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DEA-BASED MODELS FOR BEST PARTNER SELECTION FOR MERGER

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Abstract. Mergers and Acquisitions (M&A) is a process whereby two or more companies merge into one company to improve their efficiency and strengthen their market positions. Previous studies about best partner selection for M&A simply consider one factor independently among several relevant factors. In this paper, DEA is applied to support decision making for best partner selection in M&A for decision making units (DMUs), *i.e.*, the companies. According to the different perspectives of efficiency, revenue, and cost, three models based on DEA approach are firstly introduced to select the best partner for M&A. By compositing these different perspectives, we further propose a new DEA model, which has comprehensively considered input cost, output revenue and efficiency to select the best partner among many candidates. 0–1 integer linear programming models are built to implement the process. Finally, an example is given to verify the applicability to this model.

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1. Introduction

Mergers and Acquisitions (M&A) is a process whereby two or more companies merge into one company to improve their efficiency and strengthen their market positions (see Buono and Bowditch [7]). In real life, there always exist instances of M&A among, for example, plants, supermarket chains, banks, and manufacturers. The significance of M&A can be seen from the following aspects. Firstly, Ceausescu [8] indicated that merger has played a significant role in corporate history and has been a vital part of any healthy economy. More importantly, it has become one of the primary ways that companies are able to provide returns to owners and investors. Secondly, Dietrich and Sorensen [13] implied that for the target companies, merger usually produces cost saving and profit increase through economies of scale. For the candidate companies, the key contribution usually includes revenue enhancement, cost reduction, economies scale, gaining a foothold in a new geographic market, and so on. Thirdly, Gugler and Konrad [21] pointed out that an appropriate merger could alter the market structure, augment market power, generate economies of scale and other synergies, have tax advantages, and serve managerial ambitions. Moreover, M&A has become a more and more important topic in the management and development of companies (see Ragothaman *et al.* [32]).

Keywords. Merger and acquisitions, data envelopment analysis, efficiency, decision making units, 0-1integer linear programming.

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Although there has been much research on M&A, the resulting publications are mostly based on estimating the efficiency after M&A. Much less research has discussed the best partner selection. Selecting the best partner(s) from many alternatives is still an open challenge. Therefore, it is imperative to develop an efficient and reasonable approach to find the best partner among alternatives for M&A.

The focus of this paper is to select the best partner for a specified company for the purposes of M&A. We try to deal with this problem using the non-parametric data envelopment analysis (DEA) approach. Developed by Charnes *et al.* [9], DEA is a non-parametric mathematical approach which is used to evaluate the relative performance of a group of homogenous decision making units (DMUs), especially a group with multiple inputs and multiple outputs (see Lewin and Morey [25], Cooper *et al.* [10], Yang *et al.* [40], Amirteimoori and Yang [3], Wu and Liang [37], Wu *et al.* [39]). As a nonparametric technique, DEA does not require a known functional form of the production frontier. For this reason, it is not limited by the functional form and also does not require the many assumptions that arise from the use of statistical methods for function estimation and efficiency measurement (see Yang *et al.* [41], Amirteimoori and Emrouznejad [2], Liang *et al.* [26], Sahoo and Tone [33], Zhu *et al.* [42]). Cooper *et al.* [12] indicated that DEA has been extensively applied in performance evaluation and benchmarking of hospitals, universities, cities, and other areas, even extending to large-scale performance of regions and countries.

There is considerable theoretical literature on the pros and cons of mergers based on DEA, and a number of studies trying to evaluate the effects of actual mergers ex post. Bogetoft and Wang [6] studied the gains to be derived from a merger, decomposing those gains into a technical efficiency index, a harmony index, and a size index. Färe *et al.* [15] developed a dynamic network DEA model to evaluate the potential output gains from a merger of two firms. Using non-parametric production frontiers, Prior [31] evaluated the level of technical efficiency and capture potential economies of scope. Johnes and Yu [23] measured the research performance of Chinese higher education in order to quantify the impact of merger activity which has taken place in Chinese higher education. Lozano and Villa [30] used DEA as a pre-merger planning tool to estimate expected cost and profit efficiency gains. Kao and Yang [24] applied DEA to appraise the Taiwan Forestry Bureau's three alternatives for reorganizing thirteen districts and provided a better alternative. Liu [27] investigated reorganization of the credit departments of farmers' associations. In his research, he proposed two alternatives for M&A, which one is partial M&A of regional farmers' associations, and the other is at the county- and city-level. His study was based on the principle of reorganizing departments with two or three adjacent credit departments and merging the DMUs located in townships or villages within the same city. Gattoufi *et al.* [20] proposed a new inverse DEA model to required level of the inputs and outpouts for a merged bank to reach a predetermined efficiency target. Based on this new model, Amin and Al-Muharrami [1] has extended it to the situation of negative data. Halkos [22] applied DEA model to pre-evaluate technical efficiency gains from possible M&As in the Japanese regional banking sector. Shi *et al.* [36] developed a novel two-stage cost efficiency model to estimate and decompose the potential gains from M&As. However, there are fewer studies on which candidate should be chosen for merger with a given enterprise even though the issue is a very important problem in M&A. Färe *et al.* [18] did propose using DEA to help companies in making decisions about acquiring potential partners, but it just considered the objective of output. Lozano [29] also proposed DEA to choose the best potential partner organization, but his work just considered the objective of cost. Wu *et al.* [38] proposed a new approach based on context-dependent DEA model to select the best cooperative partner for input resource reallocation.

