also equal to $(x \mapsto x + c[\mathrm{Per}_\mu u](\gamma))$ for some non-zero number c, $\int_{(\iota_0)_*\gamma} \Omega = c[\mathrm{Per}_\mu u](\gamma)$. □

By changing the differentiable structure of Σ, there exists a closed 1-form $\widehat{\omega}^\sigma$ ($\sigma = s, u$) of Σ such that $\widehat{\omega}^\sigma$ defines $(\mathcal{G}^\sigma, \mu^\sigma)$ and $\widehat{\omega}\sigma = 0$ at the saddle singularities of $\mathcal{G}^\sigma$. (I.e. there is a homeomorphism ρ of Σ isotopic to the identity map such that $\rho^*(\mathcal{G}^\sigma, \mu^\sigma)$ is the measured foliation defined by $\widehat{\omega}^\sigma$.) By Lemma 4, $\int_\gamma \iota_0^*\Omega = c \int_\gamma \widehat{\omega}^u$ for any $\gamma \in \pi_1(\Sigma, y_0)$.

LEMMA 5. — *There exist embeddings $\eta_+, \eta_- : \Sigma \to \Sigma \times \mathbb{R}_+$ satisfying the following conditions :*

1) $c\widehat{\omega}^u = \eta_+^*(\iota_0^*\Omega + dt) = \eta_-^*(\iota_0^*\Omega - dt)$.

2) $\eta_\pm(\Sigma)$ *is transverse to* $\{*\} \times \mathbb{R}_+$ *for each* $* \in \Sigma$, *and* $\eta_\pm$ *is isotopic to* $\Sigma \times \{0\}$.

Proof. — By the above argument, $[\iota_0^*\Omega]$ and $[c\widehat{\omega}^u]$ are cohomologous in $H^1(\Sigma; \mathbb{R})$. Hence there is a function $\xi : \Sigma \to \mathbb{R}$ such that $\iota_0^*\Omega - c\widehat{\omega}^u = d\xi$. We define $\eta_+ : \Sigma \to \Sigma \times \mathbb{R}_+$ by $\eta_+(x) = (x, \mathrm{Max}(\xi) - \xi(x))$ and $\eta_- : \Sigma \to \Sigma \times \mathbb{R}_+$ by $\eta_-(x) = (x, \xi(x) - \mathrm{Min}(\xi))$. Then

$$\begin{aligned}
&\eta_\pm^*(p_1^*\iota_0^*\Omega \pm p_2^*dt) \quad (p_1(x,t) = x, \quad p_2(x,t) = t)\\
&= (p_1\eta_\pm)^*\iota_0^*\Omega \pm (p_2\eta_\pm)^*dt\\
&= \iota_0^*\Omega - d\xi\\
&= c\widehat{\omega}^u.
\end{aligned}$$

□

Proof of Theorem 2. — There exists a sufficiently large integer m (> 0) such that $\Gamma\eta_+(\Sigma)$ and $\Gamma\eta_-(\Sigma)$ are contained in $\Sigma \times [0, m)$. Let $q' : N \to \widehat{M}$ denote the quotient map of N by θ^m. Denote by $\widehat{p} : \widehat{M} \to M$ the finite covering satisfying $q = \widehat{p} \cdot q'$. If $\Gamma^*\Omega = \iota_0^*\Omega + dt$ (resp. $\Gamma^*\Omega = \iota_0^*\Omega - dt$), then we define $\widehat{g} : \Sigma \to \widehat{M}$ by $q'\Gamma\eta_+$ (resp. $q'\Gamma\eta_-$). Then $\widehat{g} : \Sigma \to \widehat{M}$ is an embedding isotopic to the fiber of $\widehat{M}$. Since $\widehat{g}^*\widehat{p}^*\mathcal{F}$ is defined by $(\Gamma\eta_\pm)^*\Omega = \eta_\pm^*(\iota_0^*\Omega \pm dt) = c\widehat{\omega}^u$, $\widehat{g}^*\widehat{p}^*\mathcal{F}$ is C^0 isotopic to $\mathcal{G}^u$, which is an unstable foliation of a pseudo-Anosov diffeomorphism which is C^0 isotopic to the monodromy map f^m of $\widehat{M}$. □

