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## ON IDEALS GENERATED BY BOUNDED ANALYTIC FUNCTIONS IN THE BIDISC

BY

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RÉSUMÉ. — Nous considérons une certaine généralisation du problème de la couronne dans le bidisque.

ABSTRACT. — We consider a certain generalization of the corona problem in the bidisc.

### Introduction

Let  $D$  be the unit disc in  $\mathbb{C}$  with boundary  $\partial D = T$ . We denote by  $H(D \times D)$  the analytic functions on  $D \times D$  and by  $H^p(D \times D)$  the functions in  $H(D \times D)$  with boundary values in  $L^p(T \times T)$ ; the measure on  $T$  is the normalized Lebesgue measure  $d\sigma$  and the measure on  $T \times T$  is  $d\sigma \otimes d\sigma$ .

In this note we prove that a certain generalized corona problem (for two generators) always has a solution that can be estimated by the Cauchy transform of a bounded function plus a bounded function.

Related results have been obtained by CHANG [2], LIN [3] and VAROPOULOS [4].

### 1. The $\bar{\partial}$ -problem

Suppose  $H = H_1 d\bar{z}_1 + H_2 d\bar{z}_2$  is a  $\bar{\partial}$ -closed form on  $D \times D$  with coefficients continuous on  $\bar{D} \times \bar{D}$ . Define :

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$$(1) \quad U(z_1, z_2) = \frac{1}{2\pi i} \int_D \frac{H_1(\xi_1, z_2)}{\xi_1 - z_1} d\xi_1 \wedge d\bar{\xi}_1 \\ + \frac{1}{(2\pi i)^2} \int_{T \times D} \frac{H_2(\xi_1, \xi_2) d\xi_1}{\xi_1 - z_1} \frac{d\xi_2 \wedge d\bar{\xi}_2}{\xi_2 - z_2}.$$

Then  $\bar{\partial}U = H$  for  $\partial U / \partial \bar{z}_1 = H_1$  and since  $\partial H_1 / \partial \bar{z}_2 = \partial H_2 / \partial \bar{z}_1$

$$\frac{\partial U}{\partial \bar{z}_2} = \frac{1}{2\pi i} \left\{ \int_D \frac{\partial H_2 / \partial \bar{z}_1(\xi_1, z_2)}{\xi_1 - z_1} d\xi_1 \wedge d\xi_1 + \int_T \frac{H_2(\xi_1, z_2) d\xi_1}{\xi_1 - z_1} \right\} = H_2$$

by Cauchy's general integral formula.

We now want to rewrite (1) and therefore first consider the formula

$$(2) \quad 0 = 4 \int_D \log \left| \frac{z_1 - \xi_1}{1 - \xi_1 z_1} \right|^2 \frac{\partial H_1}{\partial z_1}(\xi_1, z_2) d\xi_1 \wedge d\bar{\xi}_1 \\ + \int_D \frac{H_1(\xi_1, z_2)}{\xi_1 - z_1} d\xi_1 \wedge d\bar{\xi}_1 + \bar{z}_1 \int_D \frac{H_1(\xi_1, z_2)}{1 - \xi_1 \bar{z}_1} d\xi_1 \wedge d\bar{\xi}_1$$

which follows from the fact that the right hand side is harmonic (in  $z_1$ ) and has boundary values zero.

Hence, (1) is now

$$(3) \quad U(z_1, z_2) = -\frac{2}{\pi i} \int_D \log \left| \frac{z_1 - \xi_1}{1 - \xi_1 z_1} \right|^2 \frac{\partial H_1}{\partial z_1}(\xi_1, z_2) d\xi_1 \wedge d\bar{\xi}_1 \\ - \frac{1}{2\pi i} \bar{z}_1 \int_D \frac{H_1(\xi_1, z_2)}{1 - \xi_1 \bar{z}_1} d\xi_1 \wedge d\bar{\xi}_1 \\ - \frac{2}{(2\pi i)^2} \int_{T \times D} \log \left| \frac{z_2 - \xi_2}{1 - \xi_2 z_2} \right|^2 \frac{\partial H_2(\xi_1, \xi_2)}{\partial z_2} d\xi_2 \wedge d\bar{\xi}_2 \frac{d\xi_1}{\xi_1 - z_1} \\ - \frac{1}{(2\pi i)^2} \int_{T \times D} \frac{\bar{z}_2 H_2(\xi_1, \xi_2)}{1 - \xi_2 \bar{z}_2} d\xi_2 \wedge d\bar{\xi}_2 \frac{d\xi_1}{\xi_1 - z_1}.$$

