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## Geometry of an étale covering of the $p$-adic upper half plane

# GEOMETRY OF AN ÉTALE COVERING OF THE $p$-ADIC UPPER HALF PLANE

## by Jeremy TEITELBAUM (*)

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## Introduction.

In this paper we describe the rigid geometry of the first layer in the tower of coverings of the $p$-adic upper half plane obtained from the division points of the formal group constructed in [2]. This covering is accessible because it is abelian and in some sense "tame." Using our results, we are able to describe the stable special fiber at $p$ of Shimura curves with a very small amount of level $p$ structure.

## Preliminaries.

Let $\hat{\mathcal{H}}_p$ denote the formal scheme over $\mathbf{Z}_p$ constructed by Mumford ([4]) and commonly referred to as the $p$-adic upper half plane. Naively, $\hat{\mathcal{H}}_p$ is the complement of the $\mathbf{Q}_p$-rational points in $\mathbf{P}^1$ . We let $\mathcal{H}_p$ be the rigid analytic space associated to $\hat{\mathcal{H}}_p$.

In [2], Drinfeld shows that $\hat{\mathcal{H}}_p$ is a parameter space for two-dimensional formal groups with a certain endomorphism structure. As a

result, there is a universal family of formal groups $\mathcal{G}$ over $\hat{\mathcal{H}}_p$. The subgroups obtained as the division points of this family of formal groups yield a tower of coverings of $\hat{\mathcal{H}}_p$. The rigid spaces associated to these coverings are a family of étale coverings of $\mathcal{H}_p$. Our goal in this work is to describe the simplest of these coverings.

For a detailed description of Drinfeld's universal formal group, we refer the reader to [8]. We recall here the basic definitions which we will require.

Let $D$ be the quaternion division algebra over $\mathbf{Q}_p$, and let $\mathcal{O}_D$ be the maximal order in $D$. A formal group $G$ of dimension 2 and height 4 over a ring $R$ on which $p$ is nilpotent is called a special, formal $\mathcal{O}_D$–module (abbreviated $SFD$-module) provided that $\mathcal{O}_D$ acts on $G$ and, at each maximal ideal $m$ of $R$, both characters of the residue field of $\mathcal{O}_D$ occur in the tangent space to $G$ at $m$. Over $\overline{\mathbf{F}}_p$, all $SFD$–modules are isogenous, so fix one such module $\Phi$. With these conventions, we can state Drinfeld's theorem.

THEOREM (Drinfeld). — $\hat{\mathcal{H}}_p \otimes \mathrm{Spf}\, \hat{\mathbf{Z}}_p^{ur}$ (over $\mathbf{Z}_p$) represents the functor which assigns to a ring $R$ on which $p$ is nilpotent the set of isomorphism classes of triples $(\psi, G, \rho)$ where

   1. $\psi : \hat{\mathbf{Z}}_p^{ur}/p\hat{\mathbf{Z}}_p^{ur} \to R/p$ is a homomorphism,

   2. $G$ is an $SFD$-module over $R$,

   3. and $\rho : \psi_* \Phi \to G \otimes R/p$ is a "quasi–isogeny of height zero," which means that $\rho$ is an isogeny with a certain normalization condition which will not be important in our work.

