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GEOMETRY OF AN ÉTALE COVERING OF THE p-ADIC UPPER HALF PLANE

by Jeremy TEITELBAUM (*)

Introduction.

In this paper we describe the rigid geometry of the first layer in the tower of coverings of the p-adic upper half plane obtained from the division points of the formal group constructed in [2]. This covering is accessible because it is abelian and in some sense "tame." Using our results, we are able to describe the stable special fiber at p of Shimura curves with a very small amount of level p structure.

Preliminaries.

Let $\hat{\mathcal{H}}_p$ denote the formal scheme over $\mathbf{Z}_p$ constructed by Mumford ([4]) and commonly referred to as the p-adic upper half plane. Naively, $\hat{\mathcal{H}}_p$ is the complement of the $\mathbf{Q}_p$–rational points in $\mathbf{P}^1$. We let $\mathcal{H}_p$ be the rigid analytic space associated to $\hat{\mathcal{H}}_p$.

In [2], Drinfeld shows that $\hat{\mathcal{H}}_p$ is a parameter space for two–dimensional formal groups with a certain endomorphism structure. As a

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result, there is a universal family of formal groups $\mathcal{G}$ over $\hat{\mathcal{H}}_p$. The subgroups obtained as the division points of this family of formal groups yield a tower of coverings of $\hat{\mathcal{H}}_p$. The rigid spaces associated to these coverings are a family of étale coverings of $\mathcal{H}_p$. Our goal in this work is to describe the simplest of these coverings.

For a detailed description of Drinfeld's universal formal group, we refer the reader to [8]. We recall here the basic definitions which we will require.

Let D be the quaternion division algebra over $\mathbf{Q}_p$, and let $\mathcal{O}_D$ be the maximal order in D. A formal group G of dimension 2 and height 4 over a ring R on which p is nilpotent is called a special, formal $\mathcal{O}_D$–module (abbreviated SFD-module) provided that $\mathcal{O}_D$ acts on G and, at each maximal ideal m of R, both characters of the residue field of $\mathcal{O}_D$ occur in the tangent space to G at m. Over $\overline{\mathbf{F}}_p$, all SFD–modules are isogenous, so fix one such module Φ. With these conventions, we can state Drinfeld's theorem.

THEOREM (Drinfeld). — *$\hat{\mathcal{H}}_p \otimes$ Spf $\hat{\mathbf{Z}}_p^{ur}$ (over $\mathbf{Z}_p$) represents the functor which assigns to a ring R on which p is nilpotent the set of isomorphism classes of triples (ψ, G, ρ) where*

1. *$\psi : \hat{\mathbf{Z}}_p^{ur}/p\hat{\mathbf{Z}}_p^{ur} \to R/p$ is a homomorphism,*
2. *G is an SFD–module over R,*
3. *and $\rho : \psi_*\Phi \to G \otimes R/p$ is a "quasi–isogeny of height zero," which means that ρ is an isogeny with a certain normalization condition which will not be important in our work.*

We let $(\Psi, \mathcal{G}, \mathrm{P})$ be the universal triple over $\hat{\mathcal{H}}_p \otimes \mathrm{Spf}\, \hat{\mathbf{Z}}_p^{ur}$, and $\mathcal{G}_\Pi$ be the kernel of multiplication by Π on $\mathcal{G}$. This is a finite, flat group scheme of order p^2 over $\hat{\mathcal{H}}_p \otimes \mathrm{Spf}\, \hat{\mathbf{Z}}_p^{ur}$. If $\mathrm{PGL}_2^+(\mathbf{Q}_p)$ denotes the image in $\mathrm{PGL}_2(\mathbf{Q}_p)$ of the elements of $\mathrm{GL}_2(\mathbf{Q}_p)$ with determinants of even p-adic order, then the action of $\mathrm{PGL}_2^+(\mathbf{Q}_p)$ on $\hat{\mathcal{H}}_p$ extends to an action on the universal triple over $\hat{\mathcal{H}}_p \otimes \mathrm{Spf} \hat{\mathbf{Z}}_p^{ur}$, and therefore to $\mathcal{G}_\Pi$. Furthermore, the residue field $\mathcal{O}_D/\Pi = \mathbf{F}$ acts on $\mathcal{G}_\Pi$, and this action commutes with the action of $\mathrm{PGL}_2^+(\mathbf{Q}_p)$. In fact, $\mathrm{GL}_2(\mathbf{Q}_p)$ acts on the universal triple; see [2] p. 109 for details.