## 2. A corona problem for two generators

In this section, we prove the following theorem :

**THEOREM.** — Assume that  $g, f_1$  and  $f_2 \in H^\infty(D \times D)$  with

$$|g| \leq (|f_1|^2 + |f_2|^2)^{1+\varepsilon}$$

for some  $\varepsilon > 0$ . Then there is a function

$$a(z_1, z_2) = \int_T \frac{K(\xi_1, z_2)}{\xi_1 - z_1} d\sigma(\xi_1) + L(z_1, z_2)$$

where  $K \in L^\infty(T \times D)$  and  $L \in L^\infty(D \times D)$  so that

$$\frac{g\bar{f}_1}{|f_1|^2 + |f_2|^2} - f_2 a = g_1 \in H(D \times D)$$

$$\frac{g\bar{f}_2}{|f_1|^2 + |f_2|^2} + f_1 a = g_2 \in H(D \times D).$$

In particular,  $f_1 g_1 + f_2 g_2 = g$ .

*Proof.* — Consider

$$g_1 = \frac{g\bar{f}_1}{|f_1|^2 + |f_2|^2} - f_2 a, \quad g_2 = \frac{g\bar{f}_2}{|f_1|^2 + |f_2|^2} + f_1 a.$$

If we could find  $a \in C^1(D \times D)$  so that  $\bar{\partial} g_1 = 0$ , then

$$0 = \bar{\partial} g = \bar{\partial}(f_1 g_1 + f_2 g_2) = f_2 \bar{\partial} g_2,$$

which means that  $\bar{\partial}(g_2) = 0$  outside an analytic set, hence  $\bar{\partial} g_2 = 0$  on  $D \times D$  which proves that  $g_2$  is also analytic.

Set

$$\Phi_i = f_2 \frac{\partial f_1}{\partial z_i} - f_1 \frac{\partial f_2}{\partial z_i}.$$

That  $\bar{\partial} g_1 = 0$  means that :

$$H = \bar{\partial} a = \frac{g\bar{\Phi}_1}{(|f_1|^2 + |f_2|^2)^2} d\bar{z}_1 + \frac{g\bar{\Phi}_2}{(|f_1|^2 + |f_2|^2)^2} d\bar{z}_2$$

$$= H_1 d\bar{z}_1 + H_2 d\bar{z}_2.$$

To avoid regularity problems, we can dilatate  $g$ ,  $f_1$  and  $f_2$ , prove uniform estimates and use weak convergence and normal family arguments. Moreover, we can assume that :

$$\|f_1\|_{L^\infty} + \|f_2\|_{L^\infty} \leq 1.$$

We now claim that there exist two functions  $K_1 \in L^\infty(T \times D)$  and  $K_2 \in L^\infty(T \times T)$  such that  $\|K_1\|_{L^\infty} + \|K_2\|_{L^\infty} \leq C$

$$\int_{T \times D} A(\xi_1, \xi_2) K_1(\xi_1, \xi_2) d\sigma(\xi_1) d\xi_2 \wedge d\bar{\xi}_2$$

$$= \int_{D \times D} A(\xi_1, \xi_2) H_1(\xi_1, \xi_2) d\xi_1 \wedge d\bar{\xi}_1 d\xi_2 \wedge d\bar{\xi}_2$$

and

$$\begin{aligned} \int_{T \times T} B(\xi_1, \xi_2) K_2(\xi_1, \xi_2) \, d\xi_1 \, d\sigma(\xi_2) \\ = \frac{1}{(2\pi i)} \int_{T \times D} B(\xi_1, \xi_2) H_2(\xi_1, \xi_2) \, d\xi_1 \, d\xi_2 \wedge d\bar{\xi}_2 \end{aligned}$$

for every  $A \in L^1(T \times D)$ , analytically extendable in the first variable and every  $B \in L^1(T \times T)$ , analytic extendable in the second variable.