Surveying this prior research, we find that the existing DEA approaches toward M&A are mostly based on efficiency estimation after M&A, while little has considered how to choose from among candidate target companies for a bidder company. The little research there is rarely considers sufficiently the factors of the bidder company itself and the target companies. However, in real life there are many companies who select their merger partners based on factors such as cost, revenue, and efficiency. Examples can be listed as follows. On July 31, 1997, American Boeing Company merged American Merton Company, a merger which was mainly based on revenue perspective. On January 10, 2001, America Online (AOL), the largest Internet service provider (ISP) in the world, merged the global giant Time Warner of entertainment and media. The merger was mainly based on revenue-orientation, with a lesser consideration of the cost factors. However, on April 28, 1994, the M&A between Shanghai Building Materials, the biggest shareholder of Lingguang Industrial, and Hengtong Group was mainly motivated by cost saving. Based on these examples, enterprises should be willing to consider more factors about themselves and target companies when choosing candidates for M&A.

In order to consider more factors about the enterprise itself, this paper explores the methods of mergers using different models based on varying perspectives. Specifically, the problem we will deal with is how to select the best partner for a specified bidder enterprise. Based on the three different perspectives of efficiency, revenue, and cost, corresponding models are introduced to find the best partners for M&A. By compositing these perspectives, a comprehensive DEA model is proposed. The proposed approaches are rather flexible, being able to select the best partner based on different perspectives. Therefore, this research differs from previous studies on several counts. Firstly, we choose the best partner from different perspectives like efficiency, revenue, and cost. Secondly, give more consideration to the given corporate capital for the enterprise, that is, in different conditions, we may use different models to determine the optimal efficiency and benefits. For example, when capital is tight we can use cost model to select the best partner, but large-scale enterprises may care more about revenue models. Further, the comprehensive model can be used to select the best partner(s) by synthesizing different perspectives.

The structure of the paper is as follows. The traditional Russell measure in M&A is given in Section 2. In Section 3 our proposed DEA models based on different perspectives are formulated, in particular an efficiency model, revenue model, cost model, and comprehensive model. In Section 3, we explore a real world case to illustrate the proposed approach. Finally, conclusions are made in Section 4.

2. Traditional russell measure in M&A

Assume that there are n $DMUs$ to be considered, each DMU consuming varying amounts of $m$different inputs to produce$s$ different outputs. Specifically, DMU_j consumes amount x_{ij} of input i and produces amount $y_{rj}$of output r. We assume that $x_{ij} \geqslant 0x_{ij} \geqslant 0$ and $y_{rj} \geqslant 0y_{rj} \geqslant 0$, and further assume that each DMU has at least one positive input and one positive output value. In this section we firstly review the traditional Russell measure for best partner selection in M&A.

The traditional model to measure the efficiency of $DMU_0 = DMU_{j,}$can be denoted as follows based on Färe *et al.* [18],

$$G\left(x_0\right) = \text{Max}\left\{\sum_{k=1}^{n} r_k y_k : \sum_{k=1}^{n} r_k \mathrm{x}_k \leqslant x_0, r_k \geqslant 0, k = 1, 2, \ldots n\right\}$$

where $G\left(x_0\right) G\left(x_0\right)$ expresses the maximum output that could be produced by DMU_0.

Therefore, the efficiency of DMU_0 is the ratio:

$$\rho_0 = \frac{y_0}{G\left(x_0\right)}.$$

Definition 2.1. *DMUo* is said to be *efficient* if and only if $\rho_0 = 1$.