Remark. - The foliation $\mathcal{H}$ obtained by cutting $\widehat{p}^*\mathcal{F}$ along $\widehat{g}(\Sigma)$ is a C^0 foliation of $\Sigma \times I$ with a transverse invariant measure with full support such that $\mathcal{H}|(\Sigma \times \{0\})$ is the (un-)stable foliation of a pseudo-Anosov diffeomorphism which is C^0 isotopic to f^m. If we choose the pseudo-Anosov diffeomorphism as the monodromy map of $\widehat{M}$, then $\mathcal{H}|(\Sigma \times \{0\})$ is equal to $\mathcal{H}|(\Sigma \times \{1\})$. (Here $\mathcal{H}$ is not a foliation at the saddle singularities of $\mathcal{H}|(\Sigma \times \partial I)$ by the ordinary definition of foliations. Such foliations are called pseudo-foliations in [9]. However, in this paper, we call them also foliations.)

3. Foliations of $\Sigma \times I$ with transverse invariant measures.

By Theorems 1 and 2 (see also Remark of Section 2), the main theorem obviously follows from the following Theorem 3.

THEOREM 3. — *Let Σ be a closed orientable surface with genus greater than 1. Let f be a pseudo-Anosov diffeomorphism with an (un-)stable foliation $(\mathcal{G}^\sigma, \mu^\sigma)$ $(\sigma = s, u)$. Suppose that $\mathcal{H}$ is a transversely orientable C^0 foliation of $\Sigma \times I$ $(I = [0,1])$ satisfying the following conditions :*

1) *$\mathcal{H}$ has a transverse invariant measure ν with full support.*

2) $\mathcal{H}|(\Sigma \times \{0\}) = \mathcal{H}|(\Sigma \times \{1\}) = \mathcal{G}^\sigma$.

Then $\mathcal{H}$ is C^0 isotopic to $\widehat{\mathcal{H}}(\sigma, \alpha, \mathcal{G}^\sigma, \mu^\sigma)|(\Sigma \times I)$ with the boundary fixed for some non-zero number α, where $\widehat{\mathcal{H}}(\sigma, \alpha, \mathcal{G}^\sigma, \mu^\sigma)$ is the foliation of $\Sigma \times \mathbb{R}$ defined in Section 1.

In order to prove Theorem 3, we need some consideration.

First we consider some properties of singular foliations of Σ. Let $\mathcal{G}$ be a singular foliation of Σ (all the singularities of $\mathcal{G}$ are saddle ones). A leaf L of $\mathcal{G}$ is called *ordinary* if L is neither a saddle singularity nor a separatrix, and $\mathcal{G}$ is called *minimal* if all the leaves except for the saddle singularities are dense in Σ. The next lemma is the generalization of Levitt's pantalon decomposition theorem ([7]) to singular foliations having saddle singularities with many separatrices.

LEMMA 6. — *Let $\mathcal{G}$ be a transversely orientable minimal singular foliation of Σ. Then there exist disjoint simple closed curves γ_i $(1 \le i \le n)$ satisfying the following conditions :*

1) *γ_i $(1 \le i \le n)$ is transverse to $\mathcal{G}$. Denote by S_j $(1 \le j \le m)$ the connected components obtained by cutting Σ along $\bigcup_{i=1}^{n} \gamma_i$. Then,*

2) *$\mathcal{G}|S_j$ $(1 \le j \le m)$ is a singular foliation transverse to ∂S_j with a unique saddle singularity whose separatrices reach ∂S_j.*

3) *All the ordinary leaves of $\mathcal{G}|S_j$ are properly embedded arcs which connect different boundaries of S_j, and there are ordinary leaves $\beta_1^j, \beta_2^j, \beta_3^j, \ldots, \beta_{p_j}^j$ which cut S_j into a 2-disk.*

Proof. — Suppose that disjoint submanifolds S_j $(1 \leq j \leq q \leq m)$ satisfying the conditions 2) and 3) of Lemma 6 are constructed. Denote by N the closure of $\Sigma - \bigcup_{j=1}^{q} S_j$.