Let us fix a map $\psi_0 : \hat{\mathbf{Z}}_p^{ur} \to \hat{\mathbf{Z}}_p^{ur}$ and consider the fiber of the induced projection map $\hat{\mathcal{H}}_p \otimes \mathrm{Spf}\, \hat{\mathbf{Z}}_p^{ur} \to \mathrm{Spf}\, \hat{\mathbf{Z}}_p^{ur}$. The resulting formal scheme is isomorphic to $\hat{\mathcal{H}}_p \otimes \mathrm{Spf}\, \hat{\mathbf{Z}}_p^{ur}$ viewed as a formal scheme over $\hat{\mathbf{Z}}_p^{ur}$ with structure map ψ_0. The geometry of the formal scheme obtained in this way does not depend on the choice of ψ_0, so for the remainder of this paper we will abuse notation, suppress reference to ψ_0, and denote by $\hat{\mathcal{H}}_p$ and $\mathcal{H}_p$ respectively the formal and rigid p-adic upper half planes over $\mathbf{Z}_p$ (resp. $\mathbf{Q}_p$), base–changed up to $\hat{\mathbf{Z}}_p^{ur}$ (resp $\hat{\mathbf{Q}}_p^{ur}$.) Similarly, we base change the covering $\mathcal{G}_\Pi$ to obtain a finite flat group scheme of order p^2 over (our base–changed) $\hat{\mathcal{H}}_p$. We let $\hat{\Sigma}$ be the complement of the zero section in this group scheme, and Σ be the associated rigid space. This paper is devoted to describing the rigid geometry of Σ.

Classification of Σ as μ_{p^2-1}–torsor.

The action of the endomorphism ring of $\mathcal{G}$ induces an action of $\mathbf{F}^\times$ on the covering Σ, as we mentioned before. The two embeddings

$$\sigma_0, \sigma_1 : \mathbf{F} \to \hat{\mathbf{Z}}_p^{ur}/p\hat{\mathbf{Z}}_p^{ur}$$

induce two different actions of μ_{p^2-1} on our fixed SFD–module Φ over $\overline{\mathbf{F}}_p$, and therefore, via the universal isogeny P, two different actions on Σ. This allows us to view Σ as a μ_{p^2-1} torsor in two different ways.

DEFINITION 1. — *Let*

$$c_i(\Sigma) \in H^1_{et}(\mathcal{H}_p \otimes \operatorname{Spf} \mathbf{C}_p, \mu_{p^2-1})$$

be the class representing the covering $\Sigma \otimes \operatorname{Spf} \mathbf{C}_p$ viewed as a μ_{p^2-1} torsor via the embedding

$$\tilde{\sigma}_i : W(\mathbf{F}) \to \hat{\mathbf{Z}}_p^{ur} \subset \mathbf{C}_p$$

induced by σ_i .

The following lemma relates the two classes.

LEMMA 2. — $c_i(\Sigma) = pc_{i+1}(\Sigma)$, *reading subscripts mod* 2 .

Proof. — Changing the choice of σ_i twists the μ_{p^2-1} action on Σ by $\zeta \mapsto \zeta^p$. □

Our goal now is to determine the classes c_i precisely. Let $\mathcal{T}$ be the tree of $\mathrm{SL}_2(\mathbf{Q}_p)$. We fix a reduction map $r : \mathcal{H}_p \to \mathcal{T}$ which is compatible with the action of $\mathrm{SL}_2(\mathbf{Q}_p)$. For one of many detailed descriptions of the relation between $\mathcal{T}$ and $\mathcal{H}_p$, see [8], pp. 648–660.

The following theorem of Drinfeld ([1]) relates the cohomology of $\mathcal{H}_p$ to the tree $\mathcal{T}$.