For a given DMU_0 which wants to merge with potential partners, called $DMU_j (j \in I,\ I = 1, \ldots, n\ j \neq o)$, the optimal outputs brought by the merger members can be illustrated as:

$$\begin{aligned} G\left(x_{0I}\right) = \text{Max}\Bigg\{ & \sum_{k=1}^{n} r_k^0 y_k + \sum_{i=1, i\neq 0}^{n} \sum_{k=1}^{n} \theta_i r_k^i y_k : \\ & \sum_{k=1}^{n} r_k^0 x_k \leqslant x_0,\ \sum_{k=1}^{n} \theta_i r_k^i x_k \leqslant x_i,\ \sum_{k=1}^{n} \theta = N, \theta_i \in \{0, 1\} \\ & r_k^0 \geqslant 0, r_k^i \geqslant 0, k = 1, 2, \ldots n, i = 1, 2, \ldots n, i \neq 0 \Bigg\} \end{aligned}$$

where N denotes the number of partners in the final merger.

To simplify the problem, now suppose that DMU_j and DMU_h merge to form a new unit DMU_{jh}, then the new optimal output could be solved by the following program.

$$G(x_{jh}) = \text{Max}\left\{ \sum_{k=1}^{n} r_k^j y_k + \sum_{k=1}^{n} r_k^h y_k : \sum_{k=1}^{n} r_k^j \mathrm{x}_k \leqslant x_j, \sum_{k=1}^{n} r_k^h x_k \leqslant x_h, r_k^j \geqslant 0, r_k^h \geqslant 0, k = 1, 2, \ldots n \right\}.$$

In the model, we have known the relationship of$x_{jh} = x_j + x_h$, which is obtained from the additive assumption in Bogetoft and Wang [6] and Färe [14].

Formulating a horizontal cooperation between DMU_j and DMU_h would be beneficial if $G(x_{jh}) > G(x_j) + G(x_h)$. Alternatively, one could obtain at least a weaker condition for cooperation formation in this condition as $G(x_{jh}) > y_j + y_h G(x_{jh}) > y_j + y_h$ where y_j and $y_h y_h$ are the observed outputs of $DMUj$ and DMU_h, respectively.

3. Proposed models

In order to learn more about the profile of the bidder company and target company, in this section, we propose models to merge partners who can bring the optimum results from four different perspectives.

3.1. Partners selection based on efficiency perspective

In the case of multiple inputs and outputs, the efficiency of a *DMU*, say DMU_i, is often measured by the Farrell's [19] measures.

$$E_i = \text{Min}\left\{E \in \Re_0 |\ (Ex_i, y_i) \in T\ \right\}$$
$$F_i = \text{Max}\left\{F \in \Re_0 |\ (x_i, Fy_i) \in T\ \right\}.$$

E_i is the maximal minification of all inputs and F_i is the maximal expansion of all outputs for the specific DMU_i. T is the underlying production possibility set, which can be clarified as

$$T = \left\{\ (x, y) \in \Re_0^{p+q} |\ \exists \lambda \in \Re_0^n : x \geqslant \sum_i \lambda_i x_i, y \leqslant \sum_i \lambda_i y_i, \lambda_i \in \Delta(s) \right\}.$$

Different models need different assumptions about the production possibility set. The initial CCR model based on constant returns to scale (CRS) was presented by Charnes *et al.* [9]. The FGL model based on nonincreasing returns to scale (NIRS) was presented by Fare *et al.* [16,17] and the BCC model based on variable returns to scale (VRS) was presented by Banker *et al.* [5]. Later Seiford and Thrall [34] had proposed the ST approach based on nondecreasing returns to scale (NDRS).

Under the different models $\Delta(s)$ can be expressed as

$$\Delta(CRS) = \Re_0^N,\ \Delta(VRS) = \left\{\lambda \in \Re_0^N |\ \sum_i \lambda_i = 1, \lambda_i \geqslant 0\right\}$$
$$\Delta(NIRS) = \left\{\lambda \in \Re_0^N |\ \sum_i \lambda_i \leqslant 1 \lambda_i \geqslant 0\right\}$$
$$\Delta(NDRS) = \left\{\lambda \in \Re_0^N |\ \sum_i \lambda_i \geqslant 1, \lambda_i \geqslant 0\right\}.$$

In most situations, the production possibility set is assumed as VRS. Let us suppose that it has the organizational sense to merge the J-DMU_S *i.e.*, the DMU_S with indexes $j \in J \subseteq \{1, 2, \ldots, n\}$. Direct combination of the inputs and outputs gives a unit that consumes $\sum_{j \in J} x_j$ to produce $\sum_{j \in J} y_j$.