By Stokes' formula (cf. [1, Thm 2]), we have for almost all  $\xi_2 \in D$  :

$$\left| \int_D \frac{Ag \bar{\Phi}_1}{2\xi_1 (|f_1|^2 + |f_2|^2)^2} \, d\xi_1 \wedge d\bar{\xi}_1 \right| \leq C \int_T |A(\xi_1, \xi_2)| \, d\sigma(\xi_1).$$

Applying this to  $\xi_1 A$

$$\begin{aligned} \left| \int_{D \times D} A(\xi_1, \xi_2) H_1(\xi_1, \xi_2) \, d\xi_1 \wedge d\bar{\xi}_1 \, d\xi_2 \wedge d\bar{\xi}_2 \right| \\ \leq C \left| \int_{T \times D} |A(\xi_1, \xi_2)| \, d\sigma(\xi_1) \, d\xi_2 \wedge d\bar{\xi}_2 \right|, \end{aligned}$$

so the linear functional

$$A \mapsto \int_{D \times D} AH_1$$

can, by the Hahn-Banach theorem, be extended to  $L^1(T \times D)$  without increase the norm. Since the dual is  $L^\infty(T \times D)$  we have proved that  $K_1$  exist.

We also have from [1] that

$$\left| \int_D \frac{Bg \bar{\Phi}_2}{2\xi_2 (|f_1|^2 + |f_1|^2)^2} \, d\xi_2 \wedge d\bar{\xi}_2 \right| \leq C \int_T |B(z_1, \xi_2)| \, d\sigma(\xi_2)$$

for almost all  $z_1 \in T$ , so therefore

$$\begin{aligned} \left| \int_{T \times D} \frac{Bg \bar{\Phi}_2}{2\xi_2 (|f_1|^2 + |f_2|^2)^2} \, d\xi_1 \, d\xi_2 \wedge d\bar{\xi}_2 \right| \\ \leq C \int_{T \times T} |B(\xi_1, \xi_2)| \, d\sigma(\xi_1) \, d\sigma(\xi_2) \end{aligned}$$

so the linear functional

$$B \mapsto \int_{T \times D} BH_2$$

can be extended to  $L^1(T \times T)$  without increase the norm and since the dual of  $L^1(T \times T)$  is  $L^\infty(T \times T)$  the claim is proved.

Note that if  $f$  is analytic in a neighborhood of  $\bar{D}$ , then

$$\int_T f(\xi_1)K_1(\xi_1, \xi_2) d\sigma(\xi_1) = \int_D f(\xi_1)H_1(\xi_1, \xi_2) d\xi_1 \wedge d\bar{\xi}_1, \quad \xi_2 \in D;$$

$$\int_T f(\xi_2)K_2(\xi_1, \xi_2) d\sigma(\xi_2) = \int_D f(\xi_2)H_2(\xi_1, \xi_2) d\xi_2 \wedge d\bar{\xi}_2, \quad \xi_1 \in T.$$

