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THE SCHOTTKY-JUNG THEOREM FOR MUMFORD CURVES

by Guido VAN STEEN

Introduction.

The classical Schottky relations for theta functions are relations which are valid for theta functions on the Jacobian variety of a Riemann surface. These relations are derived from a theorem by Schottky and Jung.

In [6] Mumford gives a purely algebraic geometrical version of this theorem. However, in the case of a complete non-archimedean valued base field there exists a theory of theta functions on analytic tori which is very similar to the complex theory, *cf.* [3].

In this paper we use these theta functions to prove the Schottky-Jung theorem in the particular case that the torus is the Jacobian variety of a Mumford curve. In Section 2 we prove a slightly weaker version of the theorem. In Section 3 we prove the stronger version in the particular case of hyperelliptic curves. In Section 3 we prove the theorem in the general case using the technique of analytic families of curves.

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Notations.

i) k is an algebraically closed complete non-archimedean valued field, $\mathrm{char}(k) \neq 2, 3$.

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ii) $\mathbf{P}^1$ is the projective line over k .

1. Theta functions and the Riemann Vanishing Theorem.

Let $\Gamma \subset PGL(2,k)$ be a Schottky group of rank $g+1$. Let $X_\Gamma = \Omega/\Gamma$ be the corresponding Mumford curve; $\Omega \subset \mathbf{P}^1$ the set of ordinary points of Γ . The Jacobian variety J_Γ of X_Γ can be identified with an analytic torus; *cf.* [4]. We recall briefly how this is done.

If $a,b \in \Omega$ we define $u_{a,b}(z) = \prod_{\gamma\in\Gamma} \dfrac{z-\gamma(a)}{z-\gamma(b)}$; $z \in \Omega$.

This product defines a meromorphic function on Ω which satisfies a functional equation $c_{a,b}(\gamma)\cdot u_{a,b}(\gamma z) = u_{a,b}(z)$ with $\gamma \in \Gamma$ and $c_{a,b} \in \mathrm{Hom}(\Gamma, k^*)$. If $b \notin \Gamma(a)$ then $u_{a,b}$ has zeroes in the orbit $\Gamma(a)$ and poles in the orbit $\Gamma(b)$. If $b = \gamma(a)$ with $\gamma \in \Gamma$, then $u_{a,b}$ does not depend on a . In this case we denote $u_\gamma = u_{a,b}$ and $c_\gamma = c_{a,b}$. The function u_γ has no zeroes or poles.

Let $G_\Gamma = \mathrm{Hom}(\Gamma, k^*)$. This group can be identified with $(k^*)^{g+1}$ and hence has an analytic structure. The subgroup $\Lambda_\Gamma = \{c_\gamma \mid \gamma \in \Gamma\}$ is a free abelian group of rank $g+1$ and is discrete in G_Γ .

With a divisor $D = \sum_{i=1}^{n}(\bar{a}_i - \bar{b}_i)$ on X_Γ with $\deg(D) = 0$ corresponds a homomorphism $c = \prod_{i=1}^{n} c_{a_i,b_i} \in G_\Gamma$; $a_i, b_i \in \Omega$. This correspondence induces an analytic isomorphism from J_Γ onto the quotient G_Γ/Λ_Γ .

Let $p \in \Omega$ be a fixed point. Define $t_\Gamma : \Omega \to G_\Gamma$ by $t_\Gamma(x) = c_{x,p}$. The induced map $\bar{t}_\Gamma : X_\Gamma \to J_\Gamma$ is the canonical embedding of X_Γ into J_Γ with base point $\bar{p}$. This map is extended to divisors in a canonical way.

The dual variety $\widehat{J}_\Gamma$ of J_Γ can also be represented as an analytic torus. One has $\widehat{J}_\Gamma = \widehat{G}_\Gamma/\widehat{\Lambda}_\Gamma$ with $\widehat{G}_\Gamma = \mathrm{Hom}(\Lambda_\Gamma, k^*)$ and

$\widehat{\Lambda}_\Gamma = \{d \in \widehat{G}_\Gamma \mid \exists \alpha \in \Gamma \text{ such that } d(c_\gamma) = c_\alpha(\gamma) \text{ for all } c_\gamma \in \Lambda_\Gamma\}$.

The group Λ_Γ acts on

$\mathbf{O}^*(G_\Gamma) = \{f \mid f \text{ holomorphic and nowhere vanishing function on } G_\Gamma\}$.

For $f \in \mathbf{O}^*(G_\Gamma)$, $c_\gamma \in \Lambda_\Gamma$ and $c \in G_\Gamma$ one defines $f^{c_\gamma}(c) = f(c_\gamma c)$. If $\xi \in \mathbf{Z}^1(\Lambda_\Gamma, \mathbf{O}^*(G_\Gamma))$ is a 1-cocycle then we denote

$\mathbf{L}(\xi) = \{h \mid h \text{ holomorphic function on } G_\Gamma\ ,\ \ h(c) = \xi_{c_\gamma}(c)h(c_\gamma c)$

$\text{for all } c_\gamma \in \Lambda_\Gamma\}$.

Elements of $\mathbf{L}(\xi)$ are called holomorphic theta functions of type ξ .

Let $\lambda_\xi : G_\Gamma \to \widehat{G}_\Gamma$ be defined by $\lambda_\xi(c)(c_\gamma) = c(\gamma)$. This morphism induces a morphism $\bar{\lambda}_\xi : J_\Gamma \to \widehat{J}_\Gamma$.

If $\mathbf{L}(\xi) \neq 0$, then $\bar{\lambda}_\xi$ is an isogeny and $\dim(\mathbf{L}(\xi)) = [\operatorname{Ker}\bar{\lambda}_\xi : \overline{\operatorname{Ker}\lambda_\xi}]$ where $\overline{\operatorname{Ker}\lambda_\xi}$ is the image in J_Γ of $\operatorname{Ker}\lambda_\xi \subset G_\Gamma$; *cf.* [3], [11].

A canonical 1-cocycle can be defined in the following way. Let

$$p_\Gamma : \Lambda_\Gamma \times \Lambda_\Gamma \to k^*$$

be a symmetric bilinear form such that $p_\Gamma^2(c_\gamma, c_\delta) = c_\gamma(\delta)$ for all $\gamma, \delta \in \Gamma$. Define ξ_Γ by $\xi_{\Gamma,c_\gamma}(c) = p_\Gamma(c_\gamma, c_\gamma)c(\gamma)$; $c_\gamma \in \Lambda_\Gamma$, $c \in G_\Gamma$. In this case $\bar{\lambda}_{\xi_\Gamma}$ is an isomorphism and hence $\dim(\mathbf{L}(\xi_\Gamma)) = 1$. In fact $\mathbf{L}(\xi_\Gamma)$ is generated by the Riemann theta function $\theta_\Gamma(c) = \sum_{c_\gamma \in \Lambda_\Gamma} \xi_{\Gamma,c_\gamma}(c)$. The divisor of θ_Γ is Λ_Γ-invariant and hence induces a divisor on J_Γ . This divisor defines a polarization Θ_Γ on J_Γ .