THEOREM 3 (Drinfeld [1]). — *If N is an integer prime to p, there is an isomorphism*

$$\partial : H^1_{et}(\mathcal{H}_p \otimes \mathrm{Spf}\ \mathbf{C}_p, \mu_N) \to C^1_{har}(\mathcal{T}, \mathbf{Z}/N\mathbf{Z})$$

where $\mathcal{T}$ is the tree of SL_2 and $C^1_{har}(\mathcal{T}, \mathbf{Z}/N\mathbf{Z})$ is the group of harmonic 1-cochains on $\mathcal{T}$ — that is, the set of functions f on the edges of $\mathcal{T}$ such that, for each vertex v, f satisfies

$$\sum_{e \mapsto v} f(e) = 0$$

where the sum is over the oriented edges of $\mathcal{T}$ meeting v.

Let us briefly recall how the map ∂ of the theorem is constructed. Suppose Y is a torsor over $\mathcal{H}_p \otimes \mathrm{Spf}\ \mathbf{C}_p$. Let U be the admissible open set in $\mathcal{H}_p$ corresponding to a vertex v in $\mathcal{T}$, together with its bounding edges. It follows from Lemma 2 of [8] that U is a $\mathrm{GL}_2(\mathbf{Q}_p)$ translate of the standard open set

$$V = \{P \in \mathcal{H}_p : 1/p < |z(P)| < p\} - \bigcup_{i=1}^{p-1} B^+_{1/p}(i)$$

where $B^+_r(i)$ denotes the closed ball centered at i of radius r. Therefore $\mathrm{Pic}(U) = 0$ and so

$$\mathcal{O}_Y = \mathcal{O}_U[T]/(T^{p^2-1} - f)$$

where μ_{p^2-1} acts by multiplication on T, and f is uniquely determined up to $(p^2-1)^{st}$ powers.

Let $e = \{v, v'\}$ be an (oriented) edge leaving v. To evaluate $\partial(Y)(e)$, choose a coordinate function z on U such that if $P \in U$ reduces to v then $|z(P)| = 1$ while if $P \in U$ reduces to e then

$$|p| < |z(P)| < 1 .$$

Then we let

$$\partial(Y)(e) = \mathrm{Res}_e df/f \pmod{p^2-1}$$

where Res_e denotes the rigid analytic "annular residue" computed with respect to the selected coordinate z (which determines the sign of Res.)

Since $\mathrm{PGL}_2^+(\mathbf{Q}_p)$ acts on Σ, the class $c_i(\Sigma)$ is invariant by $\mathrm{PGL}_2^+(\mathbf{Q}_p)$. As we see in the next lemma, this is a very strong condition.

LEMMA 4. — *Suppose Y is a $\mathrm{PGL}_2^+(\mathbf{Q}_p)$-invariant torsor over $\mathcal{H}_p \otimes \mathrm{Spf}\ \mathbf{C}_p$, and $\partial(Y)$ is the associated harmonic 1-cocycle. Then $\partial(Y)$ satisfies the following conditions:*

1. $\partial(Y)(e) = \partial(Y)(\gamma e)$ *for all* $\gamma \in \mathrm{PGL}_2^+(\mathbf{Q}_p)$ *and all oriented edges* e *of* $\mathcal{T}$.

2. $\partial(Y)(e) \equiv 0 \pmod{p-1}$ *on all edges* e *of* $\mathcal{T}$.

Proof. — The first property is a consequence of the invariance of Y by $\mathrm{PGL}_2^+(\mathbf{Q}_p)$ and the fact that $\mathrm{PGL}_2^+(\mathbf{Q}_p)$ preserves the orientation of edges in $\mathcal{T}$. For the second, observe that all the edges e leaving any vertex v are permuted transitively by $\mathrm{PGL}_2^+(\mathbf{Q}_p)$. Therefore $\partial(Y)(e)$ is a constant x on all edges e leaving v. From the harmonicity condition we obtain

$$\sum_{e \mapsto v} \partial(Y)(e) = (p+1)x \equiv 0 \pmod{p^2-1}$$

since there are $p+1$ edges leaving v. This proves the lemma. □

In order to give a precise statement of our theorem, we must invoke the relationship between orientations on $\mathcal{T}$ and the embeddings σ_i. As Drinfeld shows, and we explain in Lemma 14 of [8], the action of Π on the tangent space T to $\mathcal{G}$ allows us to partition the vertices of $\mathcal{T}$ into two classes labeled with the σ_i. To describe this partition, first decompose T into σ_i–eigenspaces for the action for the quadratic unramified extension of $\mathbf{Z}_p$ inside $\mathcal{O}_D$. A vertex v is labelled with σ_i if $\Pi T^i \subset pT^{i+1}$ over the affinoid reducing to v where T^i is the σ_i–eigenspace in T. Vertices of the two classes alternate in the tree.