Using radial output-based measure of the potential overall gains from merging the J-DMU_S gives us the following.

$$F_J = \text{Max}\left\{F \in \Re_0 \middle| \left(\sum\nolimits_{j\in J} x_j, F\left[\sum\nolimits_{j\in J} y_j\right]\right) \in T\right\}$$

F_J is the maximal proportional expansion of the aggregate output $\sum_{j\in J} y_j$ which is feasible in the merged unit with aggregate input $\sum_{j\in J} x_j$.

Definition 3.1. The merger is said to be *efficient* if and only if $F_J \geqslant 1$. When $F_J > 1$ it is called to be *strongly efficient* and when $F_J = 1$ it is called to be *weekly efficient*.

The problem can be solved by inserting a DEA estimate of the underlying production possibility set (see Bogetoft and Wang [6]).

$$\begin{aligned}
&\text{Max} \quad F_J \\
s.t. &\left[\sum_{j\in J} x_j\right] \geqslant \sum_j \lambda_j x_j \\
&F_J\left[\sum_{j\in J} y_j\right] \leqslant \sum_j \lambda_j y_j \\
&\lambda_j \in \Delta(s), j = 1, 2, \ldots, n.
\end{aligned} \tag{3.1}$$

This paper gives two propositions that can judge the efficiency of the merger intuitively, neither of which is not pointed out by Bogetoft and Wang [6].

Proposition 3.2. *If the merged inputs and outputs are still in the production possibility set, that is,* $\left(\sum_{j\in J} x_j, \sum_{j\in J} y_j\right) \in T$, *then the merger is efficient.*

Proposition includes two parts: one is that if the merged inputs and outputs are on the efficient frontier of the production possibility set then the merger is weakly efficient, the other is that if the merged inputs and outputs are inside of the production possibility set (not on the efficient frontier) then the merger is strongly efficient.

Proof. From model (3.1), we know that the right-hand side expressions from the first and second constraints denote the efficient frontier of the production possibility set.

For the fixed input $\sum_{j\in J} x_j$, if $(\sum_{j\in J} x_j, \sum_{j\in J} y_j) \in T$, then there exists an F which is bigger than 1 that makes $\left(\sum_{j\in J} x_j, F\left[\sum_{j\in J} y_j\right]\right)$ be on the efficient frontier of the production possibility set.

If $F = 1$, then $\left(\sum_{j\in J} x_j, \sum_{j\in J} y_j\right)$ is on the efficient frontier of the production possibility set. The inputs and outputs of the new merged DMU are the addition of the merged inputs and outputs, that is, their efficiency has neither increased nor decreased. Thus, it is weakly efficient.

If $F > 1$, the potential increase ratio of the merged output is greater than 1, so the merger is strongly efficient. □

Proposition 3.3. *If the merged inputs and outputs are out of the production possibility set, that is,* $\left(\sum_{j\in J} x_j, \sum_{j\in J} y_j\right) \notin T$, *then the merger is inefficient.*

Proof. If $\left(\sum_{j\in J} x_j, \sum_{j\in J} y_j\right) \notin T$, then there exist an F which is smaller than 1 that makes $\left(\sum_{j\in J} x_j, F\left[\sum_{j\in J} y_j\right]\right)$ be on the frontier of the production possibility set. The potential increase ratio of the merged outputs is smaller than 1, so the merger is inefficient. □

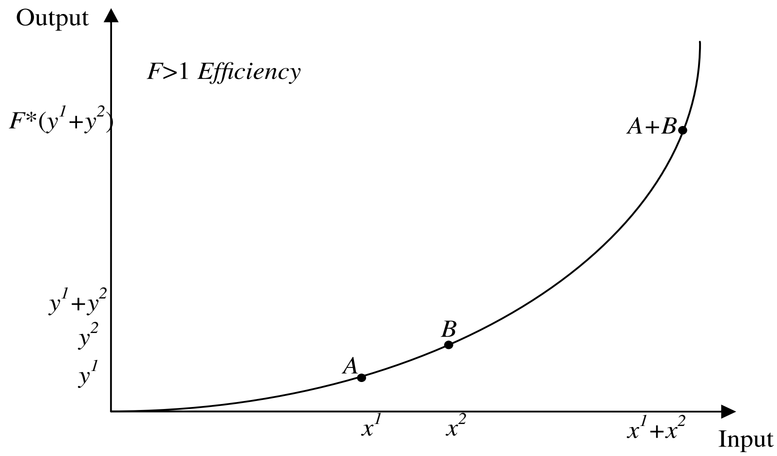

FIGURE 1. Efficient merger.