The isogeny form J_Γ onto $\widehat{J}_\Gamma$ which can be associated with a polarization is in this case $\bar{\lambda}_{\xi_\Gamma}$. Since this is an isomorphism, Θ_Γ is a principal polarization. In fact Θ_Γ is the canonical principal polarization which exists on a Jacobian variety. This follows from :

THEOREM 1.1 (Riemann Vanishing Theorem).

i) *The holomorphic function $\theta_\Gamma \circ t_\Gamma$ has a Γ-invariant divisor which, regarded as a divisor on X_Γ , has degree $g+1$.*

ii) *If the map $\bar{t}_\Gamma : X_\Gamma \to J_\Gamma$ is based at the point $p \in \Omega$, and if $K_\Gamma = (\operatorname{div}(\theta \circ t_\Gamma) - p) \bmod \Gamma \in \operatorname{Div}(X_\Gamma)$, then $2K_\Gamma$ is a canonical divisor. Furthermore, the class of K_Γ under linear equivalence of divisors does not depend on the choice of p .*

iii) *If $c \in G_\Gamma$ then $\theta_\Gamma(c) = 0$ if and only if $\bar{c} = \bar{t}_\Gamma(D - K_\Gamma)$ for some positive divisor D of degree g . The order of vanishing of θ_Γ at c is equal to $i(D)$, the index of speciality of D .*

Proof. — The divisor θ_Γ is calculated in [4]. The other assertions are easily proved in a similar way as in the complex case; e.g. the proof such as given in [1] can easily be adapted. □

2. The Schottky-Jung theorem.

Let X_Γ be as in Section 1. Let $\pi : X \to X_\Gamma$ be an analytic covering of X_Γ ; X a curve of genus $2g+1$.

The condition of π being analytic is stronger than being just unramified, *cf.* [8]. In particular this condition implies that X is a Mumford curve corresponding to a Schottky group Δ with Δ a subgroup of Γ with $[\Gamma : \Delta] = 2$. Since Δ is normal in Γ , both groups have the same set of ordinary points. So $X = X_\Delta = \Omega/\Delta$. Moreover, the map π is given by

$$\pi(\Delta\text{-orbit of } x) = (\Gamma\text{-orbit of } x) \ ; \ x \in \Omega \ .$$

The Jacobian variety of X_Δ is constructed in the same way as J_Γ . We keep the same notations as in Section 1 but to indicate that we work with respect to Δ we will denote

$$\tilde{u}_{a,b}(z) = \prod_{\beta \in \Delta} \frac{z-\beta(a)}{z-\beta(b)} \ ; \ \ \tilde{c}_{a,b}(\delta) = \frac{\tilde{u}_{a,b}(z)}{\tilde{u}_{a,b}(\delta(z))} \ , \ \ \tilde{c}_\delta = \tilde{c}_{a,\delta(a)}, \dots$$

We take a symmetric bilinear form $p_\Delta : \Lambda_\Delta \times \Lambda_\Delta \to k^*$ such that $p_\Delta^2(\tilde{c}_\alpha, \tilde{c}_\beta) = \tilde{c}_\alpha(\beta)$. The canonical 1-cocycle $\xi_\Delta \in \mathbf{Z}^1(\Lambda_\Delta, \mathbf{O}^*(G_\Delta))$ is defined by $\xi_{\Delta,\tilde{c}_\delta}(\tilde{c}) = p_\Delta(\tilde{c}_\delta, \tilde{c}_\delta)\tilde{c}(\delta)$; $\tilde{c}_\delta \in \Lambda_\Delta$ and $\tilde{c} \in G_\Delta$. The Riemann theta function on G_Δ is defined by

$$\theta_\Delta(\tilde{c}) = \sum_{\tilde{c}_\delta \in \Lambda_\Delta} \xi_{\Delta,\tilde{c}_\delta}(\tilde{c}) \ ; \ \ \tilde{c} \in G_\Delta \ .$$

Let $(\gamma_0, \gamma_1, \dots, \gamma_g)$ be a free basis for the group Γ . We may assume $\gamma_0 \notin \Delta$ and $\gamma_i \in \Delta$ for $i = 1, \dots, g$.

So Δ has a free basis $\delta_0, \delta_1, \dots, \delta_g, \delta_{-1}, \dots, \delta_{-g}$ with $\delta_0 = \gamma_0^2$, $\delta_i = \gamma_i$, $\delta_{-i} = \gamma_0\gamma_i\gamma_0^{-1}$; $i = 1, \dots, g$. The bilinear forms can be normalized such that

i) $p_\Delta(\tilde{c}_{\delta_0}, \tilde{c}_{\delta_0}) = c_{\gamma_0}(\gamma_0)$,

ii) $\forall \alpha, \beta \in \Delta : p_\Delta(c_{\alpha|_\Delta}, \tilde{c}_\beta) = p_\Gamma(c_\gamma, c_\beta)$.

($c_{\alpha|_\Delta}$ is the restriction of c_γ to Δ .)

Let $\pi^* : J_\Gamma \to J_\Delta$ be the dual map of π . This map is defined by

$$\pi^*(c \bmod(\Lambda_\Gamma)) = c|_\Delta \bmod(\Lambda_\Delta) \ .$$

Since π is unramified $\operatorname{Ker}\pi^*$ has order 2. The non-trivial element of $\operatorname{Ker}\pi^*$ is $\bar{c}_0$ with $c_0 \in G_\Gamma$ defined by $c_0(\gamma_0) = -1$ and $c_0(\gamma_i) = 1$;

$i = 1, \ldots, g$. More relations between J_Γ and J_Δ can be found in [11]. The relation between θ_Γ and θ_Δ is given by

THEOREM 2.1 (Schottky-Jung relation). — *There exists a homomorphism $e_0 \in G_\Gamma$ such that $e_0^2 = c_0$ and such that*

$$\frac{\theta_\Delta(c|_\Delta)}{\theta_\Gamma(e_0 c) \cdot \theta_\Gamma(e_0^{-1} c)}$$

is a constant function in $c \in G_\Gamma$.

In this Section we will prove only that e_0 satisfies $e_0^2 \equiv c_0 \bmod(\Lambda_\Gamma)$. This weaker version of the theorem is basically the same as the algebraic geometrical result given in [6].

Meromorphic functions on X_Γ or X_Δ can be lifted to Γ-invariant or Δ-invariant meromorphic functions on Ω .

