

# ANNALES SCIENTIFIQUES DE L'É.N.S.

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**Correction to “Cohomology of line bundles on  $G/B$ ”**

*Annales scientifiques de l'É.N.S. 4<sup>e</sup> série*, tome 8, n° 3 (1975), p. 421

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Correction to

## COHOMOLOGY OF LINE BUNDLES ON $G/B$

BY LAKSHMI BAI, C. MUSILI AND C. S. SESHADRI

(*Ann. Scient. Éc. Norm. Sup.*, 4<sup>e</sup> série, t. 7, 1974, p. 89 à 138.)

G. Kempf has pointed out that the computation of the line bundle  $K_r$  on  $X(w_n)_r$  [cf. § 3, B, type  $B_n$ , 6, 7 (b) and 8; p. 115 to 121] is incorrect and that in fact it turns out to be the trivial line bundle. However this does not affect the proof of the main theorem of paragraph 3, Type  $B_n$  (Theorem B. 11), in fact the proof of the essential step I on p. 121 now becomes immediate after writing the exact cohomology sequence. Further as we shall now see, the proof that  $K_r$  is trivial also turns out to be simpler than the considerations of the paper for computing  $K_r$ .

Thus one has to make the following correction: In place of Proposition B. 9 (p. 119) one has

**PROPOSITION.** —  $K_r$  is isomorphic to the trivial line bundle and in particular, we have the exact sequence.

$$0 \rightarrow \mathcal{O}_{X(w_n)_r} \rightarrow \mathcal{O}_{Z_r} \rightarrow \mathcal{O}_{X(w_n)_r} \rightarrow 0.$$

*Proof.* — Let  $P = P_{\hat{g}}, T, B$  be the subgroups of  $G = \mathrm{SO}(2n+1) \hookrightarrow \mathrm{GL}(2n+1)$  and identify  $P \backslash G$  with the quadric  $Q \equiv x_1 y_n + \dots + x_n y_1 + z^2 = 0$  in  $P^{2n} = \{(x_1, \dots, x_n, z, y_1, \dots, y_n)\}$  as in the paper. The coordinate functions  $x_1, \dots, x_n, z, y_1, \dots, y_n$  can be canonically identified with functions on  $G$ , namely the entries of the last row. We have the ideals  $I = (x_1, \dots, x_n, z)$  and  $J = (x_1, \dots, x_n)$  in  $A = k[G]$ . Take the action of  $G$  on  $A$  induced by right translation. Recall that  $I$  and  $J$  are  $B$ -stable ideals. Further notice that the element  $z$  is  $B$ -invariant modulo  $J$  (not merely  $B$ -stable modulo  $J$ , we see that  $B$  acts on  $z \bmod J$  through the trivial character).

Let  $K = I/J$  as in the paper. Let  $R = A/I$ ; then  $R = k[X(w_n)]$ . Since  $I^2 \subset J$ ,  $I/J$  acquires a  $B$ -action consistent with the canonical  $B$ -action on  $R$  ( $B$ -actions induced by right multiplication). To prove that  $K_r$  is the trivial line bundle on  $X(w_n)_r$ , we have to show that (as  $R$ -module)  $I/J$  is  $B$ -isomorphic to  $R$ ,  $R$  being considered as a module over itself. Since  $K_r$  is a line bundle, we know that  $I/J$  is a projective  $R$ -module of rank 1. Hence it suffices to show that there exists  $m \in I/J$  such that: 1°  $m$  generates  $I/J$  over  $R$  and 2°  $m$  is  $B$ -invariant. For  $m$  we take the image in  $I/J$  of  $z \in I$ . Since  $z^2 \in J$  it follows that  $z$  generates  $I/J$  over  $R$  and we have seen that  $z \bmod J$  is a  $B$ -invariant element.

Q. E. D.