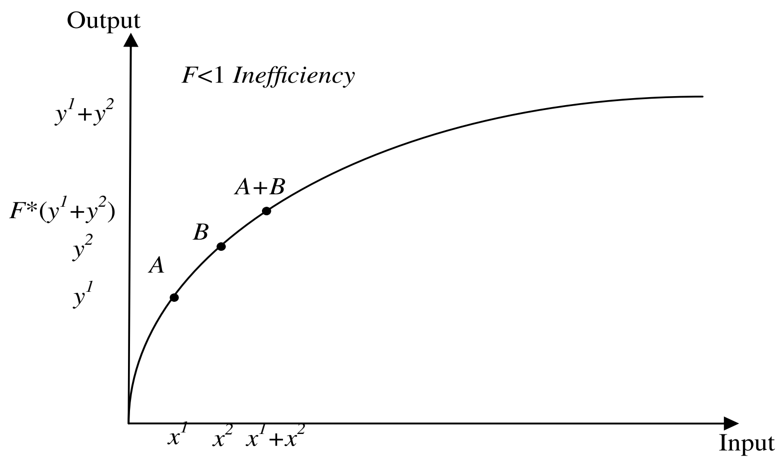

FIGURE 2. Inefficient merger.

The following Figures 1 and 2 illustrate the situations of efficient and inefficient merger.

Considering a given DMU_0 which wants to merge with other partner(s), the maximum proportional expansion of the efficiency can be written as following the program,

$$\begin{aligned} & \text{Max} \quad F_J \\ s.t. \quad & x_{i0} + \sum_{j=1, j\neq 0}^{n} \theta_j x_{ij} \geqslant \sum_{j=1}^{n} \lambda_j x_{ij} \qquad i = 1, 2, \ldots, m \\ & F_J(y_{r0} + \sum_{j=1, j\neq 0}^{n} \theta_j y_{rj}) \leqslant \sum_{j=1}^{n} \lambda_j y_{rj} \; r = 1, 2, \ldots, s \\ & \sum_{j=1, i\neq 0}^{n} \theta_j = N, \;\; \lambda_i \in \Delta(s), \;\; \theta_i \in \{0, 1\}\,. \end{aligned} \tag{3.2}$$

θ_j is a binary variable of value 0 or 1 where $\theta_j = 1$ if and only if DMU_j merges with DMU_0. The parameter Nmeans the number of the other $DMUs$ which merge with DMU_0.

Model (3.2) is an output-oriented efficiency measure. The corresponding input-oriented model is as follows.

$$\begin{aligned}
&\text{Min} \quad E_J \\
s.t. \quad & E_J(x_{i0} + \sum_{j=1, j\neq 0}^{n} \theta_j x_{ij}) \leqslant \sum_{j=1}^{n} \lambda_j x_{ij} \quad i = 1, 2, \ldots, m \\
& y_{r0} + \sum_{j=1, j\neq 0}^{n} \theta_j y_{rj} \geqslant \sum_{j=1}^{n} \lambda_j y_{rj} \qquad r = 1, 2, \ldots, s \\
& \sum_{j=1, j\neq 0}^{n} \theta_j = N, \lambda_i \in \Delta(s), \theta_j \in \{0, 1\}\,.
\end{aligned} \tag{3.3}$$

3.2. Partners selection based on revenue perspective

Since merger advantages are often expressed in the revenue terms, many enterprises may give more consideration to revenue brought by a merger. In real life, there are many instances that are revenue-oriented. For example, on July 31, 1997, American Boeing Company merged American Merton Company, a merger mainly based on revenue perspective. On January 10, 2001, America Online (AOL), the largest Internet service provider (ISP) in the world merged with the global giant Time Warner of entertainment and media. The merger was mainly based on revenue-orientation, with a lesser consideration about the cost factors.

In the multi-output formulation, assuming that output prices $(p_r, r = 1, 2, \ldots, s)$ are known, one may maximize revenues by maximizing $\sum_{r=1}^{s} p_r y_r$ subject to a technological constraint. In addition, we also suppose DMU_0 wants to merge with partners called DMU_i $(i \in I)$. The maximum joint revenue of the merger can be estimated by following program:

$$\begin{aligned}
R_{0,I} = Max \quad & \sum_{r=1}^{s} p_r \hat{y}_r \\
s.t. \quad & \hat{x}_i \geqslant \sum_{j=1}^{n} \lambda_j x_{ij} \qquad i = 1, 2, \ldots, m \\
& \hat{y}_r \leqslant \sum_{j=1}^{n} \lambda_j y_{rj} \qquad r = 1, 2, \ldots, s \\
& \hat{x}_i \leqslant x_{i0} + \sum_{j=1, j\neq 0}^{n} \theta_j x_{ij} \quad i = 1, 2, \ldots, m \\
& \sum_{i=1, i\neq 0}^{n} \theta_i = N, \theta_i \in \{0,\, 1\}\,, \lambda_i \in \Delta(s).
\end{aligned} \tag{3.4}$$