A similar correspondence holds for divisors on X_Γ and X_Δ . We make no difference between divisors on X_Γ (or X_Δ) and their lifts to Ω . If D is a divisor on X_Γ then denote

$\mathbf{L}_\Gamma(D) = \{f | f, \Gamma\text{-invariant meromorphic function on } \Omega \text{ with } \operatorname{div}(f) + D \geqq 0\}$.

(Similar meaning for $\mathbf{L}_\Delta$.)

PROPOSITION 2.2. — *Let D be a divisor on X_Γ with* $\deg(D) = g$ *and let $\pi^*(D)$ be the reciprocal image of D on X_Δ . The following sequence is exact :*

$$0 \to \mathbf{L}_\Gamma(D) \xrightarrow{\alpha} \mathbf{L}_\Delta(\pi^*(D)) \xrightarrow{\beta} \mathbf{L}_\Gamma(D - D_0) \to 0$$

with :

i) $D_0 = \operatorname{div}(f_0)$ *and f_0 a meromorphic function on Ω such that $c_0(\gamma) f_0(\gamma c) = f_0(c)$ for all $\gamma \in \Gamma$*

ii) $\alpha(f) = f$ *for all* $f \in \mathbf{L}_\Gamma(D)$

iii) $\beta(g) = \dfrac{g - g \circ \gamma_0}{2} \cdot f_0$ *for all* $g \in \mathbf{L}_\Delta(\pi^*(D))$.

Proof. — It is easy to verify that these maps are well defined. If $g \in \operatorname{Ker} \beta$ then $g = g \circ \gamma_0$ and g is Δ-invariant. So g is Γ-invariant and in fact g is an element of $\mathbf{L}_\Gamma(D)$. If $f \in \mathbf{L}(D - D_0)$ then $f = \beta(f/f_0)$. So β is surjective. □

Let $p \in \Omega$. We have canonical maps $\bar{t}_\Gamma : X_\Gamma \to J_\Gamma$ and $\bar{t}_\Delta : X_\Delta \to J_\Delta$ with $\bar{t}_\Gamma(\bar{x}) = \bar{c}_{x,p}, \bar{t}_\Delta(\bar{x}) = \bar{\bar{c}}_{x,p}$. These maps are extended to divisors.

Define K_Γ and K_Δ as in Section 1. According to the Riemann Vanishing Theorem $2K_\Gamma$ and $2K_\Delta$ are canonical divisors on X_Γ and X_Δ . Since π is unramified $\pi^*(2K_\Gamma)$ and $2K_\Delta$ are linear equivalent. Hence $\pi^*(K_\Gamma) = K_\Delta + E$ where E is a divisor of degree 0 such that $2E$ is principal.

Let $\varepsilon \in G_\Delta$ such that $\bar{t}_\Delta(E) = \bar{\varepsilon}$, (ε is defined up to periods in Λ_Δ). We have the following

LEMMA 2.3. — $\dfrac{\theta_\Delta(c|_\Delta \cdot \varepsilon)}{\theta_\Gamma(c) \cdot \theta_\Gamma(cc_0)}$ *is a nowhere vanishing holomorphic function on* G_Γ .

Proof. — If $\theta_\Gamma(c) = 0$ then $\bar{c} = \bar{t}_\Gamma(D - K_\Gamma)$; D a positive divisor on X_Γ with $\deg(D) = g$. Hence $\pi^*(\bar{c}) = \overline{c|_\Delta} = \overline{t_\Delta}(\pi^*(D) - \pi^*(K_\Gamma))$ and consequently $\pi^*(\bar{c}) \cdot \bar{\varepsilon} = \overline{t_\Delta}(\pi^*(D) - K_\Delta)$. It follows that $\theta_\Delta(c|_\Delta \cdot \varepsilon) = 0$. In a similar way we find that $\theta_\Delta(c|_\Delta \cdot \varepsilon) = 0$ if $\theta_\Gamma(cc_0) = 0$. Furthermore the vanishing order of $\theta_\Delta(c|_\Delta \cdot \varepsilon)$ is the sum of the vanishing orders of $\theta_\Gamma(c)$ and $\theta_\Gamma(cc_0)$. This follows from 2.2 and the Riemann Vanishing Theorem. □

LEMMA 2.4. — *K_Δ and $\gamma_0(K_\Delta)$ are linear equivalent.*

Proof. — It follows from the definition of K_Δ that

$$\gamma_0(K_\Delta) = \mathrm{div}(\theta_\Delta \circ t_\Delta \circ \gamma_0) - \gamma_0(p) .$$

If $x \in \Omega$ we have $t_\Delta(\gamma_0(x)) = \tilde{c}_{\gamma_0(x),p} = \tilde{c}_{\delta_0} \cdot \tilde{c}_{\gamma_0^{-1}(x),p} = \tilde{c}_{\delta_0} \cdot \tilde{c}^{\gamma_0}_{x_0,\gamma_0(p)}$, *cf.* [10]. (If $\tilde{c} \in G_\Delta$, then $\tilde{c}^{\gamma_0}$ is defined by $\tilde{c}^{\gamma_0}(\delta) = \tilde{c}(\gamma_0\delta\gamma_0^{-1})$.)

Since $\dfrac{\theta_\Delta(\tilde{c}_\delta \tilde{c})}{\theta_\Delta(\tilde{c})} \in \mathbf{O}^*(G_\Delta)$ and since $\theta_\Delta(\tilde{c}^{\gamma_0}) = \theta_\Delta(\tilde{c})$, we find that $\gamma_0(K_\Delta) = \mathrm{div}(\theta_\Delta(\tilde{c}_{x,\gamma_0(p)})) - \overline{\gamma_0(p)}$. It follows from 1.1 that $\gamma_0(K_\Delta)$ and K_Δ are linear equivalent. □

As a consequence $\gamma_0(E)$ and E are linear equivalent and hence $\varepsilon^{\gamma_0}\varepsilon^{-1} \in \Lambda_\Delta$. Since $\varepsilon^{\gamma_0}\varepsilon^{-1}$ is γ_0-anti-invariant, we have $\varepsilon^{\gamma_0}\varepsilon^{-1} = \tilde{c}_\delta^{\gamma_0}\tilde{c}_\delta^{-1}$ for some $\delta \in \Delta$, *cf.* [11]. Hence, after replacing ε by $\varepsilon\tilde{c}_\delta^{-1}$, we may assume that ε is invariant under the action of γ_0 . It follows that $\varepsilon = \pi^*(e_0)$ for some $e_0 \in G_\Gamma$.

We have the following weaker version of Theorem 2.1.

PROPOSITION 2.5.

i) $e_0^2 \equiv c_0 \bmod \Lambda_\Gamma$

ii) $\dfrac{\theta_\Delta(\pi^*(c))}{\theta_\Gamma(ce_0)\cdot\theta_\Gamma(ce_0^{-1})}$ *is constant in* c .

