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Guy Casale, Lucia Di Vizio and Jean-Pierre Ramis

Volume à la mémoire de Hiroshi Umemura: "Équations de Painlevé et théories de Galois différentielles"

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Hiroshi Umemura, Kyoto 2014

Préface

This special volume of the "Annales de la Faculté des Sciences de Toulouse" contains several contributions in memory of Hiroshi Umemura.

Umemura died on March 8th, 2019, at the age of 74, in Nagoya, where he graduated and spent the greatest part of his life, as a professor at Nagoya University. He started working in algebraic geometry. More precisely, he studied the subgroups of Cremona groups, subject that made him acquainted with Paul Painlevé. He rapidly became a pillar of the Japanese school on Painlevé equations. In 1996, he published his first article on Galois theory for non-linear differential equations, which has been a turning point for the domain. His last article, which closes this volume, is in its direct continuity and shows how one can weaken the definition of Galois group of a functional equation and hence make quantum groups naturally appear in the framework.

Hiroshi was friendly and very enthusiastic when sharing knowledge or ideas (mathematical or not). For this reason, he was deeply appreciated by all colleagues. The contributions to this volume confirm that his vision of mathematics will be a source of inspiration and of open problems for next generations of mathematicians.

The articles published below touch several aspects of Umemura long and rich activity.

Souvenirs and overall presentation of Umemura work. The first two texts present the academic career of Umemura, his visits to France and his main mathematical achievements.

J.-P. Ramis has written an historical survey on the relations of Umemura with the French mathematical community. It can also be read as an introduction to the difficult work of Umemura on differential Galois theory, starting from the seminal work of E. Vessiot and presenting open questions and possible expected applications.

In the second article, K. Okamoto and Y. Ohyama give an account of Umemura mathematical career from his first work on Fano 3-fold to his definition of his Galois infinitesimal group **Inf-gal**(L/K) of a differential field extension.

Noriko et Hiroshi Umemura avec Monique Ramis, Carcassonne 2015

Hiroshi Umemura, Université de Nagoya 2013

Umemura polynomials. K. Okamoto and Y. Ohyama introduce the subsequent article written by Umemura in 1996, for the proceedings of the conference *Theory of nonlinear special functions: the Painlevé transcendents*, organised at Montréal. This work was never published, differently from the paper *Special polynomials associated with the Painlevé equations II* by Noumi, Okada, Okamoto and Umemura, that is its natural prolongation. Hiroshi was interested in rational solutions of Painlevé equations. After an introduction to Yablonskii–Vorobiev polynomials, he defined a family of polynomials related to Painlevé equations and gave the first step toward a conjectural combinatorial description of those polynomials, that nowadays we call Umemura polynomials.

In the next paper, M. Noumi generalizes the constructions of Umemura polynomials, extending both presentations given in Umemura's article. The first construction is given in term of Bäcklund transformations of the 6th Painlevé equation, while the second one is completely combinatorial. Noumi conjectures that the polynomials obtained actually coincide. In the case of Umemura polynomials, this conjecture is a theorem by Taneda, which, following Umemura, confirm that the Painlevé transcendents are special functions.

Discrete Painlevé equations. Y. Ohyama, J. Sauloy and J.-P. Ramis explore some q-analogs of the notion of isomonodromic deformations. This is a first step toward the discrete Riemann-Hilbert correspondence. On one side of the Riemann–Hilbert correspondence, we have the spaces of initial conditions, classified by Sakai. On the other side, we must have the q-version of the character varieties. To defined it, the authors restrict their interest to a family of linear difference systems rich enough to obtain the celebrated Jimbo-Sakai Lax pair, that produces the q-P_{VI} equation. Then, using a new tool called Mano decomposition, they describe the algebraic structure of the character variety.

The Hamiltonian presentation of Painlevé equations is one of their essential properties. The associated Newton polygon gives the weights used to prove the irreducibility of Painlevé transcendents. The article written by T. Mase, A. Nakamura et H. Sakai gives a discrete version of this Hamiltonian. In a case by case computation, they obtain Hamiltonians for the discrete Painlevé equations. In contrast with the differential case, the discrete Hamiltonians not only are not rational but they contain logarithmic and dilogarithmic terms.

Hiroshi Umemura avec Daniel Bertrand, 2004

Hiroshi Umemura avec Pierre Cartier, Université de Nagoya 2013

“Unlikely intersections” in the functional setting. The reader may compare the form given to P_{VI} and the following Proposition 1 in D. Bertrand’s article and the Schwarzian equation and the Proposition 4.10 in D. Blàzquez Sanz, G. Casale, J. Freitag and J. Nagloo paper. They study the differential equation satisfied by the uniformization of triangle groups. They show the strong minimality of the set of solutions of this equation and explore some consequences of this property. Strong minimality is a notion from Model Theory generalising the celebrated (J) condition of Umemura. From a Galoisian perspective, an equation with this property should have a Galois group whose Lie algebra has no algebraic sub Lie-algebra. To give a proof of the strong minimality of Schwarzian equations computing their Umemura groups **Inf-gal** is a non-trivial open problem. . . The authors used other technics from model theory.

The article written by D. Bertrand is a survey on Manin map and Manin kernel. Different maps are introduced and, following Manin, Coleman, Chai and André, D. Bertrand shows the equality of the different kernels. In a first part, he recalls the presentation of P_{VI} using Manin map and show how properties of its kernel give informations on solutions of the sixth Painlevé equation.

Quantum groups as Galois groups. The volume ends with Umemura’s last article in collaboration with A. Masuoka et K. Saito. They develop a new point of view on the construction of Picard–Vessiot extensions starting from the Galois theory devised by Umemura in his seminal article *Differential Galois theory of infinite dimension*. The functional equations studied are given by an action of a non-cocommutative bialgebra. The weakening of the construction gives rise to quantum groups. The foundations of this new theory are detailed on three examples and for equations with constant coefficients.

Hiroshi Umemura’s interests for Painlevé equations and differential Galois theory have brought him to France very often. Hence this volume is published in France to honor his longstanding mathematical collaboration and the profound friendship that he has left behind.

Guy Casale, Lucia Di Vizio, Jean-Pierre Ramis