If $\mathcal{G}|N$ has no saddle singularities, then N is the disjoint union of annuli, say A_i $(1 \leq i \leq n)$, and each $\mathcal{G}|A_i$ is the product foliation $\left\{D^1 \times \{*\}; * \in S^1\right\}$. Denote by γ_i one of the boundaries of A_i. Then γ_i's $(1 \leq i \leq n)$ satisfy the conditions of Lemma 6.

Next suppose that $\mathcal{G}|N$ has a saddle singularity s. Denote by $\sigma_1, \sigma_2, \sigma_3, \ldots, \sigma_{2r}$ the separatrices of s in the clockwise order. Since the singular foliation $\mathcal{G}$ is minimal, σ_{2k} $(k = 1, 2, 3, \ldots, r)$ intersects ∂N. Hence there exist pairwise disjoint closed transversals ρ_k $(k = 1, 2, 3, \ldots, r)$ contained in the interior of N and intersecting $\sigma_{2k} - \{s\}$. Let z_k denote the point of $\sigma_{2k} \cap (\bigcup_{l=1}^{r} \rho_\ell)$ nearest to s along σ_{2k}. The closed transversal ρ_ℓ containing z_k is denoted by ρ'_k and the restriction of σ_{2k} to $[s, z_k]$ is denoted by w_k. Then there exists a sufficiently small closed neighborhood S_{q+1} $(\subset \operatorname{int} N)$ of $\bigcup_{k=1}^{r} (w_k \cup \rho'_k)$ whose boundary is transverse to $\mathcal{G}$. The singular foliation $\mathcal{G}|S_{q+1}$ satisfies the conditions 2) and 3) of Lemma 6. By induction on the number of the saddle singularities of $\mathcal{G}| \bigcup_{j=1}^{q} S_j$, Lemma 6 holds. □

Next we prove the following lemmas about foliations obtained by cutting $\mathcal{H}$ along $\bigcup_{i=1}^{n} (\gamma_i \times I)$.

Let S be an orientable surface with boundary. A transversely orientable C^0 foliation $\mathcal{U}$ of $S \times I$ having a transverse invariant measure ν with full support is called a *unit foliation* if it satisfies the following conditions :

1) $(\mathcal{U}, \nu)|(S \times \{0\})$ is a measured foliation of S transverse to ∂S satisfying the conditions 2) and 3) of Lemma 6.

2) $(\mathcal{U}, \nu)|(S \times \{1\}) = (\mathcal{U}, \nu)|(S \times \{0\})$.

3) $\mathcal{U}$ is transverse to $\partial S \times I$.

LEMMA 7. — *Let $(\mathcal{U}, \nu)$ be a unit foliation. Then $\mathcal{U}|(\partial S \times I)$ has no vertical leaves, where a leaf of $\mathcal{U}|(\partial S \times I)$ is called vertical if it is isotopic to $\{*\} \times I$ with $\{*\} \times \partial I$ fixed.*

Proof. — If $\mathcal{U}|(\partial S \times I)$ has a vertical leaf, then all the leaves of the component of $\mathcal{U}|(\partial S \times I)$ containing the vertical leaf are vertical because $\mathcal{U}$ has the transverse invariant measure ν.

Let ℓ be a vertical leaf of $\mathcal{U}|(\partial S \times I)$ such that $\partial \ell$ is not contained in any separatrix of $\mathcal{U}|(S \times \partial I)$. Let x_0 (resp. x_1) denote the endpoint of ℓ contained in $\partial S \times \{0\}$ (resp. $\partial S \times \{1\}$). Denote by β_{x_0} (resp. β_{x_1}) the ordinary leaf of $\mathcal{U}|(S \times \partial I)$ containing x_0 (resp. x_1), and denote by y_0 (resp. y_1) the other endpoint of β_{x_0} (resp. β_{x_1}). Since $\mathcal{U}|(\partial S \times I)$ has no holonomy, $\mathcal{U}|(\partial S \times I)$ contains no interior compact leaves. Hence there exists a properly embedded arc α ($\subset \partial S \times I$) connecting y_0 and y_1 and isotopic to $\{*\} \times I$ ($* \in \partial S$) with $\{*\} \times \partial I$ fixed such that α is either transverse or tangent to $\mathcal{U}|(\partial S \times I)$.