Proof. — Since $\dfrac{\theta_\Delta(\pi^*(c)\varepsilon)}{\theta_\Gamma(c)\theta_\Gamma(cc_0)} \in \mathbf{O}^*(G_\Gamma)$ it has a decomposition of the form $\lambda\cdot v_\alpha$ with $\lambda\in k^*$, $\alpha\in\Gamma$ and $v_\alpha(c)=c(\alpha)$, *cf.* [4].

But as a quotient of theta functions $\dfrac{\theta_\Delta(\pi^*(c)\varepsilon)}{\theta_\Gamma(c)\theta_\Gamma(cc_0)}$ itself is a theta function of type $\xi\in\mathbf{Z}^1(\Lambda_\Gamma,\mathbf{O}^*(G_\Gamma))$ with $\xi_{c_\gamma}(c)=\dfrac{e_0^2(\gamma)}{c_0(\gamma)}$. On the other hand $\lambda v_\alpha(c_\gamma c)=c_\gamma(\alpha)\cdot\lambda v_\alpha(c)=c_\alpha(\gamma)\cdot\lambda v_\alpha(c)$. Hence $\dfrac{e_0^2}{c_0}=c_\alpha^{-1}\in\Lambda_\Gamma$ and we find that

$$\frac{\theta_\Delta(\pi^*(c))}{\theta_\Gamma(ce_0^{-1})\theta_\Gamma(ce_0)}=\frac{\theta_\Delta(\pi^*(ce_0^{-1})\varepsilon)}{\theta_\Gamma(ce_0^{-1})\theta_\Gamma(ce_0^{-1}c_0c_\alpha^{-1})}$$
$$=\xi_{\Gamma,c_\gamma^{-1}}(ce_0^{-1}c_0)\cdot\lambda v_\alpha(ce_0^{-1}) .$$

So $\dfrac{\theta_\Delta(\pi^*(c))}{\theta_\Gamma(ce_0^{-1})\theta_\Gamma(ce_0)}=\lambda p_\Gamma(c_\alpha,c_\alpha)c_0(\alpha)^{-1}$. This expression is constant in c . □

Remark. — The homomorphism e_0 is only defined up to periods in Λ_Γ . If one replaces e_0 by e_0c_γ with $\gamma\in\Gamma$, then $e_0^2=c_0c_{\alpha^{-1}\gamma^2}$. So α is only defined up to squares in Γ .

In the following sections we will prove that e_0 can be chosen such that $\alpha=1$.

3. The case of hyperelliptic curves.

We take $\pi: X_\Delta\to X_\Gamma$ as in Section 2, but we now assume that X_Δ is hyperelliptic. So there exists an element s in the normaliser of Δ in $PGL(2,k)$ such that $s\delta s^{-1}\equiv\delta^{-1}\bmod[\Delta,\Delta]$ for all $\delta\in\Delta$, *cf.* [9].

Since $\gamma^2\in\Delta$ for all $\gamma\in\Gamma$ and since $\Gamma/[\Gamma,\Gamma]$ is a free abelian group we find that $s\gamma s^{-1}\equiv\gamma^{-1}\bmod[\Gamma,\Gamma]$. Hence X_Γ is also hyperelliptic. We may assume that s has order 2. Furthermore there exists a free basis $\gamma_0,\ldots,\gamma_g$ for Γ such that $s\gamma_is^{-1}=\gamma_i^{-1}$; $i=0,\ldots,g$; *cf.* [9]. We also may assume that $\gamma_0\notin\Delta$. If $\gamma_i\notin\Delta$ $(i=1,\ldots,g)$, then $\gamma_i\gamma_0\in\Delta$ and $\gamma_0\gamma_i\in\Delta$. But $s(\gamma_i\gamma_0)\cdot(\gamma_0\gamma_i)^{-1}s^{-1}=\gamma_i^{-1}\gamma_0^{-1}\gamma_i\gamma_0\equiv(\gamma_i\gamma_0)(\gamma_0\gamma_i)^{-1}\bmod[\Delta,\Delta]$. This contradicts the fact that $s\delta s^{-1}\equiv\delta^{-1}\bmod[\Delta,\Delta]$ for all $\delta\in\Delta$. This means that $\gamma_0, \ldots, \gamma_g$ satisfy the assumptions of Section 2 and that Δ has a free basis $\delta_0, \delta_1, \ldots, \delta_g, \delta_{-1}, \ldots, \delta_{-g}$ with $\delta_0 = \gamma_0^2$, $\delta_i = \gamma_i$ and $\delta_{-i} = \gamma_0\gamma_i\gamma_0^{-1}$; $i = 1, \ldots, g$.

Let $\mu_{-i} = \delta_{-i}\delta_0 = \gamma_0\gamma_i\gamma_0$. So $\delta_0, \delta_1, \ldots, \delta_g, \mu_{-1}, \ldots, \mu_{-g}$ is a basis for Δ and $s\delta_0 s^{-1} = \delta_0^{-1}$, $s(\delta_i)s^{-1} = \delta_i^{-1}$ and $s\mu_{-i}s^{-1} = \mu_{-i}^{-1}$, $i = 1, \ldots, g$.

Let a and b be the fixpoints of s and let a_i and b_i be the fixpoints of $s\gamma_i$; $i = 0, \ldots, g$. The fixpoints of $s\delta_0$ are then $\gamma_0^{-1}(a)$ and $\gamma_0^{-1}(b)$ and the fixpoints of $s\mu_{-i}$ are $\gamma_0^{-1}(a_i)$ and $\gamma_0^{-1}(b_i)$.

All these fixpoints are ordinary points. The double coverings

$$X_\Gamma \to \mathbf{P}^1(k) \quad \text{and} \quad X_\Delta \to \mathbf{P}^1(k)$$

are ramified in the points $\bar{a}, \bar{b}, \bar{a}_0, \bar{b}_0, \ldots, \overline{a_g}, \overline{b_g} \in X_\Gamma$ and $\bar{a}, \bar{b}, \bar{a}_1, \bar{b}_1, \ldots, \overline{a_g}, \overline{b_g}, \overline{\gamma_0^{-1}(a)}, \overline{\gamma_0^{-1}(b)}, \overline{\gamma_0^{-1}(a_1)}, \overline{\gamma_0^{-1}(b_1)}, \ldots, \overline{\gamma_0^{-1}(a_g)}, \overline{\gamma_0^{-1}(b_g)} \in X_\Delta$ respectively; cf. [9].

We will now calculate K_Γ and K_Δ . The linear equivalence classes of these divisors do not depend on the base point of the canonical maps $\bar{t}_\Gamma : X_\Gamma \to J_\Gamma$ and $\bar{t}_\Delta : X_\Delta \to J_\Delta$. We may assume that this base point is a .

The $\bar{t}_\Gamma$-images of the ramification points of $X_\Gamma \to \mathbf{P}^1(k)$ are calculated in [10].

