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Corrections to : “A conjecture in the arithmetic theory of differential equations”

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Corrections to

**A CONJECTURE IN THE ARITHMETIC THEORY
OF DIFFERENTIAL EQUATIONS (*)**

BY

NICHOLAS M. KATZ

I am indebted to O. GABBER for pointing out the following errors.

Section 10.6. — Delete the sentence “the Lie algebra $\text{Lie}(G_{\text{gal}})$ is the smallest algebraic Lie sub-algebra of $M(n)$ which contains the endomorphism

$$\begin{pmatrix} 2\pi i\lambda_1 & & 0 \\ & \ddots & \\ 0 & & 2\pi i\lambda_n \end{pmatrix}''$$

It is both false (think of the case of all $\lambda_i \in \mathbb{Z}$), and irrelevant to the correct calculation which follows.

Theorem 11.2 is incomplete in both hypotheses and proof as it is stated in the text. Here is a correct version.

THEOREM 11.2. — *Let (M, ∇) be a rank two equation on an arbitrary X , whose determinant becomes trivial on a finite étale covering of X . Then we have $\mathcal{G} = \text{Lie}(G_{\text{gal}})$ if any of the following conditions holds.*

- (A) *The Lie algebra $\mathcal{G}$ contains non-nilpotent endomorphisms.*
- (B) *(M, ∇) has non-nilpotent ψ_p for infinitely many p .*
- (C) *(M, ∇) does not have regular singular points.*
- (D) *(M, ∇) has regular singular points but it does not have rational exponents at infinity i. e., it does not have quasi-unipotent local monodromy at infinity.*

(*) Addendum à l'article de N. M. KATZ paru dans le fascicule II, tome 110, 1982, p. 203-239.

(E) $\mathcal{G} \neq 0$, and Grothendieck's conjecture holds for all rank one equations on all open subsets of X .

Proof. — Recall first that both (D), (C) imply (B), and (B) implies (A). If (A) holds, then the proof in the text is complete. If (A) does *not* hold, but (E) holds, then $\mathcal{G}$ is the unipotent radical $\mathcal{U}$ of a Borel. (This is the case overlooked in the text.) Then there exists a unique line $L \subset M \otimes \mathbb{C}(X)$ which is $\mathcal{G}$ -stable, and this line is killed by $\mathcal{G}$. As in case 2 in the text, this L must be horizontal. Then both L and M modulo L have nilpotent, hence zero, ψ_p for almost all p . Applying Grothendieck's conjecture to L and to $M \bmod L$, we find that $\text{Lie}(G_{\text{gal}})$ lies in $\mathcal{U}$.

Q.E.D.

Example 11.3. — Replace “of infinite order” (true but irrelevant) by “not quasi-unipotent”.