We let $(\Psi, \mathcal{G}, \mathrm{P})$ be the universal triple over $\hat{\mathcal{H}}_p \otimes \mathrm{Spf}\, \hat{\mathbf{Z}}_p^{ur}$, and $\mathcal{G}_\Pi$ be the kernel of multiplication by $\Pi$ on $\mathcal{G}$. This is a finite, flat group scheme of order $p^2$ over $\hat{\mathcal{H}}_p \otimes \mathrm{Spf}\, \hat{\mathbf{Z}}_p^{ur}$. If $\mathrm{PGL}_2^+(\mathbf{Q}_p)$ denotes the image in $\mathrm{PGL}_2(\mathbf{Q}_p)$ of the elements of $\mathrm{GL}_2(\mathbf{Q}_p)$ with determinants of even $p$-adic order, then the action of $\mathrm{PGL}_2^+(\mathbf{Q}_p)$ on $\hat{\mathcal{H}}_p$ extends to an action on the universal triple over $\hat{\mathcal{H}}_p \otimes \mathrm{Spf}\hat{\mathbf{Z}}_p^{ur}$, and therefore to $\mathcal{G}_\Pi$. Furthermore, the residue field $\mathcal{O}_D/\Pi = \mathbf{F}$ acts on $\mathcal{G}_\Pi$, and this action commutes with the action of $\mathrm{PGL}_2^+(\mathbf{Q}_p)$. In fact, $\mathrm{GL}_2(\mathbf{Q}_p)$ acts on the universal triple; see [2] p. 109 for details.

Let us fix a map $\psi_0 : \hat{\mathbf{Z}}_p^{ur} \to \hat{\mathbf{Z}}_p^{ur}$ and consider the fiber of the induced projection map $\hat{\mathcal{H}}_p \otimes \mathrm{Spf}\, \hat{\mathbf{Z}}_p^{ur} \to \mathrm{Spf}\, \hat{\mathbf{Z}}_p^{ur}$. The resulting formal scheme is isomorphic to $\hat{\mathcal{H}}_p \otimes \mathrm{Spf}\, \hat{\mathbf{Z}}_p^{ur}$ viewed as a formal scheme over $\hat{\mathbf{Z}}_p^{ur}$ with structure map $\psi_0$. The geometry of the formal scheme obtained in this

way does not depend on the choice of $\psi_0$, so for the remainder of this paper we will abuse notation, suppress reference to $\psi_0$, and denote by $\hat{\mathcal{H}}_p$ and $\mathcal{H}_p$ respectively the formal and rigid $p$-adic upper half planes over $\mathbf{Z}_p$ (resp. $\mathbf{Q}_p$), base–changed up to $\hat{\mathbf{Z}}_p^{ur}$ (resp $\hat{\mathbf{Q}}_p^{ur}$.) Similarly, we base change the covering $\mathcal{G}_\Pi$ to obtain a finite flat group scheme of order $p^2$ over (our base–changed) $\hat{\mathcal{H}}_p$. We let $\hat{\Sigma}$ be the complement of the zero section in this group scheme, and $\Sigma$ be the associated rigid space. This paper is devoted to describing the rigid geometry of $\Sigma$.

## Classification of $\Sigma$ as $\mu_{p^2-1}$–torsor.

The action of the endomorphism ring of $\mathcal{G}$ induces an action of $\mathbf{F}^\times$ on the covering $\Sigma$, as we mentioned before. The two embeddings

$$\sigma_0, \sigma_1 : \mathbf{F} \to \hat{\mathbf{Z}}_p^{ur}/p\hat{\mathbf{Z}}_p^{ur}$$

induce two different actions of $\mu_{p^2-1}$ on our fixed $SFD$–module $\Phi$ over $\overline{\mathbf{F}}_p$, and therefore, via the universal isogeny $P$, two different actions on $\Sigma$. This allows us to view $\Sigma$ as a $\mu_{p^2-1}$ torsor in two different ways.

DEFINITION 1. — *Let*

$$c_i(\Sigma) \in H^1_{et}(\mathcal{H}_p \otimes \mathrm{Spf}\, \mathbf{C}_p, \mu_{p^2-1})$$

*be the class representing the covering $\Sigma \otimes \mathrm{Spf}\, \mathbf{C}_p$ viewed as a $\mu_{p^2-1}$ torsor via the embedding*

$$\tilde{\sigma}_i : W(\mathbf{F}) \to \hat{\mathbf{Z}}_p^{ur} \subset \mathbf{C}_p$$

*induced by $\sigma_i$ .*

The following lemma relates the two classes.