We now use formula (3) and get

$$\begin{aligned} U(z_1, z_2) &= -\frac{2}{\pi i} \int_D \log \left| \frac{z_1 - \xi_1}{1 - \bar{\xi}_1 z_1} \right|^2 \frac{\partial H_1}{\partial z_1}(\xi_1, z_2) d\xi_1 \wedge d\bar{\xi}_1 \\ &\quad - \frac{1}{2\pi i} \bar{z}_1 \int_D \frac{H_1(\xi_1, z_2)}{1 - \xi_1 \bar{z}_1} d\xi_1 \wedge d\bar{\xi}_1 \\ &\quad - \frac{2}{(2\pi i)^2} \int_{T \times D} \log \left| \frac{z_2 - \xi_2}{1 - \bar{\xi}_2 z_2} \right|^2 \frac{\partial H_2}{\partial z_2}(\xi_1, \xi_2) d\xi_2 \wedge d\bar{\xi}_2 \frac{d\xi_1}{\xi_1 - z_1} \\ &\quad - \frac{1}{(2\pi i)^2} \bar{z}_2 \int_{T \times D} \frac{H_2(\xi_1, \xi_2)}{1 - \xi_2 \bar{z}_2} d\xi_2 \wedge d\bar{\xi}_2 \frac{d\xi_1}{\xi_1 - z_1} \\ &= -\frac{2}{\pi i} \int_D \log \left| \frac{z_1 - \xi_1}{1 - \bar{\xi}_1 z_1} \right|^2 \frac{\partial H_1}{\partial z_1}(\xi_1, z_2) d\xi_1 \wedge d\bar{\xi}_1 \\ &\quad - \frac{\bar{z}_1}{2\pi i} \int_T \frac{K_1(\xi_1, z_2)}{1 - \xi_1 \bar{z}_1} d\sigma(\xi_1) \\ &\quad - \frac{2}{(2\pi i)^2} \int_T \left[ \int_D \log \left| \frac{z_2 - \xi_2}{1 - \bar{\xi}_2 z_2} \right|^2 \frac{\partial H_2}{\partial z_2}(\xi_1, \xi_2) d\xi_2 \wedge d\bar{\xi}_2 \right] \frac{d\xi_1}{\xi_1 - z_1} \\ &\quad - \frac{1}{2\pi i} \int_T \left[ \bar{z}_2 \int_T \frac{K_2(\xi_1, \xi_2)}{1 - \xi_2 \bar{z}_2} d\sigma(\xi_2) \right] \frac{d\xi_1}{\xi_1 - z_1} \\ &= -\frac{2}{\pi i} \int_D \log \left| \frac{z_1 - \xi_1}{1 - \bar{\xi}_1 z_1} \right|^2 \frac{\partial H_1}{\partial z_1}(\xi_1, z_2) d\xi_1 \wedge d\bar{\xi}_1 \\ &\quad - \frac{1}{2\pi i} \int_T \frac{1 - |z_1|^2}{\xi_1 |\xi_1 - z_1|^2} K_1(\xi_1, z_2) d\sigma(\xi_1) \\ &\quad + \frac{1}{2\pi i} \int_T \frac{K_1(\xi_1, z_2)}{\xi_1 - z_1} d\sigma(\xi_1) \\ &\quad - \frac{2}{(2\pi i)^2} \int_T \left[ \int_D \log \left| \frac{z_2 - \xi_2}{1 - \bar{\xi}_2 z_2} \right|^2 \frac{\partial H_2}{\partial z_2}(\xi_1, \xi_2) d\xi_2 \wedge d\bar{\xi}_2 \right] \frac{d\xi_1}{\xi_1 - z_1} \\ &\quad - \frac{1}{(2\pi i)} \int_T \left[ \int_T \frac{1 - |z_2|^2}{\xi_2 |\xi_2 - z_2|^2} K_2(\xi_1, \xi_2) d\sigma(\xi_2) \right] \frac{d\xi_1}{\xi_1 - z_1} \\ &\quad + \frac{1}{2\pi i} \int_{T \times T} \frac{K_2(\xi_1, \xi_2)}{(\xi_2 - z_2)(\xi_1 - z_1)} d\xi_1 d\sigma(\xi_2). \end{aligned}$$

- We estimate the first term as follows. Since

$$H_1 = \frac{g \bar{\Phi}_1}{(|f_1|^2 + |f_2|^2)^2}$$

we get :

$$\frac{\partial H_1}{\partial z_1} = \frac{\partial g}{\partial z_1} \frac{\bar{\Phi}_1}{(|f_1|^2 + |f_2|^2)^2} - 2g \frac{(\partial f_1 / \partial z_1 \bar{f}_1 + \partial f_2 / \partial z_1 \bar{f}_2) \bar{\Phi}_1}{(|f_1|^2 + |f_2|^2)^3}.$$

Choose  $\delta$  so that  $\frac{1 - \delta/2}{1 + \varepsilon/2} (1 + \varepsilon) = 1$ . Then