Since $\partial_i(\Sigma)$ is $\mathrm{PGL}_2^+(\mathbf{Q}_p)$-invariant, and this group permutes the edges of $\mathcal{T}$ transitively, it suffices to specify the value of $\partial_i(\Sigma)$ on a single edge. This is the content of our theorem.

THEOREM 5. — *Let* $e = [v, v']$ *be an edge of* $\mathcal{T}$. *Suppose that* v *is labeled with* σ_i. *Then*

$$\partial_i(\Sigma)(e) = p - 1 \ .$$

Proof. — Notice first of all that, by Lemmas 2 and 4,

$$\begin{aligned}\partial_i(\Sigma)([v,v']) &= -\partial_i(\Sigma)([v',v]) \\ &= p\partial_i(\Sigma)([v',v]) \\ &= \partial_{i+1}(\Sigma)([v',v])\end{aligned}$$

and therefore we may assume that v is labeled with σ_0. Let U be the affine open set of $\hat{\mathcal{H}}_p$ corresponding to e. It follows from Lemma 2 of [8] that U is a $\mathrm{PGL}_2^+(\mathbf{Q}_p)$ translate of the standard open set

$$U(1) = \{P \in \mathcal{H}_p : 1/p \leq |z(P)| \leq 1\} - \bigcup_{i=1}^{p-1} (B_1(i) \cup B_{1/p}(pi))$$

where $B_r(i)$ denotes the open disc of radius r centered at i. Therefore the coordinate ring of U is isomorphic to

$$\hat{R} = \varprojlim R/p^n R$$

where

$$R = \frac{\mathbf{Z}[z_0, z_1]}{(z_0 z_1 - p)} \left[\frac{1}{1 - z_0^{p-1}}, \frac{1}{1 - z_1^{p-1}} \right] .$$

As we recalled prior to stating this theorem, the tangent space T to $\mathcal{G}$ over U is free of rank 2, and it carries a grading coming from the action of the maximal unramified extension of $\mathbf{Z}_p$ in $\mathcal{O}_D$. Write $T = \hat{R}t_0 \oplus \hat{R}t_1$. Referring to [8] p. 656, we see that the Π action on T is $\Pi t_0 = z_1 t_1$ and $\Pi t_1 = z_0 t_0$. With these conventions, the vertex v of $\mathcal{T}$ labeled with σ_0 corresponds to the region where z_0 is a unit; the vertex v' labeled with σ_1 corresponds to the region where z_1 is a unit.

Let Ω be the cotangent space to $\mathcal{G}$. Then $\Omega_\Pi = \Omega/\Pi\Omega$ is naturally the cotangent space to $\mathcal{G}_\Pi$. If ω_0 and ω_1 generate the graded pieces of Ω, then we must have $\Pi\omega_i = z_i \omega_{i+1}$. It follows that

$$\Omega_\Pi = (\hat{R}/z_1\hat{R})\omega_0 \oplus (\hat{R}/z_0\hat{R})\omega_1 \; . \tag{1}$$

The finite flat group scheme $\mathcal{G}_\Pi$, together with its action by $\mathbf{F}$ is of the type classified by Raynaud. Applying his classification (see [5], Corollary 1.5.1) we see that the coordinate B ring of $\mathcal{G}_\Pi$ must have the form

$$B = \hat{R}[X_0, X_1]/(X_0^p - \delta_0 X_1, X_1^p - \delta_1 X_0)$$

where the functions δ_i and p/δ_i belong to $\hat{R}^\times$. In addition, Raynaud shows that the natural identification of Ω_Π with I/I^2 (I being the augmentation ideal) means that

$$\Omega_\Pi = (\hat{R}/\delta_1\hat{R})X_0 \oplus (\hat{R}/\delta_0\hat{R})X_1 \; . \tag{2}$$

Combining (1) and (2), we see that z_i and δ_i differ by a unit of $\hat{R}$.