Perhaps some further explanation is necessary at this point. Note the following:

1. N expresses the number of merger partners, not including the given DMU seeking merger;
2. for each output, all $DMUs$ face the same prices, $p_r, r = 1, 2, \ldots, s$;
3. over all i, the $\hat{x}_i$ denote the input variables of the new coalition;

4. one can compare the coalition's revenue, $R_{0,I}$, to individual revenue, $R_0, R_i, i \in I$, to determine whether a coalition is beneficial.

The partner selection based on efficiency perspective reflects the efficiency changes. However, the revenue perspective can guide the new coalition to have its production reach $(\hat{x}_i, \hat{y}_i)$, which brings the maximum revenues while using no more than the total inputs $x_{i0} + \sum\limits_{j=1, j\neq 0}^{n} \theta_j x_{ij}$.

3.3. Partners selection based on cost perspective

In addition, many companies consider addition factors about themselves, especially in relation to costs. Sometimes the companies may try their best to minimize the costs when they choose partners for merger and acquisitions. The costs of M&A include preparation costs, negotiation costs, and integration costs. However, the costs of preparation and negotiation phases are difficult to predict, so this paper mainly analyzes the input costs of integration. There are two important reasons for a cost-oriented perspective: one is that cost-orientation can save economic resources, the second is that cost-orientation can optimize the decision of cost. There are also many merger examples for cost perspective. On April 28, 1994, the M&A between Shanghai Building Materials, the biggest shareholder of Lingguang industrial, and the Hengtong Group was mainly guided by cost-orientation and cost-saving. Another example is that on September 3, 2001, the combination of Hewlett Packard (HP) and Compro was also achieved with the purpose of cost-saving. Thus, it is important to select partners for M&A based on cost-orientation.

In the multi-output formulation, assuming that input prices $(c_i, i = 1, 2, \ldots, m)$ are known, one may minimize cost by minimizing $\sum_{i=1}^{m} c_i x_i$ subject to a technological constraint. Therefore, the Mixed-Integer Linear Program which selects the best partners can be written as:

$$
\begin{aligned}
& C_{0,I} = \text{Min} \ \sum_{r=1}^{m} c_r \hat{x}_r \\
s.t. \quad & \hat{x}_i \geqslant \sum_{j=1}^{n} \lambda_j x_{ij} \qquad i = 1, 2, \ldots, m \\
& \hat{y}_r \leqslant \sum_{j=1}^{n} \lambda_j y_{rj} \qquad r = 1, 2, \ldots, s \\
& \hat{x}_i \leqslant x_{i0} + \sum_{j=1, j\neq 0}^{n} \theta_j x_{ij} \quad i = 1, 2, \ldots, m \\
& \hat{y}_i \geqslant y_{r0} + \sum_{j=1, j\neq 0}^{n} \theta_j y_{rj} \quad r = 1, 2, \ldots, s \\
& \sum_{i=1, i\neq 0}^{n} \theta_i = N, \theta_i \in \{0, 1\}, \lambda_i \in \Delta(s).
\end{aligned}
\tag{3.5}
$$

Different from model (3.4), we should give the restriction $\hat{y}_i \geqslant y_{r0} + \sum_{j=1, j\neq 0}^{n} \theta_j y_{rj} \quad r = 1, 2, \ldots, s$ in model (3.5), that is, we should guarantee the minimum outputs to be produced by the planned joint-venture facility. Otherwise, we may obtain zero from the optimum solution of model (3.5), that is, nothing would be produced in order to minimize the whole costs.