We have

1. $c_{ba}(\gamma_i) = -1$; $i = 0, \ldots, g$

2. $c_{a_i a}^2 = c_{b_i a}^2 = c_{\gamma_i}$; $c_{b_i a} = c_{b_i a_i} \cdot c_{a_i a}$; $c_{b_i a_i}(\gamma_i) = -1$ and $c_{b_i a_i}(\gamma_j) = 1$ for all $j \neq i$; $i = 0, \ldots, g$.

LEMMA 3.1. — *Let $c \in G_\Gamma$ such that $c^2 = c_\gamma \in \Lambda_\Gamma$ with $\gamma \notin [\Gamma, \Gamma]$ and such that $c(\gamma) = -p_\Gamma(c_\gamma, c_\gamma)$. Then $\theta_\Gamma(c) = 0$.*

Proof. — $\theta_\Gamma(c) = \theta_\Gamma(c^{-1}c_\gamma) = \xi_{\Gamma, c_\gamma}^{-1}(c^{-1})\theta_\Gamma(c^{-1})$.

But $\xi_{\Gamma, c_\gamma}(c^{-1}) = p_\Gamma(c_\gamma, c_\gamma) \cdot c(\gamma)^{-1} = -1$ and since θ_Γ is an even function the assertion follows. □

Since $c_{b_i a}(\gamma_i) = -c_{a_i a}(\gamma_i) = \pm p_\Gamma(c_{\gamma_i}, c_{\gamma_i})$ we find that $\theta_\Gamma \circ t_\Gamma$ has a zero in a_i or in b_i for each $i = 0, \ldots, g$.

In a similar way we find that $\theta_\Delta \circ t_\Delta$ has a zero in $\gamma_0^{-1}(a)$ or in $\gamma^{-1}(b)$, in a_i or in b_i and in $\gamma_0^{-1}(a_i)$ or in $\gamma_0^{-1}(b_i)$ for each $i = 1, \ldots, g$.

An easy calculation shows that $\tilde{c}_{\gamma_0^{-1}(a)a}(\delta_0) = p_\Delta(\tilde{c}_{\delta_0}, \tilde{c}_{\delta_0})$ and hence $\theta_\Delta \circ t_\Delta(\gamma_0^{-1}(b)) = 0$. After an eventual interchanging of a_i and b_i we may assume that $\theta_\Delta(\tilde{c}_{a_i a}) = 0$ for $i = 1, \ldots, g$.

PROPOSITION 3.2. — $K_\Delta = \overline{\gamma_0^{-1}(b)} + \sum_{i=1}^{g} \overline{a_i} + \overline{\gamma_0^{-1}(b_i)} - \bar{a}$.

Proof. — We only have to show that $\theta_\Delta(t_\Delta(\gamma_0^{-1}(b_i))) = 0$ for $i = 1, \ldots, g$. Assume that $\gamma_1, \ldots, \gamma_g$ are numbered such that

$$\theta_\Delta(t_\Delta(\gamma_0^{-1}(b_i))) = 0$$

for $i = 1, \ldots, k$ and $\theta_\Delta(t_\Delta(\gamma_0^{-1}(a_i))) = 0$ for $i = k+1, \ldots, g$ with $1 \leqq k < g$. We have

$$K_\Delta = \overline{\gamma_0^{-1}(b)} + \sum_{i=1}^{k} \overline{a_i} + \overline{\gamma_0^{-1}(a_i)} + \sum_{i=k+1}^{g} \overline{a_i} + \overline{\gamma_0^{-1}(b_i)} - \bar{a} .$$

We find that $\bar{t}_\Delta(K_\Delta - \gamma_0(K_\Delta)) = \bar{c}$ with $c \in G_\Delta$ and

$$c = \tilde{c}_{\gamma_0(b)a} \cdot \tilde{c}_{ba} \cdot \prod_{i=k+1}^{g} \tilde{c}_{\gamma_0(b_i)\gamma_0(a_i)} \cdot \tilde{c}_{b_i, a_i} .$$

Hence $c(\delta_i) = c(\mu_{-i}) = c(\delta_0) = 1$ for $i = k+1, \ldots, g$ and

$$c(\delta_i) = c(\mu_{-i}) = -1 \quad \text{for} \quad i = 1, \ldots, k .$$

It follows that $c^2 = 1$ and $c \neq 1$. So $c \notin \Lambda_\Gamma$ and K_Δ is not linear equivalent with $\gamma_0(K_\Delta)$. This contradicts 2.4. □

We can number $\gamma_1, \ldots, \gamma_g$ and choose a_0 and b_0 such that $\theta_\Gamma(t_\Gamma(a_i)) = 0$ for $i = 0, \ldots, k$ and $\theta_\Gamma(t_\Gamma(b_i)) = 0$ for $i = k+1, \ldots, g$ with $k \geqq 0$. We have

$$K_\Gamma = \sum_{i=0}^{k} \overline{a_i} + \sum_{i=k+1}^{g} \overline{b_i} - \bar{a}$$

and $\bar{t}_\Delta(\pi^*(K_\Gamma) - K_\Delta) = \bar{\varepsilon}$ with

$$\varepsilon = \tilde{c}_{a_0, \gamma_0^{-1}(a)} \cdot \tilde{c}_{\gamma_0^{-1}(a_0), \gamma_0^{-1}(b)} \cdot \prod_{i=1}^{k} \tilde{c}_{\gamma_0^{-1}(a_i), \gamma_0^{-1}(b_i)} \cdot \prod_{i=k+1}^{g} \tilde{c}_{b_i, a_i} .$$

We find

$$\varepsilon^2 = \left(\tilde{c}_{a_0, \gamma_0^{-1}(a)} \cdot \tilde{c}_{\gamma_0^{-1}(a_0), \gamma_0^{-1}(b)}\right)^2 = \left(\frac{\tilde{c}_{a_0, a} \cdot \tilde{c}_{\gamma_0^{-1}(a_0), \gamma_0^{-1}(a)}}{\tilde{c}_{\gamma_0^{-1}(a)a} \cdot \tilde{c}_{\gamma_0^{-1}(a), \gamma_0^{-1}(b)}}\right)^2 .$$

Since $(\tilde{c}_{a_0, a} \cdot \tilde{c}_{\gamma_0^{-1}(a_0), \gamma_0^{-1}(a)})^2 = c^2_{a_0 a|\Delta} = c_{\gamma_0|_\Delta} = \tilde{c}_{\delta_0} = \tilde{c}^2_{\gamma_0^{-1}(a), a}$ we have $\varepsilon^2 = 1$. In Section 2, we found that $\varepsilon = e_{0|\Delta}$ with $e_0^2 = c_0 c_{\alpha^{-1}}$; $\alpha \in \Gamma$. Since $\varepsilon^2 = 1$ we have $c_{\alpha^{-1}} = 1$. This proves Theorem 2.1 in this special case.