If α is transverse to $\mathcal{U}|(\partial S \times I)$, then there exists a null-homotopic closed transversal near $\ell \cup \beta_{x_0} \cup \alpha \cup \beta_{x_1}$. Since this contradicts the existence of the transverse invariant measure ν with full support, α is tangent to $\mathcal{U}|(\partial S \times I)$.

By Roussarie's theorem ([11], see also [9] for foliations with saddle singularities in the boundary), a null-homotopic simple closed curve $\ell \cup \beta_{x_0} \cup \alpha \cup \beta_{x_1}$ bounds a leaf of $\mathcal{U}$ homeomorphic to the 2-disk D^2. By Reeb's global stability theorem, there exists an immersion $\psi : D^2 \times [-1,1] \to S \times I$ satisfying the following conditions 1), 2) and 3) :

1) $\psi(D^2 \times \{t\})(t \in (-1,1))$ is a leaf of $\mathcal{U}$.

2) $\psi|(D^2 \times (-1,1))$ is an embedding.

3) Both $\psi(\partial D^2 \times \{1\})$ and $\psi(\partial D^2 \times \{-1\})$ contain two saddle singularities of $\mathcal{U}|(S \times \partial I)$.

By considering the transverse orientation of $\mathcal{U}|(S \times \{0\})$ in the neighborhood of the saddle singularity of $\mathcal{U}|(S \times \{0\})$, there exists a number $t_0 \in (-1,1)$ sufficiently near 1 or -1 such that $\psi(D^2 \times \{t_0\})$ contains a properly embedded short arc crossing the saddle singularity of $\mathcal{U}|(S \times \{0\})$ (Fig. 3). However this contradicts the non-existence of saddle connections of $\mathcal{U}|(S \times \{0\})$.

Thus $\mathcal{U}|(\partial S \times I)$ has no vertical leaves. □

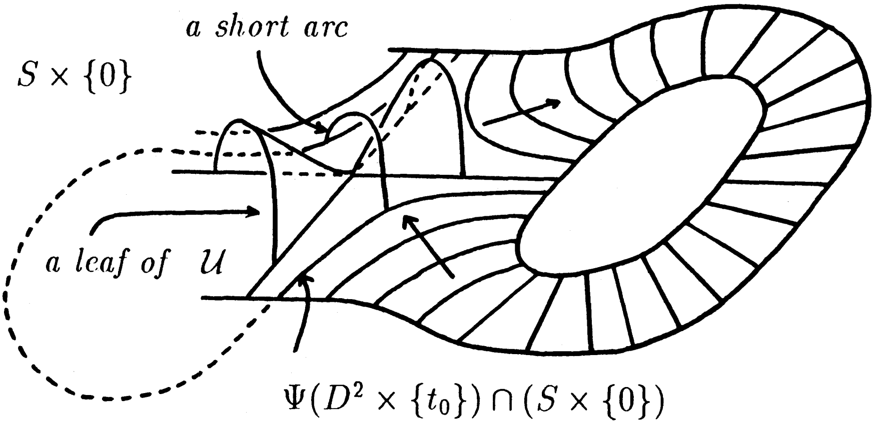

Figure 3

Remark. — The original proof of Roussarie's theorem demands that the foliations are of class C^r ($r \geq 2$). However it has already been known that his theorem is true for C^0 foliations (see [3], [5], [13]).

A unit foliation $(\mathcal{U}, \nu)$ is called *normalized* if $\mathcal{U}|(\partial S \times I)$ is transverse to $\{*\} \times I$ for any $* \in \partial S$.

LEMMA 8. — *Let $(\mathcal{U}, \nu)$ be a normalized unit foliation. For any $x, y \in \partial S$, $\nu(\{x\} \times I) = \nu(\{y\} \times I)$ and the orientation of $\{x\} \times I$ induced by the transverse orientation of $\mathcal{U}$ coincides with that of $\{y\} \times I$.*

Proof. — If x and y are contained in the same connected component of ∂S, then $\nu(\{x\} \times I) = \nu(\{y\} \times I)$ and the orientation of $\{x\} \times I$ induced by the transverse orientation of $\mathcal{U}$ coincides with that of $\{y\} \times I$.