LEMMA 2. — $c_i(\Sigma) = pc_{i+1}(\Sigma)$, *reading subscripts mod 2 .*

*Proof.* — Changing the choice of $\sigma_i$ twists the $\mu_{p^2-1}$ action on $\Sigma$ by $\zeta \mapsto \zeta^p$ .                                       □

Our goal now is to determine the classes $c_i$ precisely. Let $\mathcal{T}$ be the tree of $SL_2(\mathbf{Q}_p)$. We fix a reduction map $r : \mathcal{H}_p \to \mathcal{T}$ which is compatible with the action of $SL_2(\mathbf{Q}_p)$. For one of many detailed descriptions of the relation between $\mathcal{T}$ and $\mathcal{H}_p$, see [8], pp. 648–660.

The following theorem of Drinfeld ([1]) relates the cohomology of $\mathcal{H}_p$ to the tree $\mathcal{T}$.

THEOREM 3 (Drinfeld [1]). — *If $N$ is an integer prime to $p$, there is an isomorphism*

$$\partial : H^1_{et}(\mathcal{H}_p \otimes \operatorname{Spf} \mathbf{C}_p, \mu_N) \to C^1_{har}(\mathcal{T}, \mathbf{Z}/N\mathbf{Z})$$

*where $\mathcal{T}$ is the tree of $\operatorname{SL}_2$ and $C^1_{har}(\mathcal{T}, \mathbf{Z}/N\mathbf{Z})$ is the group of harmonic 1-cochains on $\mathcal{T}$ — that is, the set of functions $f$ on the edges of $\mathcal{T}$ such that, for each vertex $v$, $f$ satisfies*

$$\sum_{e \mapsto v} f(e) = 0$$

*where the sum is over the oriented edges of $\mathcal{T}$ meeting $v$.*

Let us briefly recall how the map $\partial$ of the theorem is constructed. Suppose $Y$ is a torsor over $\mathcal{H}_p \otimes \operatorname{Spf} \mathbf{C}_p$. Let $U$ be the admissible open set in $\mathcal{H}_p$ corresponding to a vertex $v$ in $\mathcal{T}$, together with its bounding edges. It follows from Lemma 2 of [8] that $U$ is a $\operatorname{GL}_2(\mathbf{Q}_p)$ translate of the standard open set

$$V = \{P \in \mathcal{H}_p : 1/p < |z(P)| < p\} - \bigcup_{i=1}^{p-1} B^+_{1/p}(i)$$

where $B^+_r(i)$ denotes the closed ball centered at $i$ of radius $r$. Therefore $\operatorname{Pic}(U) = 0$ and so

$$\mathcal{O}_Y = \mathcal{O}_U[T]/(T^{p^2-1} - f)$$

where $\mu_{p^2-1}$ acts by multiplication on $T$, and $f$ is uniquely determined up to $(p^2-1)^{st}$ powers.

Let $e = \{v, v'\}$ be an (oriented) edge leaving $v$. To evaluate $\partial(Y)(e)$, choose a coordinate function $z$ on $U$ such that if $P \in U$ reduces to $v$ then $|z(P)| = 1$ while if $P \in U$ reduces to $e$ then

$$|p| < |z(P)| < 1 .$$

Then we let

$$\partial(Y)(e) = \operatorname{Res}_e df/f \pmod{p^2 - 1}$$

where $\operatorname{Res}_e$ denotes the rigid analytic "annular residue" computed with respect to the selected coordinate $z$ (which determines the sign of Res.)

Since $\operatorname{PGL}^+_2(\mathbf{Q}_p)$ acts on $\Sigma$, the class $c_i(\Sigma)$ is invariant by $\operatorname{PGL}^+_2(\mathbf{Q}_p)$. As we see in the next lemma, this is a very strong condition.