$$\begin{aligned} & \left| \frac{1}{2\pi} \int_D \log \left| \frac{z_1 - \xi_1}{1 - \bar{\xi}_1 z_1} \right|^2 \frac{\partial H_1}{\partial z_1} d\xi_1 \wedge d\bar{\xi}_1 \right| \\ & \leq \left| \frac{1}{2\pi} \int_D \log \left| \frac{z_1 - \xi_1}{1 - \bar{\xi}_1 z_1} \right|^2 \frac{|\partial g / \partial z_1|}{|g|^{1-\delta/2}} \frac{|g|^{1-\delta/2}}{(|f_1|^2 + |f_2|^2)^{1+\varepsilon/2}} \right. \\ & \quad \times \left. \frac{(|\partial f_1 / \partial z_1|^2 + |\partial f_2 / \partial z_1|^2)^{1/2}}{(|f_1|^2 + |f_2|^2)^{1/2-\varepsilon/2}} \right| d\xi_1 \wedge d\bar{\xi}_1 \\ & + \left| \frac{1}{2\pi} \int_D \log \left| \frac{z_1 - \xi_1}{1 - \bar{\xi}_1 z_1} \right|^2 \frac{|g|}{(|f_1|^2 + |f_2|^2)^{1+\varepsilon}} \right. \\ & \quad \times \left. \frac{|\partial f_1 / \partial z_1|^2 + |\partial f_2 / \partial z_1|^2}{(|f_1|^2 + |f_2|^2)^{1-\varepsilon}} d\xi_1 \wedge d\bar{\xi}_1 \right| \\ & \leq \left| \frac{1}{2\pi} \int_D \log \left| \frac{z_1 - \xi_1}{1 - \bar{\xi}_1 z_1} \right|^2 \frac{|\partial g / \partial z_1|^2}{|g|^{2-\delta}} d\xi_1 \wedge d\bar{\xi}_1 \right|^{1/2} \\ & \quad \times \left| \frac{1}{2\pi} \int_D \log \left| \frac{z_1 - \xi_1}{1 - \bar{\xi}_1 z_1} \right|^2 \left[ \frac{|\partial f_1 / \partial z_1|^2}{|f_1|^{2-2\varepsilon}} + \frac{|\partial f_2 / \partial z_1|^2}{|f_2|^{2-2\varepsilon}} \right] d\xi_1 \wedge d\bar{\xi}_1 \right|^{1/2} \\ & + \left| \frac{1}{2\pi} \int_D \log \left| \frac{z_1 - \xi_1}{1 - \bar{\xi}_1 z_1} \right|^2 \left[ \frac{|\partial f_1 / \partial z_1|^2}{|f_1|^{2-2\varepsilon}} + |\partial f_2 / \partial z_1|^2 |f_2|^{2-2\varepsilon} \right] d\xi_1 \wedge d\bar{\xi}_1 \right| \\ & = \frac{1}{2\pi} \left| \frac{1}{\delta^2} \int_D \log \left| \frac{z_1 - \xi_1}{1 - \bar{\xi}_1 z_1} \right|^2 \delta |g|^\delta \right|^{1/2} \\ & \quad \times \left| \frac{1}{4\varepsilon^2} \int_D \log \left| \frac{z_1 - \xi_1}{1 - \bar{\xi}_1 z_1} \right|^2 (\Delta |f_1|^{2\varepsilon} + \Delta |f_2|^{2\varepsilon}) \right|^{1/2} \\ & + \frac{1}{2\pi} \left| \frac{1}{4\varepsilon^2} \int_D \log \left| \frac{z_1 - \xi_1}{1 - \bar{\xi}_1 z_1} \right|^2 (\Delta |f_1|^{2\varepsilon} + \Delta |f_2|^{2\varepsilon}) \right| \end{aligned}$$

$$\begin{aligned}
 &= \frac{1}{2\delta\varepsilon} \left| \int_T \frac{1 - |z_1|^2}{|\xi_1 - z_1|^2} |g(\xi_1, z_2)|^\delta d\sigma(\xi_1) - |g(z_1, z_2)^\delta| \right|^{1/2} \\
 &\quad \times \left| \int_T \frac{1 - |z_1|^2}{|\xi_1 - z_1|^2} (|f_1|^{2\varepsilon} + |f_2|^{2\varepsilon}) d\sigma(\xi_1) - |f_1(z_1, z_2)|^{2\varepsilon} - |f_2(z_1, z_2)|^{2\varepsilon} \right|^{1/2} \\
 &+ \frac{1}{4\varepsilon^2} \left[ \int_T \frac{1 - |z_1|^2}{|z_1 - \xi_1|^2} (|f_1|^{2\varepsilon} + |f_2|^{2\varepsilon}) d\sigma(\xi_1) - |f_1(z_1, z_2)|^{2\varepsilon} - |f_2(z_1, z_2)|^{2\varepsilon} \right] \\
 &\leq \frac{1}{\delta\varepsilon} + \frac{1}{\varepsilon^2}
 \end{aligned}$$

by Riesz representation theorem.