Let $\mathcal{G}$ denote $\mathcal{U}|(S \times \{0\})$. Suppose that an ordinary leaf β of $\mathcal{G}$ connects x and y ($x, y \in \partial S$). Since $\{x\} \times I$ is homotopic to $(\beta \times \{1\}) \cup (\{y\} \times I) \cup (\beta \times \{0\})$, $\nu(\{x\} \times I)$ is equal to $\nu(\{y\} \times I)$. If the orientation of $\{x\} \times I$ induced by the transverse orientation of $\mathcal{U}$ is opposite to that of $\{y\} \times I$, then there is a null-homotopic closed transversal, which contradicts the existence of the transverse invariant measure ν with full support.

Let γ and γ' be connected components of ∂S. Denote by σ and σ' the separatrices of $\mathcal{G}$ intersecting γ and γ', respectively. Then there exists a series of separatrices $\sigma = \sigma_1, \sigma_2, \sigma_3, \ldots, \sigma_k = \sigma'$ where σ_i is adjacent to σ_{i+1} for each i. Since there is an ordinary leaf of $\mathcal{G}$ near $\sigma_i \cup \sigma_{i+1}$ for each

i, $\nu(\{x\} \times I)$ $(x \in \gamma)$ is equal to $\nu(\{y\} \times I)$ $(y \in \gamma')$, and the orientation of $\{x\} \times I$ coincides with that of $\{y\} \times I$. □

LEMMA 9. — *Let $(\mathcal{U}_1, \nu_1)$ and $(\mathcal{U}_2, \nu_2)$ be normalized unit foliations of $S \times I$ satisfying $(\mathcal{U}_1, \nu_1)|\partial(S \times I) = (\mathcal{U}_2, \nu_2)|\partial(S \times I)$, then there exists a homeomorphism $h : S \times I \to S \times I$ such that $h|\partial(S \times I) = \mathrm{id}$ and $h(\mathcal{U}_1, \nu_1) = (\mathcal{U}_2, \nu_2)$.*

Proof. — Let $\mathcal{G}$ denote $\mathcal{U}_1|(S \times \{0\})$, and let β_j $(1 \le j \le p)$ be the ordinary leaves of $\mathcal{G}$ which cut S into a 2-disk. By Roussarie's theorem ([11]), there are pairwise disjoint properly embedded disks D_j (resp. D'_j) transverse to $\mathcal{U}_1$ (resp. $\mathcal{U}_2$) and bounded by $\partial(\beta_j \times I)$. Since $\mathcal{U}_1|D_j$ and $\mathcal{U}_2|D'_j$ are foliations whose leaves are properly embedded arcs, there is a homeomorphism $h : \partial(S \times I) \cup \left(\bigcup_{j=1}^{p} D_j\right) \to \partial(S \times I) \cup \left(\bigcup_{j=1}^{p} D'_j\right)$ such that $h(\mathcal{U}_1, \nu_1) = (\mathcal{U}_2, \nu_2)$.

Let $\widehat{\mathcal{U}}_1$ (resp. $\widehat{\mathcal{U}}_2$) denote the foliation of D^3 obtained by cutting $\mathcal{U}_1$ (resp. $\mathcal{U}_2$) along $\bigcup_{j=1}^{p} D_j$ (resp. $\bigcup_{j=1}^{p} D'_j$) (Fig. 4). $\widehat{\mathcal{U}}_i$ $(i = 1, 2)$ has $2p$ collapsing leaves homeomorphic to I and two saddle singularities in the boundary. The leaves of $\widehat{\mathcal{U}}_i$ near the collapsing leaves are all homeomorphic to D^2. By Poincaré-Bendixson's theorem, the ordinary leaves of $\partial\widehat{\mathcal{U}}_i$ are all homeomorphic to S^1 and the union of the leaves of $\partial\widehat{\mathcal{U}}_i$ containing a saddle singularity is a bouquet. Hence the leaves of $\widehat{\mathcal{U}}_i$ containing no saddle singularities of $\partial\widehat{\mathcal{U}}_i$ are homeomorphic to the 2-disks, and the union of the leaves of $\widehat{\mathcal{U}}_i$ containing the saddle singularity is the union of 2-disks whose intersection point is the saddle singularity. Therefore h extends to a homeomorphism of $S \times I$ which satisfies the conditions of Lemma 9. □