We now have enough information to compute the class of Σ. Indeed, Σ is defined over U by the equation

$$X_0^{p^2-1} - \delta_0^p \delta_1 = 0 \; .$$

The group $\mathbf{F}^\times$ acts on X_0 through the embedding σ_0. Let us write $f = \delta_0^p \delta_1$. Then

$$\partial_0(\Sigma)(e) = \mathrm{Res}_{z_0} df/f \pmod{p^2 - 1} \; .$$

Our results above tell us that $\delta_0^p \delta_1 = p z_0^{p-1} h(z)$ where h is a unit in $\hat{R}$. This residue is clearly $p - 1$. □

Thanks to this theorem, we can construct Σ over $\mathbb{C}_p$. Let X be the non-singular projective curve over $\mathbb{C}_p$ defined by the affine equation

$$Y^{p+1} = z - z^p \ . \tag{3}$$

Let $W \subset X$ be the admissible open set of points P on X such that

$$|p^{1/(p+1)}| < |Y(P)| < |p^{-p/(p+1)}| \ . \tag{4}$$

COROLLARY 6. — *Over $\mathbb{C}_p$, Σ consists of $p-1$ isomorphic connected components. Each such component has a covering by admissible open sets isomorphic to W. The nerve of this covering is the tree $\mathcal{T}$. If W_1 and W_2 are two elements of the covering, and $E = W_1 \cap W_2 \subset W_1$, then E is one of the boundary annuli of W_1 .*

Proof. — Let U be the subset of $\mathcal{H}_p \otimes \mathrm{Spf}\ \mathbb{C}_p$ consisting of one vertex (say, labeled with σ_0), and its bounding edges. Then by Theorem 5, Σ over U is obtained by extracting the $p^2 - 1$ root of a function with order congruent to $p-1$ mod p^2-1 on each bounding annulus. If z is a coordinate on U, then the function $f(z) = (z - z^p)^{p-1}$ clearly meets this condition. Thus Σ is defined over U by the equation

$$Y_0^{p^2-1} = (z_0 - z_0^p)^{p-1}$$

where z_0 is an appropriate parameter on U. Notice first that this equation factors, so that Σ consists of $p-1$ connected components, and is built up out of pieces of the curve in (3) as claimed. It is a simple matter to check that the subset of W satisfying the inequality (4) has genus $(p^2 - p)/2$. Thus the reduction of Σ consists of curves meeting in double points, like the reduction of $\mathcal{H}_p$, except that the rational curves which appear in the reduction of $\mathcal{H}_p$ are replaced by the curves of equation (3) .

With somewhat more care one can determine the equations for Σ over $\hat{\mathbb{Z}}_p^{ur}$, instead of just over $\mathbb{C}_p$. Examining the end of the proof of Theorem 5 we see that over $\hat{\mathbb{Z}}_p^{ur}$, we can take $\delta_0^p\delta_1 = p(z - z^p)^{p-1}$ and therefore Σ is defined over U by the equation

$$Y_0^{p^2-1} = p(z_0 - z_0^p)^{p-1} \ .$$

From this one can obtain a minimal regular model for Σ over $\hat{\mathbb{Z}}_p^{ur}$ by blowing–up. This is a straightforward computational problem whose solution we omit, although we do point out that the minimal model has no components of multiplicity one.

Finally, notice that the action of $\mathrm{PGL}_2^+(\mathbb{Q}_p)$ on Σ extends to an action of $\mathrm{PGL}_2(\mathbb{Q}_p)$. Indeed, choose $\tau \in \mathrm{PGL}_2(\mathbb{Q}_p) - \mathrm{PGL}_2^+(\mathbb{Q}_p)$. Then

$c_i(\tau^*\Sigma) = -c_i(\Sigma)$ since τ reverses orientations. It follows that $\tau^*\Sigma$ is isomorphic to Σ as a rigid space but that the action of μ_{p^2-1} on $\tau^*\Sigma$ is twisted by Frobenius. This could have been deduced, of course, from the general construction of Drinfeld.

Application to Shimura curves.

Now we examine the implications of Theorem 5 for the geometry of Shimura curves. Let Δ be an indefinite quaternion algebra over $\mathbf{Q}$ ramified at p and let L be a maximal order in Δ. Suppose $n \geq 3$ is prime to p. Let S_n be the scheme representing the functor which associates to a scheme S the set of isomorphism classes of abelian surfaces over S with an L action and a level n-structure, and let S_n^{an} be the associated p-adic rigid space.