3.4. Partners selection based on comprehensive perspective

However, the above two models are only based on one perspective, revenue or cost, without considering both revenue and cost simultaneously. In addition, efficiency factors are also not considered after M&A. Thus, we should comprehensively consider input cost, output revenue and efficiency when choosing partners. In real life, many enterprises will have a certain cost budgets for M&A. The comprehensive measure can be expressed as follows:

$$
\begin{aligned}
P_{0,I} = \text{Max} \ & \sum_{r=1}^{s} p_r \hat{y}_r - \sum_{i=1}^{m} c_i \hat{x}_i \\
\text{s.t.} \quad & \hat{x}_i \geqslant \sum_{j=1}^{n} \lambda_j x_{ij} \qquad i = 1, 2, \ldots, m \\
& \hat{y}_r \leqslant \sum_{j=1}^{n} \lambda_j y_{rj} \qquad r = 1, 2, \ldots, s \\
& \hat{x}_i \leqslant x_{i0} + \sum_{j=1, j\neq 0}^{n} \theta_j x_{ij} \quad i = 1, 2, \ldots, m \\
& \frac{\sum\limits_{r=1}^{s} p_r y_{r0}}{\sum\limits_{i=1}^{m} c_i x_{i0}} \leqslant \frac{\sum\limits_{r=1}^{s} p_r \hat{y}_r}{\sum\limits_{i=1}^{m} c_i \hat{x}_i} \\
& C_1 \leqslant \sum_{i=1}^{m} c_i \hat{x}_i \leqslant C_2 \\
& \sum_{i=1, i\neq 0}^{n} \theta_i = N, \theta_i \in \{0, 1\}, \lambda_i \in \Delta(s)
\end{aligned}
\tag{3.6}
$$

where C_1 and C_2 denote the lower and upper limitation of cost budgets, respectively. The fourth additive constraint $\frac{\sum_{r=1}^{s} p_r y_{r0}}{\sum_{i=1}^{m} c_i x_{i0}} \leqslant \frac{\sum_{r=1}^{s} p_r \hat{y}_r}{\sum_{i=1}^{m} c_i \hat{x}_i}$ indicates that the efficiency of the given $DMUo$ should be improved after M&A, where p_i and c_i can denoted the common weights of the output and input. Model (3.6) is a nonlinear program which can be changed to following linear program:

$$
\begin{aligned}
P_{0,I} = \text{Max} \ & \sum_{r=1}^{s} p_r \hat{y}_r - \sum_{i=1}^{m} c_i \hat{x}_i \\
\text{s.t.} \quad & \hat{x}_i \geqslant \sum_{j=1}^{n} \lambda_j x_{ij} \qquad i = 1, 2, \ldots, m \\
& \hat{y}_r \leqslant \sum_{j=1}^{n} \lambda_j y_{rj} \qquad r = 1, 2, \ldots, s \\
& \hat{x}_i \leqslant x_{i0} + \sum_{j=1, j\neq 0}^{n} \theta_j x_{ij} \quad i = 1, 2, \ldots, m \\
& \sum_{r=1}^{s} p_r y_{r0} \sum_{i=1}^{m} c_i \hat{x}_i \leqslant \sum_{i=1}^{m} c_i x_{i0} \sum_{r=1}^{s} p_r \hat{y}_r \\
& C_1 \leqslant \sum_{i=1}^{m} c_i \hat{x}_i \leqslant C_2 \\
& \sum_{i=1, i\neq 0}^{n} \theta_i = N, \theta_i \in \{0, 1\}, \lambda_i \in \Delta(s).
\end{aligned}
\tag{3.7}
$$

TABLE 1. Raw data of the example.

DMU	Input1	Input2	Input3	Input4	Input5	Output1	Output2
1	30 993	1084	9068	4777	1 285 960	300 169	2268
2	55 791	2433	22 979	8524	4 380 319	584 884	7987
3	48 712	2219	20 335	13 295	3 668 196	680 003	15 666
4	31 807	1340	14 193	8203	1 431 969	316 002	7889
5	75 147	3147	25 908	21 569	5 864 270	1 063 917	10 609
6	51 372	1905	14 937	17 994	1 959 879	659 072	6059
7	41 273	1438	9618	16 464	1 576 976	409 362	2665
8	50 915	2082	17 168	16 050	3 313 708	703 789	8086
9	58 153	2276	19 323	13 285	3 603 381	734 310	10 072
10	44 331	1511	10 124	17 920	1 583 251	464 729	2954
11	17 376	527	3510	4758	308 496	96 949	592
12	22 218	725	5984	6061	479 660	139 633	1498
13	26 980	878	6616	5837	745 277	206 291	2662
14	17 004	397	2652	1626	104 181	36 218	220
15	45 894	1996	18 901	6682	3 686 447	725 917	5734
16	33 197	1292	12 458	5967	2 409 244	393 068	3625
17	41 985	1813	17 074	7693	4 554 469	608 018	7742
18	32 925	1204	10 383	11 214	1 207 182	302 673	2282
19	44 150	1634	12 733	10 605	2 120 539	518 447	10 405
20	37 609	1365	9401	17 000	1 429 529	417 632	2455
21	24 680	784	5680	7423	658 903	197 725	2422
22	26 577	1031	9495	10 180	797 733	240 866	4316
Price	8	14	12	18	4	32	46

This comprehensive model (3.7) can be used to find the best partners considering revenue, cost and efficiency simultaneously.