LEMMA 4. — *Suppose $Y$ is a $\operatorname{PGL}^+_2(\mathbf{Q}_p)$-invariant torsor over $\mathcal{H}_p \otimes \operatorname{Spf} \mathbf{C}_p$, and $\partial(Y)$ is the associated harmonic 1-cocycle. Then $\partial(Y)$ satisfies the following conditions:*

1. $\partial(Y)(e) = \partial(Y)(\gamma e)$ for all $\gamma \in \mathrm{PGL}_2^+(\mathbf{Q}_p)$ and all oriented edges $e$ of $\mathcal{T}$.

2. $\partial(Y)(e) \equiv 0 \pmod{p-1}$ on all edges $e$ of $\mathcal{T}$.

*Proof.* — The first property is a consequence of the invariance of $Y$ by $\mathrm{PGL}_2^+(\mathbf{Q}_p)$ and the fact that $\mathrm{PGL}_2^+(\mathbf{Q}_p)$ preserves the orientation of edges in $\mathcal{T}$. For the second, observe that all the edges $e$ leaving any vertex $v$ are permuted transitively by $\mathrm{PGL}_2^+(\mathbf{Q}_p)$. Therefore $\partial(Y)(e)$ is a constant $x$ on all edges $e$ leaving $v$. From the harmonicity condition we obtain

$$\sum_{e \mapsto v} \partial(Y)(e) = (p+1)x \equiv 0 \pmod{p^2 - 1}$$

since there are $p + 1$ edges leaving $v$. This proves the lemma.          $\square$

In order to give a precise statement of our theorem, we must invoke the relationship between orientations on $\mathcal{T}$ and the embeddings $\sigma_i$. As Drinfeld shows, and we explain in Lemma 14 of [8], the action of $\Pi$ on the tangent space $T$ to $\mathcal{G}$ allows us to partition the vertices of $\mathcal{T}$ into two classes labeled with the $\sigma_i$. To describe this partition, first decompose $T$ into $\sigma_i$–eigenspaces for the action for the quadratic unramified extension of $\mathbf{Z}_p$ inside $\mathcal{O}_D$. A vertex $v$ is labelled with $\sigma_i$ if $\Pi T^i \subset pT^{i+1}$ over the affinoid reducing to $v$ where $T^i$ is the $\sigma_i$–eigenspace in $T$. Vertices of the two classes alternate in the tree.

Since $\partial_i(\Sigma)$ is $\mathrm{PGL}_2^+(\mathbf{Q}_p)$-invariant, and this group permutes the edges of $\mathcal{T}$ transitively, it suffices to specify the value of $\partial_i(\Sigma)$ on a single edge. This is the content of our theorem.

THEOREM 5. — *Let $e = [v, v']$ be an edge of $\mathcal{T}$. Suppose that $v$ is labeled with $\sigma_i$. Then*

$$\partial_i(\Sigma)(e) = p - 1 .$$

*Proof.* — Notice first of all that, by Lemmas 2 and 4,
$$\partial_i(\Sigma)([v, v']) = -\partial_i(\Sigma)([v', v])$$
$$= p\partial_i(\Sigma)([v', v])$$
$$= \partial_{i+1}(\Sigma)([v', v])$$

and therefore we may assume that $v$ is labeled with $\sigma_0$. Let $U$ be the affine open set of $\hat{\mathcal{H}}_p$ corresponding to $e$. It follows from Lemma 2 of [8] that $U$ is a $\mathrm{PGL}_2^+(\mathbf{Q}_p)$ translate of the standard open set

$$U(1) = \{P \in \mathcal{H}_p : 1/p \le |z(P)| \le 1\} - \bigcup_{i=1}^{p-1} (B_1(i) \cup B_{1/p}(pi))$$

where $B_r(i)$ denotes the open disc of radius $r$ centered at $i$. Therefore the coordinate ring of $U$ is isomorphic to

$$\hat{R} = \varprojlim R/p^n R$$

where

$$R = \frac{\mathbf{Z}[z_0, z_1]}{(z_0 z_1 - p)} \left[ \frac{1}{1 - z_0^{p-1}}, \frac{1}{1 - z_1^{p-1}} \right] .$$