Proof of Theorem 3. — Let γ_i $(1 \le i \le n)$ denote the disjoint simple closed curves transverse to $\mathcal{G}^\sigma$ constructed by Lemma 6, and let S_j $(1 \le j \le m)$ denote the connected components obtained by cutting Σ along $\bigcup_{i=1}^{n} \gamma_i$. Since $\mathcal{H}$ has the transverse invariant measure ν with full support, $\mathcal{H}$ has no interior compact leaves. By Roussarie's theorem ([11]), $\gamma_i \times I$ can be taken by an isotopy of $\Sigma \times I$ with $\Sigma \times \partial I$ fixed so that $\gamma_i \times I$ is transverse to $\mathcal{H}$. Since all the leaves of $\mathcal{H}|(\gamma_i \times I)$ are properly embedded arcs, $\nu(\gamma_i \times \{0\})$ is equal to $\nu(\gamma_i \times \{1\})$. By the unique ergodicity of the (un-)stable foliation

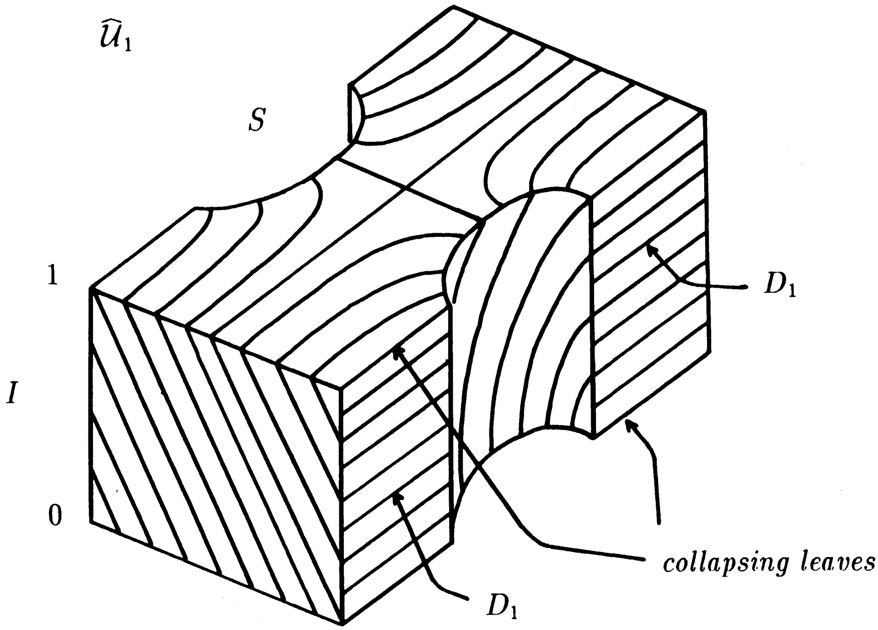

Figure 4

of the pseudo-Anosov diffeomorphism ([1]), $\nu|(\Sigma \times \{0\}) = \nu|(\Sigma \times \{1\})$. Therefore $(\mathcal{H}|(S_j \times I), \nu|(S_j \times I))$ is a unit foliation.

By Lemma 7, $\mathcal{H}|(S_j \times I)$ has no vertical leaves. We change $\Sigma \times I$ again by an isotopy with $\Sigma \times \partial I$ fixed so that $\{*\} \times I$ is transverse to $\mathcal{H}$ for any $* \in \bigcup_{i=1}^{n} \gamma_i$. Then $(\mathcal{H}|(S_j \times I), \nu|(S_j \times I))$ is a normalized unit foliation.

We take the transverse orientation of $\mathcal{H}$ so that the transverse orientation of $\mathcal{H}|(\Sigma \times \{0\})$ coincides with that of $\mathcal{G}^\sigma$. Since all the leaves of $\mathcal{H}|(\gamma_i \times I)$ are properly embedded arcs, the transverse orientation of $\mathcal{H}|(\Sigma \times \{1\})$ also coincides with that of $\mathcal{G}^\sigma$.