4. Analytic families of Mumford curves.

Let S be a connected analytic space and let $\rho : \mathbf{P}^1 \times S \to S$ be the projection on S . Let $\mathrm{Aut}_S(\mathbf{P}^1 \times S)$ be the group of analytic automorphisms u of $\mathbf{P}^1 \times S$ which satisfy $\rho \circ u = \rho$.

Let Γ be a free group of rank $g+1$ and let $\psi : \Gamma \to \mathrm{Aut}_S(\mathbf{P}^1 \times S)$ be a family of Schottky groups.

If $s \in S$ define then $\nu_S : \mathrm{Aut}_S(\mathbf{P}^1 \times S) \to \mathrm{Aut}(\mathbf{P}^1)$ by $\nu_s(u)(x) = y$ if and only $u(x,s) = (y,s)$; $u \in \mathrm{Aut}_S(\mathbf{P}^1 \times S)$, $x, y \in \mathbf{P}^1$.

The map $\nu_s \circ \psi$ is then injective and $\Gamma_s = \mathrm{Im}(\nu_s \circ \psi)$ is a Schottky group. If $\gamma \in \Gamma$ and $s \in S$ then denote $\gamma(s) = \nu_s \circ \psi(\gamma)$.

There exists an analytic subdomain $\Omega \subset \mathbf{P}^1 \times S$ such that for all $s \in S$ the set $\Omega_s = \{x \in \mathbf{P}^1 \mid (x,s) \in \Omega\}$ is the set of ordinary points of Γ_s . This result is proved in [7].

The group Γ acts in a canonical way on Ω . Let $\mathbf{X}_\Gamma = \Omega/\Gamma$ be the quotient space and let $\bar{\rho} : \mathbf{X}_\Gamma \to S$ be the map induced by ρ . For all $s \in S$ the fiber $\mathbf{X}_{\Gamma,s} = \bar{\rho}^{-1}(s)$ is then isomorphic to the Mumford curve X_{Γ_s} .

The Jacobians of the curves X_{Γ_s} can be regarded as fibers of an analytic family over S .

Let $G_\Gamma = \mathrm{Hom}(\Gamma, k^*)$, $\mathbf{G}_\Gamma = G_\Gamma \times S$ and $\tau : \mathbf{G}_\Gamma \to S$ be the projection on S . If $\gamma \in \Gamma$ then define $\lambda_\gamma : \mathbf{G}_\Gamma \to \mathbf{G}_\Gamma$ by $\lambda_\gamma(c,s) = (d,s)$ with $d(\delta) = c(\delta)c_{\gamma(s)}(\delta(s))$.

PROPOSITION.

i) λ_γ *is an analytic automorphism*

ii) λ_γ *has a fixpoint* $\Longleftrightarrow$ λ_γ *is the identity* $\Longleftrightarrow$ $\gamma \in [\Gamma, \Gamma]$.

Proof.

i) S admits an admissible covering by affinoids S_i , $(i \in I)$, such that each S_i admits analytic sections $x_0, x_1 : S_i \to \Omega$ such that $x_0(s) \neq x_1(s)$ for all $s \in S_i$, *cf.* [2]. If $s \in S_i$ then

$$c_{\gamma(s)}(\delta(s)) = \frac{u_{\delta,x_1}(x_0(s),s)}{u_{\delta,x_1}(\gamma(x_0(s),s))}$$

with $u_{\delta,x_1}(z,s) = \prod_{\gamma \in \Gamma} \frac{z - \sigma \circ \gamma(x_1(s))}{z - \sigma \circ \gamma\delta(x_1(s))}$ where $\sigma : \mathbf{P}^1 \times S \to \mathbf{P}^1$ is the projection on $\mathbf{P}^1$. The function u_{δ,x_1} is analytic on $\Omega \cap (\mathbf{P}^1 \times S_i)$. It follows that the restriction of λ_γ to $G_\Gamma \times S_i$ is analytic. Hence λ_γ is everywhere analytic.

ii) $\lambda_\gamma(c,s) = (c,s)$ if and only if $c_{\gamma(s)}(\delta(s)) = 1$ for all $\delta \in \Gamma$. This means that $\gamma(s) \in [\Gamma_s, \Gamma_s]$. □

Let $\Lambda = \{\lambda_\gamma \mid \gamma \in \Gamma\}$. We can make the quotients space $\mathbf{J}_\Gamma = \mathbf{G}_\Gamma/\Lambda$. Let $\bar{\tau} : \mathbf{J}_\Gamma \to S$ be induced by $\tau : \mathbf{G}_\Gamma \to S$.

PROPOSITION 4.2. — *For all $s \in S$ the fiber $\mathbf{J}_{\Gamma,s} = \bar{\tau}^{-1}(s)$ is isomorphic to the Jacobian variety J_{Γ_s} of X .*

Proof. — Define $\alpha : \mathbf{J}_{\Gamma,s} \to J_{\Gamma_s} = \mathrm{Hom}(\Gamma_s, k^*)/\Lambda_{\Gamma_s}$ by $\alpha(\overline{c,s}) = \overline{c_s}$ with $c_s(\gamma(s)) = c(\gamma)$. This map is an isomorphism. □

Let $\Delta \subset \Gamma$ be a subgroup of index 2. We can find a basis $\gamma_0, \ldots, \gamma_g$ for Γ such that $\gamma_0 \notin \Delta$ and $\gamma_1, \ldots, \gamma_g \in \Delta$. The group Δ has a basis $\delta_0, \delta_1, \ldots, \delta_g, \delta_{-1}, \ldots, \delta_{-g}$ with $\delta_0 = \gamma_0^2$, $\delta_i = \gamma_i$ and $\delta_{-i} = \gamma_0\gamma_i\gamma_0^{-1}$; $i = 1, \ldots, g$. For $s \in S$ we denote $\Delta_s = \{\delta(s) \in \Gamma_s \mid \delta \in \Delta\}$. So Δ_s is a Schottky group and Γ_s and Δ_s satisfy the conditions of Section 2. For data which refer to these groups we keep the same notations as in Section 2.

We have an analytic family of Mumford curves $\bar{\rho} : \mathbf{X}_\Delta = \Omega/\Delta \to S$ and for each $s \in S$ the fiber $\mathbf{X}_{\Delta,s}$ is isomorphic to the Mumford curve X_{Δ_s} .

Let $\pi : \mathbf{X}_\Delta \to \mathbf{X}_\Gamma$ the canonical map induced by the identity on Ω .

Define $\mathbf{J}_\Delta$ in a similar way as $\mathbf{J}_\Gamma$. We have a dual map $\pi^* : \mathbf{J}_\Gamma \to \mathbf{J}_\Delta$ with $\pi^*(\overline{c,s}) = (\overline{c_{|\Delta}, s})$.