As we recalled prior to stating this theorem, the tangent space $T$ to $\mathcal{G}$ over $U$ is free of rank 2, and it carries a grading coming from the action of the maximal unramified extension of $\mathbf{Z}_p$ in $\mathcal{O}_D$. Write $T = \hat{R}t_0 \oplus \hat{R}t_1$. Referring to [8] p. 656, we see that the $\Pi$ action on $T$ is $\Pi t_0 = z_1 t_1$ and $\Pi t_1 = z_0 t_0$. With these conventions, the vertex $v$ of $T$ labeled with $\sigma_0$ corresponds to the region where $z_0$ is a unit; the vertex $v'$ labeled with $\sigma_1$ corresponds to the region where $z_1$ is a unit.

Let $\Omega$ be the cotangent space to $\mathcal{G}$. Then $\Omega_\Pi = \Omega/\Pi\Omega$ is naturally the cotangent space to $\mathcal{G}_\Pi$. If $\omega_0$ and $\omega_1$ generate the graded pieces of $\Omega$, then we must have $\Pi\omega_i = z_i\omega_{i+1}$. It follows that

(1)           $\Omega_\Pi = (\hat{R}/z_1\hat{R})\omega_0 \oplus (\hat{R}/z_0\hat{R})\omega_1 .$

The finite flat group scheme $\mathcal{G}_\Pi$, together with its action by $\mathbf{F}$ is of the type classified by Raynaud. Applying his classification (see [5], Corollary 1.5.1) we see that the coordinate $B$ ring of $\mathcal{G}_\Pi$ must have the form

$$B = \hat{R}[X_0, X_1]/(X_0^p - \delta_0 X_1, X_1^p - \delta_1 X_0)$$

where the functions $\delta_i$ and $p/\delta_i$ belong to $\hat{R}^\times$. In addition, Raynaud shows that the natural identification of $\Omega_\Pi$ with $I/I^2$ ($I$ being the augmentation ideal) means that

(2)           $\Omega_\Pi = (\hat{R}/\delta_1\hat{R})X_0 \oplus (\hat{R}/\delta_0\hat{R})X_1 .$

Combining (1) and (2), we see that $z_i$ and $\delta_i$ differ by a unit of $\hat{R}$.

We now have enough information to compute the class of $\Sigma$. Indeed, $\Sigma$ is defined over $U$ by the equation

$$X_0^{p^2-1} - \delta_0^p \delta_1 = 0 .$$

The group $\mathbf{F}^\times$ acts on $X_0$ through the embedding $\sigma_0$. Let us write $f = \delta_0^p \delta_1$. Then

$$\partial_0(\Sigma)(e) = \operatorname{Res}_{z_0} df/f \pmod{p^2 - 1} .$$

Our results above tell us that $\delta_0^p \delta_1 = p z_0^{p-1} h(z)$ where $h$ is a unit in $\hat{R}$. This residue is clearly $p - 1$ .                    $\square$

Thanks to this theorem, we can construct $\Sigma$ over $\mathbf{C}_p$. Let $X$ be the non-singular projective curve over $\mathbf{C}_p$ defined by the affine equation

(3) $$Y^{p+1} = z - z^p \ .$$

Let $W \subset X$ be the admissible open set of points $P$ on $X$ such that

(4) $$|p^{1/(p+1)}| < |Y(P)| < |p^{-p/(p+1)}| \ .$$

COROLLARY 6. — *Over $\mathbf{C}_p$, $\Sigma$ consists of $p-1$ isomorphic connected components. Each such component has a covering by admissible open sets isomorphic to $W$. The nerve of this covering is the tree $\mathcal{T}$. If $W_1$ and $W_2$ are two elements of the covering, and $E = W_1 \cap W_2 \subset W_1$, then $E$ is one of the boundary annuli of $W_1$ .*

