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Correction to

COHOMOLOGY OF LINE BUNDLES ON G/B

BY LAKSHMI BAI, C. MUSILI AND C. S. SESHADRI

(Ann. Scient. Éc. Norm. Sup., 4ᵉ série, t. 7, 1974, p. 89 à 138.)

G. Kempf has pointed out that the computation of the line bundle K_r on $X(w_n)_r$ [*cf.* § 3, B, type B_n, 6, 7 (*b*) and 8; p. 115 to 121] is incorrect and that in fact it turns out to be the trivial line bundle. However this does not affect the proof of the main theorem of paragraph 3, Type B_n (Theorem B. 11), in fact the proof of the essentiel step I on p. 121 now becomes immediate after writing the exact cohomology sequence. Further as we shall now see, the proof that K_r is trivial also turns out to be simpler than the considerations of the paper for computing K_r.

Thus one has to make the following correction: In place of Proposition B. 9 (p. 119) one has

PROPOSITION. — K_r *is isomorphic to the trivial line bundle and in particular, we have the exact sequence.*

$$0 \to \mathcal{O}_{X(w_n)_r} \to \mathcal{O}_{Z_r} \to \mathcal{O}_{X(w_n)_r} \to 0.$$

Proof. — Let $P = P_{\mathfrak{a}}$, T, B be the subgroups of $G = SO(2n+1) \subsetneq GL(2n+1)$ and identify $P \backslash G$ with the quadric $Q \equiv x_1 y_n + \ldots + x_n y_1 + z^2 = 0$ in $\mathbf{P}^{2n} = \{(x_1, \ldots, x_n, z, y_1, \ldots, y_n)\}$ as in the paper. The coordinate functions $x_1, \ldots, x_n, z, y_1, \ldots, y_n$ can be canonically identified with functions on G, namely the entries of the last row. We have the ideals $I = (x_1, \ldots, x_n, z)$ and $J = (x_1, \ldots, x_n)$ in $A = k[G]$. Take the action of G on A induced by right translation. Recall that I and J are B-stable ideals. Further notice that the element z is B-*invariant* modulo J (not merely B-stable modulo J, we see that B acts on z mod J through the trivial character).

Let $K = I/J$ as in the paper. Let $R = A/I$; then $R = k[X(w_n)]$. Since $I^2 \subset J$, I/J acquires a B-action consistent with the canonical B-action on R (B-actions induced by right multiplication). To prove that K_r is the trivial line bundle on $X(w_n)_r$, we have to show that (as R-module) I/J is B-isomorphic to R, R being considered as a module over itself. Since K_l is a line bundle, we know that I/J is a projective R-module of rank 1. Hence it suffices to show that there exists $m \in I/J$ such that: 1° m generates I/J over R and 2° m is B-invariant. For m we take the image in I/J of $z \in I$. Since $z^2 \in J$ it follows that z generates I/J over R and we have seen that z mod J is a B-invariant element.

Q. E. D.