The analytic space S locally admits analytic sections x_0 and x_1 with values in Ω such that $x_0(s) \neq x_1(s)$ for all s , (*cf.* Prop. 4.2). We now assume that x_0 and x_1 exist on S itself.

Let $t_\Gamma : \Omega \to \mathbf{G}_\Gamma$ and $t_\Delta : \Omega \to \mathbf{G}_\Delta$ be defined by

$$\begin{aligned} t_\Gamma(x,s) &= (c,s) \quad \text{with } c(\gamma) = c_{x,\sigma(x_0(s))}(\gamma)\ , \quad (\gamma \in \Gamma) \\ t_\Delta(x,s) &= (\tilde{c},s) \quad \text{with } \tilde{c}(\delta) = \tilde{c}_{x,\sigma(x_0(s))}(\delta)\ , \quad (\delta \in \Delta) \end{aligned}$$

($\sigma : \mathbf{P}^1 \times S \to \mathbf{P}^1$ the projection on $\mathbf{P}^1$).

These maps are analytic and induce maps $\bar{t}_\Gamma : \mathbf{X}_\Gamma \to \mathbf{J}_\Gamma$ and $\bar{t}_\Delta : \mathbf{X}_\Delta \to \mathbf{J}_\Delta$. For each $s \in S$ the restrictions of $\bar{t}_\Gamma$ and $\bar{t}_\Delta$ to the fibers over s are the canonical maps $\bar{t}_{\Gamma_s} : X_{\Gamma_s} \to J_{\Gamma_s}$ and $\bar{t}_{\Delta_s} : X_{\Delta_s} \to J_{\Delta_s}$ based at $\sigma(x_0(s))$.

Let $p_{\Gamma_s} : \Lambda_{\Gamma_s} \times \Lambda_{\Gamma_s} \to k^*$ and $p_{\Delta_s} : \Lambda_{\Delta_s} \times \Lambda_{\Delta_s} \to k^*$ be symmetric bilinear forms such as in Section 2 and assume that they are normalized as before. So we have theta functions $\theta_{\Gamma_s}, \theta_{\Delta_s}$ and divisors $K_{\Gamma_s}, K_{\Delta_s}$ and $E_s = \pi^*(K_{\Gamma_s}) - K_{\Delta_s}$. Let $\varepsilon_s \in G_{\Delta_s}$ such that $\bar{t}_{\Delta_s}(E_s) = \bar{\varepsilon}_s$ and such that $\varepsilon_s^{\gamma_0(s)} = \varepsilon_s$. So $\varepsilon_s = \pi^*(e_{0,s})$ with $e_{0,s} \in G_{\Gamma_s}$.

Define $e_0 : S \to \mathbf{G}_\Gamma$ and $\varepsilon : S \to \mathbf{G}_\Delta$ by

$$e_0(s) = (a, s) \quad \text{with} \quad a(\gamma) = e_{0,s}(\gamma(s))$$

and

$$\varepsilon(s) = (\tilde{a}, s) \quad \text{with} \quad \tilde{a}(\delta) = \varepsilon_s(\delta(s)) \ .$$

So $\varepsilon = \pi^* \circ e_0$.

The sections e_0 and ε need not to be analytic. However, if one defines multiplication of sections in an obvious way, we can prove the following.

LEMMA 4.3. — *S admits an admissible covering $(S_i)_{i \in I}$ with the following properties :*

For each $i \in I$ one can choose the homomorphisms $e_{0,s}$ in such a way that the restriction $e_{0,i}$ of e_0 to S_i satisfies that $e_{0,i}^2$ is analytic. Furthermore, for each $i, j \in I$ there exists a $\beta_{ij} \in \Gamma$ such that for all $s \in S_i \cap S_j$, $e_{0,i} e_{0,j}^{-1}(s) = (a, s)$ with $a(\gamma) = c_{\beta_{ij}(s)}(\gamma(s))$.

Proof. — For each $s \in S$ define $d_{\Gamma_s} \in G_{\Gamma_s}$ and $d_{\Delta_s} \in G_{\Delta_s}$ by

$$d_{\Gamma_s}(\gamma_i) = p_{\Gamma_s}(c_{\gamma_i(s)}, c_{\gamma_i(s)}) \ ; \quad i = 0, \ldots, g$$

and

$$d_{\Delta_s}(\delta_i) = p_{\Delta_s}(\tilde{c}_{\delta_i(s)}, \tilde{c}_{\delta_i(s)}) \ ; \quad i = 0, \ldots, g, -1, \ldots, -g \ .$$

Define functions η_Γ and η_Δ on $\mathbf{G}_\Gamma$ and $\mathbf{G}_\Delta$ respectively by

$$\eta_\Gamma(c, s) = \theta_{\Gamma_s}(d_{\Gamma_s} \cdot c_s) \quad \text{with} \quad c_s(\gamma(s)) = c(\gamma)$$

and

$$\eta_\Delta(\tilde{c}, s) = \theta_{\Delta_s}(d_{\Delta_s} \cdot \tilde{c}_s) \quad \text{with} \quad \tilde{c}_s(\delta(s)) = \tilde{c}(\delta) \ .$$

These functions are holomorphic, (cf. [2]).

The divisors $L_\Gamma = \operatorname{div}(\eta_\Gamma \circ t_\Gamma)$ and $L_\Delta = \operatorname{div}(\eta_\Delta \circ t_\Delta)$ are invariant under the actions of Γ and Δ respectively. So they can be regarded as divisors on $\mathbf{X}_\Gamma$ and $\mathbf{X}_\Delta$.

Let $E' = \pi^*(L_\Gamma) - L_\Delta$. For each $s \in S$ the restriction E'_s of E' to the fiber $\mathbf{X}_{\Delta,s}$ has degree 0 . One has a corresponding homomorphism $\varepsilon'_s \in G_{\Delta_s}$, (defined up to periods in Λ_{Δ_s}), such that $\bar{t}_{\Delta_s}(E'_s) = \overline{\varepsilon'_s}$.

