Erratum : “On the supercuspidal representations of $GL_N$, $N$ the product of two primes”

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ERRATUM

On the supercuspidal representations of $\text{GL}_N$, $N$ the product of two primes
(Philip Kutzko and David Manderscheid)

Since the abovementioned paper appeared in print, we have carefully read Waldspurger's paper [Wa] (all references as in the original paper), something we should certainly have done far earlier! As a result of our reading we have learned two things. First, several of the results in section 5 either may be found in [Wa] or are easy consequences of results found there. Second, after reading section II of [Wa] we have learned that it is never appropriate to invoke the phrase "theory of the Heisenberg group and the oscillator representation" as a substitute for careful argumentation. To be precise, our assertions about the representations $\Lambda$ and $\Lambda_0$ found in section 5 are not properly justified there and need, to say the least, further comment. Here, then, is what needs to be done to support the assertions in section 5 (all notation as in the original).

1. The representation $\Lambda$ referred to just prior to Lemma 5.6 certainly exists but not for the reasons stated. A proof of the existence of this representation is given in Proposition II.4 of [Wa]. In order that our Lemma 5.6 hold, $\Lambda$ must have the additional properties ascribed to it in Waldspurger's Proposition II.4; our Lemma 5.6 is now a trivial consequence of Lemmas VI.1.1 and VI.1.2 of [Wa].

2. In order to obtain the representation $\Lambda_0$ referred to just prior to Lemma 5.6, one must use the construction found in section III of [Wa]. There, Waldspurger constructs a representation $\Theta$ of a group $K$, both $\Theta$ and $K$ depending on certain data. If, in his notation, this data is chosen to be $r=1$, $t=R$, $F=F_0$, $F_1=E=F'$, $\chi_1=\psi$, $\xi_1=0$ and $p'=1$, then $K$ is seen to be our group $J_0^{-1}$ and our $\Lambda_0$ should be taken to be $\Theta$. It is then necessary to show that there is a certain compatibility between $\Lambda$ and $\Lambda_0$, namely that one can choose a character $\chi$ of $F^*$ such that the representation induced by $\Lambda_0$ on the group $U(\mathfrak{A}_{0,0}) \cdot U^m(\mathfrak{A}_F)$ coincides with the restriction to that group of the representation $\Lambda \otimes \chi \cdot \det$. This last point is not trivial but a verification is not difficult; this verification as well as other details will be provided upon request.
With this choice of $\Lambda_0$, the assertions made prior to Lemma 5.7 for the case $r=0$ are now valid and Lemma 5.8 follows as in our paper or from the Hecke Algebra isomorphism given in Theorem VI.2.2 of [Wa].

3. In order to obtain the representation $\Lambda_r$, $r>0$, referred to just prior to Lemma 5.6, one must imitate Waldspurger's construction of $\Theta$ and $K$ alluded to above. With $\Lambda_r$ constructed in this way, the appropriate compatibility condition for $\Lambda$ and $\Lambda_r$ is obtained and the assertions made prior to Lemma 5.7 are now valid for arbitrary $r$. In order that our proof of Lemma 5.7 now be complete, one need only verify (using Lemma II.5 of [Wa]) that the element $x$ defined in Lemma 5.7 does indeed intertwine $\Lambda_r$.

In conclusion, we wish to apologise for any confusion the abovedescribed errors may have caused.

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