Let $\wp \subset L$ be the unique prime ideal above p. Let $S_{n,\wp}$ be the covering of S_n which classifies abelian L surfaces together with level n structure and level $\wp$ structure. As before, $S_{n,\wp}^{\mathrm{an}}$ is the associated p-adic rigid space.

Let

$$X_n = U_n \backslash (\Delta' \otimes \mathbb{A}^f)^\times / \Delta'^\times$$

where Δ' is the definite quaternion algebra obtained from Δ by interchange of invariants at p and ∞. Let U_n be the principal congruence subgroup

$$U_n \subset \prod_{l \neq p} (L \otimes \mathbf{Z}_l) .$$

With this notation, we can state (a slightly simplified form of) Drinfeld's theorem.

THEOREM (Drinfeld [2]). — *There are isomorphisms*

$$\mathrm{GL}_2(\mathbf{Q}_p) \backslash \mathcal{H}_p \otimes \mathrm{Spf}\ \hat{\mathbf{Z}}_p^{ur} \times X_n \to S_n^{\mathrm{an}} \tag{5}$$

and

$$\mathrm{GL}_2(\mathbf{Q}_p) \backslash \Sigma \times X_n \to S_{n,\wp}^{\mathrm{an}} . \tag{6}$$

Furthermore, as Drinfeld points out, the quotient in (5) is actually the union of a finite number of components, each of the form

$$\Gamma \backslash \mathcal{H}_p \otimes \mathrm{Spf}\ \hat{\mathbf{Z}}_p^{ur}$$

where Γ is a Schottky group.

Combining this with our geometric description of Σ, we obtain the following theorem.

THEOREM 8. — *Let* $S_n(\Gamma) = \Gamma\backslash\mathcal{H}_p \otimes \mathrm{Spf}\ \hat{\mathbf{Z}}_p^{ur}$ *be one of the components of* S_n^{an}. *Suppose* $\Phi = \mathcal{T}/\Gamma$ *is the intersection graph of* $S_n(\Gamma)$. *Then the covering* $S_{n,\wp}^{\mathrm{an}}$ *over* $S_n(\Gamma)$ *has a stable model over* $\mathbf{C}_p$ *consisting of* $p-1$ *components. The reduction of each such component has intersection graph* Φ, *but the vertices correspond to curves with the equation (3) rather than to rational curves.*

For the sake of concreteness, we supply an example. Suppose that Δ has discriminant 26 and that $p = 2$. Then Δ' has discriminant 13. Choose an embedding $\Delta' \otimes \mathbf{Z}_p \hookrightarrow M_2(\mathbf{Q}_p)$. Let A be a maximal $\mathbf{Z}[1/2]$ order in Δ', and let

$$\Gamma = \{\gamma \in A : nr(\gamma) = 2^k, k \text{ even}\} .$$

The Shimura curve S_1^{an} of level 1 (over $\mathbf{C}_p$) is the quotient

$$\Gamma\backslash\mathcal{H}_p \otimes \mathrm{Spf}\ \mathbf{C}_p .$$

We are allowed to consider level 1 since Δ' has no multiplicative torsion. Since Δ' has class number 1, it is not hard to check that the special fiber of S_1 consists of two rational curves meeting in 3 points — see [3] or [6]. By the theorem, the special fiber of $S_{1,\wp}$ consists of two copies of the elliptic curve $Y^3 = z - z^2$ crossing in three points.

Conclusions.

In conclusion, we mention two questions related to our subject matter. The first, rather naturally, is to obtain information on the higher coverings of the p-adic upper half plane; and, in particular, on the covering obtained from the p-torsion on Drinfeld's formal group. This is clearly a much harder problem than the one we have solved, since the higher coverings are not abelian and are in some sense "wildly ramified."

From a rigid analytic point of view, however, it would also be interesting to study the class of curves which admit a uniformization by Σ. Such curves are a type of generalized Mumford curve, and it would be worthwhile to extend the p-adic analytic theory of Mumford curves to this more general setting. In particular, the Jacobians of these curves are semi-abelian schemes, and it would be interesting to obtain some form of the Manin-Drinfeld theory of p-adic automorphic forms on Σ.

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