The section $\overline{\varepsilon'} : S \to \mathbf{J}_\Delta$ with $\overline{\varepsilon'}(s) = (\overline{\tilde{a}, s})$ and $\tilde{a}(\delta) = \varepsilon'_s(\delta(s))$ is then analytic. Let D_{Γ_s} and D_{Δ_s} be divisors on X_{Γ_s} and X_{Δ_s} such that $\bar{t}_{\Gamma_s}(D_{\Gamma_s}) = \bar{d}_{\Gamma_s}$ and $\bar{t}_{\Delta_s}(D_{\Delta_s}) = \bar{d}_{\Delta_s}$. So $\operatorname{div}(\theta_{\Gamma_s} \cdot t_{\Gamma_s}))$ is linear equivalent with $\operatorname{div}(\theta_{\Gamma_s} \circ t_{\Gamma_s}) + D_{\Gamma_s}$ and $\operatorname{div}(\theta_{\Delta_s}(d_{\Delta_s} \cdot t_{\Delta_s}))$ is linear equivalent with $\operatorname{div}(\theta_{\Delta_s} \circ t_{\Delta_s}) + D_{\Delta_s}$, *cf.* [4]. It follows that E'_s is linear equivalent with $E_s + \gamma_0(s)(D_{\Delta_s})$ and hence $\varepsilon'_s \equiv \varepsilon_s \cdot \tilde{g}_s \bmod \Lambda_{\Delta_s}$ with $\tilde{g}_s(\delta_i(s)) = p_{\Delta_s}(\tilde{c}_{\delta_i(s)}, \tilde{c}^{\gamma_0(s)}_{\delta_i(s)})$; $i = 0, \ldots, g, -1, \ldots, -g$. Since ε'_s is only defined up to periods we may assume that this congruence is an equality. Since $\tilde{g}_s^{\gamma_0(s)} = \tilde{g}_s$ we have $\varepsilon'^{\gamma_0(s)}_s = \varepsilon'_s$. So there exist $g_s, e_s \in G_{\Gamma_s}$ with $g_{s|\Delta_s} = \tilde{g}_s$ and $e_s|\Delta_s = \varepsilon'_s$.

Define sections $g : S \to \mathbf{G}_\Gamma$ with $g(s) = (a, s)$ with $a(\gamma) = g_s(\gamma(s))$ and $\bar{e} : S \to \mathbf{J}_\Gamma$ with $\bar{e}(s) = (\overline{b, s})$ with $b(\gamma) = e_s(\gamma(s))$. So $\overline{\varepsilon'} = \pi^* \circ \bar{e}$ and $\bar{e}$ is analytic. It follows that $\bar{e}$ can locally be lifted to an analytic section with values in $\mathbf{G}_\Gamma$. There exists an analytic covering $(S_i)_{i \in I}$ of S and analytic sections $e_i : S_i \to \mathbf{G}_\Gamma$ such that for each $s \in S_i$, $\overline{e_i(s)} = e(\bar{s})$.

If $s \in S_i \cap S_j$ then $e_i(x) \equiv e_j(s) \bmod \Lambda$ and since $e_i e_j^{-1}$ is analytic there exists a $\beta_{ij} \in \Gamma$ such that $\lambda_{\beta_{ij}}(e_i(s)) = e_j(s)$ for all $s \subset S_i \cap S_j$.

Define $e_{0,i} : S_i \to \mathbf{G}_\Gamma$ by $e_{0,i} = e_i \cdot g$. For each $s \in S_i$ we have $\overline{e_{0,i}(s)} = \overline{e_0(s)}$ in $\mathbf{J}_\Gamma$. Moreover, it is easy to verify that g^2 is analytic. Hence $e_{0,i}^2$ is analytic and the sections $(e_{0,i})_{i \in I}$ satisfy the required conditions. □

We proved in Section 2 that $e_{0,s}^2 \equiv c_{0,s} \bmod(\Lambda_{\Gamma_s})$ with $c_{0,s}(\gamma_0(s)) = -1$ and $c_{0,s}(\delta(s)) = 1$ for all $\delta(s) \in \Delta_s$. Define $c_0 \in G_\Gamma$ by $c_0(\gamma_0) = -1$ and $c_0(\delta) = 1$ for all $\delta \in \Delta$. The section $c : S \to \mathbf{G}_\Gamma$ which maps s onto (c_0, s) is then analytic and for all $s \in S_i$ we have $e_{0,i}^2(s) \equiv c(s) \bmod \Lambda$. Since both sections are analytic there exists a $\alpha_i \in \Gamma$ such that $e_{0,i}^2 = \lambda_{\alpha_i}(c(s))$ for all $s \in S_i$. We can sum up as follows.

PROPOSITION 4.4. — *The analytic space S admits an admissible covering $(S_i)_{i \in I}$ with the following properties :*

i) *for each $i \in I$ one can choose the homomorphisms $e_{0,s}$, $s \in S_i$, in such a way there exists a $\alpha_i \in \Gamma$ with*

$$e_{0,s}^2(\gamma(s)) = c_{\alpha_i(s)}(\gamma(s)) \text{ for all } \gamma \in \Gamma \ ;$$

ii) *for all $i, j \in I$ there exists a $\beta_{ij} \in \Gamma$ such that $\alpha_i \alpha_j^{-1} = \beta_{ij}^2$.*

Remark. — The homomorphism $c_{\alpha_i(s)}$ depends only on the class of α_i in $\Gamma/[\Gamma, \Gamma]$. Furthermore, since $e_{0,s}$ is only defined up to periods, α_i is only defined up to squares in Γ .

COROLLARY 4.5. — *If $\mathbf{X}_{\Delta,s}$ is hyperelliptic for some $s \in S$, then one can take $\alpha_i = 1$ for all $i \in I$.*

Proof. — Assume $s \in S_j$. We proved in Section 3 that $e_{0,s}$ can be chosen such that $e_{0,s}^2 = c_{0,s}$. Hence we can take $\alpha_j = 1$.

For all k such that $S_k \cap S_j \neq \emptyset$ we have $\alpha_k = \beta_{kj}^2$. Since α_k is only defined up to squares we can take $\alpha_k = 1$. This argument can be repeated. Since S is connected any S_i is reached in this way. □

We can now finish the

Proof of Theorem 2.4. — Let S be the Teichmuller space T_{g+1} . A point in T_{g+1} can be identified with an ordered set $\nu = (\nu_0, \ldots, \nu_g)$ with $\nu_i \in PGL(2,k)$ and such that :

i) $\nu_0, \ldots, \nu_g$ is a basis for a Schottky group of rank $g+1$.

ii) ν_0 has 0 and ∞ as attractive and repulsive fixpoints respectively.

iii) ν_1 has 1 as attractive fixpoint.

The space T_{g+1} has a connected analytic structure, *cf.* [5].

Now take Γ, Δ and $\gamma_0, \ldots, \gamma_g$ as in the previous part of the section and define $\psi : \Gamma \to \mathrm{Aut}_s(\mathbf{P}^1 \times S)$ by

$$\psi(\gamma_i)(x,\nu) = (\nu_i(x), \nu) \ ; \quad i = 0, \ldots, g \ .$$

For each $\nu \in S$, the Schottky group Γ_ν is then generated by $\nu_0, \ldots, \nu_g$. Furthermore, any situation as in Section 2 can be realized by taking the fibers $\mathbf{X}_{\Gamma,\nu}$ and $\mathbf{X}_{\Delta,\nu}$. In particular $\mathbf{X}_{\Delta,\nu}$ is hyperelliptic for at least one $\nu \in T_{g+1}$. So we can always choose $e_{0,\nu}$ such that $e_{0,\nu}^2 = c_{0,\nu}$.□

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