*Proof.* — Let $U$ be the subset of $\mathcal{H}_p \otimes \mathrm{Spf} \, \mathbf{C}_p$ consisting of one vertex (say, labeled with $\sigma_0$), and its bounding edges. Then by Theorem 5, $\Sigma$ over $U$ is obtained by extracting the $p^2 - 1$ root of a function with order congruent to $p-1 \bmod p^2 - 1$ on each bounding annulus. If $z$ is a coordinate on $U$, then the function $f(z) = (z - z^p)^{p-1}$ clearly meets this condition. Thus $\Sigma$ is defined over $U$ by the equation

$$Y_0^{p^2-1} = (z_0 - z_0^p)^{p-1}$$

where $z_0$ is an appropriate parameter on $U$. Notice first that this equation factors, so that $\Sigma$ consists of $p - 1$ connected components, and is built up out of pieces of the curve in (3) as claimed. It is a simple matter to check that the subset of $W$ satisfying the inequality (4) has genus $(p^2 - p)/2$. Thus the reduction of $\Sigma$ consists of curves meeting in double points, like the reduction of $\mathcal{H}_p$, except that the rational curves which appear in the reduction of $\mathcal{H}_p$ are replaced by the curves of equation (3) .

With somewhat more care one can determine the equations for $\Sigma$ over $\hat{\mathbf{Z}}_p^{ur}$, instead of just over $\mathbf{C}_p$. Examining the end of the proof of Theorem 5 we see that over $\hat{\mathbf{Z}}_p^{ur}$, we can take $\delta_0^p \delta_1 = p(z - z^p)^{p-1}$ and therefore $\Sigma$ is defined over $U$ by the equation

$$Y_0^{p^2-1} = p(z_0 - z_0^p)^{p-1} \ .$$

From this one can obtain a minimal regular model for $\Sigma$ over $\hat{\mathbf{Z}}_p^{ur}$ by blowing-up. This is a straightforward computational problem whose solution we omit, although we do point out that the minimal model has no components of multiplicity one.

Finally, notice that the action of $\mathrm{PGL}_2^+(\mathbf{Q}_p)$ on $\Sigma$ extends to an action of $\mathrm{PGL}_2(\mathbf{Q}_p)$. Indeed, choose $\tau \in \mathrm{PGL}_2(\mathbf{Q}_p) - \mathrm{PGL}_2^+(\mathbf{Q}_p)$. Then

$c_i(\tau^*\Sigma) = -c_i(\Sigma)$ since $\tau$ reverses orientations. It follows that $\tau^*\Sigma$ is isomorphic to $\Sigma$ as a rigid space but that the action of $\mu_{p^2-1}$ on $\tau^*\Sigma$ is twisted by Frobenius. This could have been deduced, of course, from the general construction of Drinfeld.

## Application to Shimura curves.

Now we examine the implications of Theorem 5 for the geometry of Shimura curves. Let $\Delta$ be an indefinite quaternion algebra over $\mathbf{Q}$ ramified at $p$ and let $L$ be a maximal order in $\Delta$. Suppose $n \geq 3$ is prime to $p$. Let $S_n$ be the scheme representing the functor which associates to a scheme $S$ the set of isomorphism classes of abelian surfaces over $S$ with an $L$ action and a level $n$-structure, and let $S_n^{\mathrm{an}}$ be the associated $p$-adic rigid space.

Let $\wp \subset L$ be the unique prime ideal above $p$. Let $S_{n,\wp}$ be the covering of $S_n$ which classifies abelian $L$ surfaces together with level $n$ structure and level $\wp$ structure. As before, $S_{n,\wp}^{\mathrm{an}}$ is the associated $p$-adic rigid space.