By Lemma 8, the orientations of $\{*\} \times I$ $(* \in \partial S_j)$ induced by the transverse orientation of $\mathcal{H}$ are either all positive or all negative. For each γ_i and γ_j, there is an arc in a leaf of $\mathcal{G}^\sigma$ connecting γ_i with γ_j by the minimality of $\mathcal{G}^\sigma$. Thus the orientations of $\{*\} \times I$ $(* \in \bigcup_{i=1}^{n} \gamma_i)$ are either all positive or all negative. If they are positive (resp. negative), then we put $\delta(\mathcal{H}) = 1$ (resp. $\delta(\mathcal{H}) = -1$).

Denote by c the positive number satisfying $c\nu|(\Sigma \times \partial I) = \mu^\sigma$. In the following, the transverse invariant measure of $\mathcal{H}$ is given by $c\nu$.

Let α denote the positive number satisfying $c\nu(\{*\}\times I) = \alpha\int_0^1 \lambda^{-\varepsilon(\sigma)t}dt$ ($* \in \gamma_i$). The foliation $\widehat{\mathcal{H}}(\sigma, \alpha\delta(\mathcal{H}), \mathcal{G}^\sigma, \mu^\sigma)$ of $\Sigma \times \mathbf{R}$ (defined by $\lambda^{\varepsilon(\sigma)t}\omega^\sigma + \alpha\delta(\mathcal{H})dt$ in $(\Sigma - K) \times \mathbf{R}$) has a transverse invariant measure $\widehat{\nu} = \left|\int(\omega^\sigma + \alpha\delta(\mathcal{H})\lambda^{-\varepsilon(\sigma)t}dt)\right|$. The transverse orientation of $\widehat{\mathcal{H}}(\sigma, \alpha\delta(\mathcal{H}), \mathcal{G}^\sigma, \mu^\sigma)$ is given by the positive orientation of $\lambda^{\varepsilon(\sigma)t}\omega^\sigma + \alpha\delta(\mathcal{H})dt$.

In the following, we construct a homeomorphism $h'' : \Sigma \times I \to \Sigma \times I$ satisfying $h''(\mathcal{H}, c\nu) = (\widehat{\mathcal{H}}(\sigma, \alpha\delta(\mathcal{H}), \mathcal{G}^\sigma, \mu^\sigma)|(\Sigma \times I), \widehat{\nu}|(\Sigma \times I))$.

First we define the homeomorphism $h : \Sigma \times \partial I \to \Sigma \times \partial I$ by the identity map. The transversely oriented measured foliations of $S^1 \times I$ transverse to both $S^1 \times \partial I$ and $\{*\} \times I$ (for any $* \in S^1$), are determined by the lengths of $S^1 \times \{0\}$ and $\{*\} \times I$, and the orientations of $S^1 \times \partial I$ and $\{*\} \times I$ ($* \in S^1$) ([1]). Hence h extends to $h' : (\Sigma \times \partial I) \cup (\bigcup_{i=1}^{n} \gamma_i \times I) \to (\Sigma \times \partial I) \cup (\bigcup_{i=1}^{n} \gamma_i \times I)$ such that $h'(\mathcal{H}, c\nu) = (\widehat{\mathcal{H}}(\sigma, \alpha\delta(\mathcal{H}), \mathcal{G}^\sigma, \mu^\sigma), \widehat{\nu})$ and $h'(\{*\} \times I) = \{*\} \times I$ for any $* \in \bigcup_{i=1}^{n} \gamma_i$. By Lemma 9, h' extends to $h'' : \Sigma \times I \to \Sigma \times I$ which brings $\mathcal{H}$ to $\widehat{\mathcal{H}}(\sigma, \alpha\delta(\mathcal{H}), \mathcal{G}^\sigma, \mu^\sigma)|(\Sigma \times I)$. Therefore $\mathcal{H}$ is C^0 isotopic to $\widehat{\mathcal{H}}(\sigma, \alpha\delta(\mathcal{H}), \mathcal{G}^\sigma, \mu^\sigma)|(\Sigma\times I)$ with the boundary fixed. □

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Hiromichi NAKAYAMA,
Department of Mathematical Sciences
College of Science and Engineering
Tokyo Denki University
Hatoyama-machi, Hiki-gun
Saitama-ken 350-03 (Japan).