• The second term is bounded by  $\|K_1\|_{L^\infty}$  and as above, we can prove that

$$\left| \frac{1}{2\pi} \int_D \log \left| \frac{z_2 - \xi_2}{1 - \bar{\xi}_2 z_2} \right|^2 \frac{\partial H_2}{\partial z_2}(z_1, \xi_2) d\xi_2 \wedge d\bar{\xi}_2 \right| \leq \frac{1}{\delta\varepsilon} + \frac{1}{\varepsilon^2}.$$

Since the last term is analytic it can be removed and therefore the proof of the theorem is complete.

REMARK

There exist  $K \in L^\infty(T)$  such that there is no  $f \in H(D \times D)$  with

$$\bar{z}_2 \int_T \frac{K(\xi)}{\xi - z_1} d\xi + f \in L^\infty(D \times D).$$

For if  $K \in L^\infty(T)$  but  $\int_T \frac{K(\xi)}{\xi - z} d\xi \notin L^\infty(D)$ , there exists for every  $m$  a harmonic polynomial  $P_m$  with

$$\int |P_m(z)| d\sigma(z) = 1 \quad \text{but} \quad \left| \int_T \int_T \frac{K(\xi)}{\xi - z} d\xi P(z) d\sigma(z) \right| \geq m$$

so if  $f$  is any function in  $H^1(D \times D)$  then

$$\begin{aligned}
 &\left| \int_T \int_T \left\{ \bar{z}_2 \int_T \frac{K(\xi)}{\xi - z_1} d\xi + f(z_1, z_2) \right\} z_2 P(z_1) d\sigma(z_1) d\sigma(z_2) \right| \\
 &= \left| \int_T \int_T \frac{K(\xi)}{\xi - z_1} d\xi P(z_1) d\sigma(\xi) \right| \geq m,
 \end{aligned}$$

so  $\bar{z}_2 \int_T \frac{K(\xi)}{\xi - z_1} + f(z_1, z_2) \notin L^\infty(D \times D)$ .



## REMARK

It follows from the proof of the theorem that the equation

$$\frac{\partial V}{\partial \bar{z}_1} = H_1$$

has a bounded solution  $V$ . (In particular,  $\frac{\partial^2 V}{\partial \bar{z}_1 \partial \bar{z}_2} = \frac{\partial H_1}{\partial \bar{z}_2}$ , cf. CHANG [2]).

Take  $V$  to be :

$$\begin{aligned} V(z_1, z_2) = & -\frac{2}{\pi i} \int_D \log \left| \frac{z_1 - \xi_1}{1 - \bar{\xi}_1 z_1} \right|^2 \frac{\partial H_1}{\partial z_1} d\xi_1 \wedge d\bar{\xi}_1 \\ & - \int_T \frac{1 - |z_1|^2}{\xi_1 |\xi_1 - z_1|^2} K_1(\xi_1, z_2) d\sigma(\xi_1). \end{aligned}$$

Then,  $V \in L^\infty(D \times D)$  and

$$\begin{aligned} \frac{\partial V}{\partial z_1} = & \frac{\partial}{\partial \bar{z}_1} \left[ -\frac{2}{\pi i} \int_D \log \left| \frac{z_1 - \xi_1}{1 - \bar{\xi}_1 z_1} \right|^2 \frac{\partial H_1}{\partial z_1}(\xi_1, z_2) d\xi_1 \wedge d\bar{\xi}_1 \right. \\ & \left. - \frac{1}{2\pi i} \int_D \frac{\bar{z}_1}{1 - \bar{\xi}_1 \bar{z}_1} H_1(\xi_1, z_1) d\xi_1 \wedge d\bar{\xi}_1 \right] \\ = & \frac{\partial}{\partial \bar{z}_1} \left[ \frac{1}{2\pi i} \int_D \frac{H_1(\xi_1, z_1)}{\xi_1 - z_1} d\xi_1 \wedge d\bar{\xi}_1 \right] = H_1. \end{aligned}$$

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