

# BULLETIN DE LA S. M. F.

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*Bulletin de la S. M. F.*, tome 89 (1961), p. 103-104

[http://www.numdam.org/item?id=BSMF\\_1961\\_\\_89\\_\\_103\\_0](http://www.numdam.org/item?id=BSMF_1961__89__103_0)

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## ON A THEOREM OF CHARLES AND ERDÉLYI ;

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The original purpose of the following was to give a short proof of a theorem of CHARLES [1]. CHARLES then indicated that the proof resembled the proof of a theorem of ERDÉLYI [2], p. 81, and that if modified slightly, would cover both theorems. The same proof however proves also a theorem of J. IRWIN and E. WALKER [3]. In the following the three theorems are combined together.

Let  $G$  be a primary group, and if  $x \in G$  let  $h(x)$  denote the ordinary height of  $x$  in  $G$ . Also let  $a$  represent either an integer or the first infinite ordinal; and if  $a$  is the first infinite ordinal, let  $p^a G$  represent any subgroup

of  $\bigcap_{n=1}^{\infty} p^n G$ . Then :

**THEOREM** (CHARLES, ERDÉLYI, IRWIN and WALKER). — Let  $M$  be a subgroup of  $G$  maximal with respect to disjointness from  $p^a G$ . Then  $M$  is pure in  $G$ .

**PROOF.** — Deny the theorem. Then there is a least positive integer  $n < a$  for which there is an equation  $p^n x = y$ ,  $y \in M$  having a solution  $x$  in  $G$  but not in  $M$ . Then there exists an integer  $m$ ,  $0 \leq m < n$ , and  $z \in M$  such that  $0 \neq p^m x + z \in p^a G$ . Then  $h(z) = h(p^m x) \geq m$ , since  $h(p^m x) \geq m$  and  $h(p^m x + z) \geq a$ . Since  $m < n$ , there is an element  $z_1 \in M$  with  $p^m z_1 = z$ . However  $p^{n-m}(p^m x + z) \in M \cap (p^a G)$ , and hence is zero. Thus  $p^{n-m}(-z) = p^n x = y$ . Thus,

$$p^n(-z_1) = p^{n-m}(p^m(-z_1)) = p^{n-m}(-z) = y, \quad \text{with } -z_1 \in M.$$

The author would like to thank J. IRWIN and E. WALKER for letting him read their manuscripts [3] and [4] which are to appear shortly in the *Pacific Journal of Mathematics*.

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(Manuscrit reçu le 25 novembre 1960.)

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