Let

$$X_n = U_n\backslash(\Delta'\otimes \mathbf{A}^f)^\times/\Delta'^\times$$

where $\Delta'$ is the definite quaternion algebra obtained from $\Delta$ by interchange of invariants at $p$ and $\infty$. Let $U_n$ be the principal congruence subgroup

$$U_n \subset \prod_{l\neq p}(L\otimes \mathbf{Z}_l) \ .$$

With this notation, we can state (a slightly simplified form of) Drinfeld's theorem.

THEOREM (Drinfeld [2]). — *There are isomorphisms*

(5) $$\mathrm{GL}_2(\mathbf{Q}_p)\backslash\mathcal{H}_p\otimes \mathrm{Spf}\ \hat{\mathbf{Z}}_p^{ur}\times X_n \to S_n^{\mathrm{an}}$$

*and*

(6) $$\mathrm{GL}_2(\mathbf{Q}_p)\backslash\Sigma\times X_n \to S_{n,\wp}^{\mathrm{an}} \ .$$

Furthermore, as Drinfeld points out, the quotient in (5) is actually the union of a finite number of components, each of the form

$$\Gamma\backslash\mathcal{H}_p\otimes \mathrm{Spf}\ \hat{\mathbf{Z}}_p^{ur}$$

where $\Gamma$ is a Schottky group.

Combining this with our geometric description of $\Sigma$, we obtain the following theorem.

THEOREM 8. — *Let* $S_n(\Gamma) = \Gamma \backslash \mathcal{H}_p \otimes \mathrm{Spf}\ \hat{\mathbb{Z}}_p^{ur}$ *be one of the components of* $S_n^{\mathrm{an}}$. *Suppose* $\Phi = \mathcal{T}/\Gamma$ *is the intersection graph of* $S_n(\Gamma)$. *Then the covering* $S_{n,\wp}^{\mathrm{an}}$ *over* $S_n(\Gamma)$ *has a stable model over* $\mathbb{C}_p$ *consisting of* $p-1$ *components. The reduction of each such component has intersection graph* $\Phi$, *but the vertices correspond to curves with the equation* (3) *rather than to rational curves.*

For the sake of concreteness, we supply an example. Suppose that $\Delta$ has discriminant 26 and that $p = 2$. Then $\Delta'$ has discriminant 13. Choose an embedding $\Delta' \otimes \mathbb{Z}_p \hookrightarrow M_2(\mathbb{Q}_p)$. Let $A$ be a maximal $\mathbb{Z}[1/2]$ order in $\Delta'$, and let

$$\Gamma = \{\gamma \in A : nr(\gamma) = 2^k, k \text{ even}\}\ .$$

The Shimura curve $S_1^{\mathrm{an}}$ of level 1 (over $\mathbb{C}_p$) is the quotient

$$\Gamma \backslash \mathcal{H}_p \otimes \mathrm{Spf}\ \mathbb{C}_p\ .$$

We are allowed to consider level 1 since $\Delta'$ has no multiplicative torsion. Since $\Delta'$ has class number 1, it is not hard to check that the special fiber of $S_1$ consists of two rational curves meeting in 3 points — see [3] or [6]. By the theorem, the special fiber of $S_{1,\wp}$ consists of two copies of the elliptic curve $Y^3 = z - z^2$ crossing in three points.

## Conclusions.

In conclusion, we mention two questions related to our subject matter. The first, rather naturally, is to obtain information on the higher coverings of the $p$-adic upper half plane; and, in particular, on the covering obtained from the $p$-torsion on Drinfeld's formal group. This is clearly a much harder problem than the one we have solved, since the higher coverings are not abelian and are in some sense "wildly ramified."

From a rigid analytic point of view, however, it would also be interesting to study the class of curves which admit a uniformization by $\Sigma$. Such curves are a type of generalized Mumford curve, and it would be worthwhile to extend the $p$-adic analytic theory of Mumford curves to this more general setting. In particular, the Jacobians of these curves are semi-abelian schemes, and it would be interesting to obtain some form of the Manin-Drinfeld theory of $p$-adic automorphic forms on $\Sigma$.

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