4. Illustration

In this section, in order to illustrate our proposed approach, we offer in Table 1 below data which originated from Lo *et al.* [28]. Twenty-two electricity distribution districts of the Taiwan Power Company in Taiwan. Each company has five inputs x_1, x_2, x_3, x_4, x_5 and two outputs y_1, y_2. In order to make use of our revenue and cost models, we also suppose the five input costs c_1, c_2, c_3, c_4, c_5 and two output prices $p_1 p_2 p_1, p_2$ are known. Since our examples are mainly used for illustrating our proposed models, we give specific numbers to each cost and prices in our paper, which does not impact our illustration. For ease of presentation, we do not consider the location, political influences, and other factors, which may also affect the merger. We suppose all DMU_S are independent, that is, they are separate organizations. Besides, the *DMUs*' mergers could be the independent assortments.

Table 2 shows the input-oriented Farrel efficiency of each *DMU* for CCR, BCC, and NIRS, as well as the scale efficiency and the return to the scale. Here CCR indicates global technical efficiency, written as E_{CCR}, and BCC indicates local pure technical efficiency, written as E_{BCC}. The scale efficiency was obtained from E_{CCR}, using the formula E_{CCR}/E_{BCC} (see Cooper *et al.* [11]). The return to the scale is identified by Färe and Grosskopf [16], Banker *et al.* [4] and Seiford and Zhu [35].

In the table, CRS denotes the constant return to the scale, IRS denotes the increasing return to the scale, and DRS denotes the decreasing return to the scale. Note that within this sample, most of DMU_S are constant or have increasing return to the scale. It is obvious that those *DMUs* are inefficient and IRS can improve each one's efficiency by merging with partners. Obviously, *DMU* 16 has global technical inefficiency, local pure technical inefficiency, and scale inefficiency. Therefore, we choose *DMU* 16 as the study object, being the one

Table 2. Efficiency scores of different styles.

DMU	CCR	BCC	NIRS	Scale efficiency	Return to scale
1	1.000	1.000	1.000	1.000	CRS
2	0.906	0.921	0.906	0.984	IRS
3	1.000	1.000	1.000	1.000	CRS
4	1.000	1.000	1.000	1.000	CRS
5	1.000	1.000	1.000	1.000	CRS
6	1.000	1.000	1.000	1.000	CRS
7	0.931	0.957	0.931	0.973	IRS
8	1.000	1.000	1.000	1.000	CRS
9	0.952	0.997	0.997	0.955	DRS
10	1.000	1.000	1.000	1.000	CRS
11	0.928	1.000	0.928	0.928	IRS
12	0.883	0.933	0.883	0.946	IRS
13	0.920	0.963	0.920	0.956	IRS
14	1.000	1.000	1.000	1.000	CRS
15	1.000	1.000	1.000	1.000	CRS
16	0.850	0.955	0.850	0.890	IRS
17	0.997	1.000	0.997	0.997	IRS
18	0.745	0.878	0.745	0.849	IRS
19	1.000	1.000	1.000	1.000	CRS
20	0.983	1.000	0.983	0.983	IRS
21	0.944	1.000	0.944	0.944	IRS
22	1.000	1.000	1.000	1.000	CRS

Table 3. Mergers from different formulation.

N	Efficiency	Revenue	Cost	Comprehensive
$N = 1$	5	5	12	5
$N = 2$	2, 5	2, 5	11, 14	5, 6
$N = 3$	2, 5, 8	2, 5, 8	11, 12, 14	5, 6, 10

which can most enlarge its efficiency and profits by M&A. It should be noted that this paper only considers how to choose the best partner companies for a bidder company. In real life, this phenomenon often occurs in many monopolistic competition markets, such as the electricity market in China.

Using the above mentioned formulations to find the best partners, we have the optimal solutions that are illustrated in Table 3.

We cannot have the cost constraints, so we just have the efficiency constraints in the comprehensive approach. From Table 3, we know that different models may have different results. Combining with Table 2, if we consider costs, inefficient *DMUs* like *DMU* 12 and 11 will be selected as the best partners. These results can also be understood because those inefficient *DMUs* may have low scale efficiency, and their return to the scales are also increasing. If the revenue perspective is considered, the DMUs with higher efficiency will be selected, such as DMUs 2, 5, and 8. Going further with this analysis, these higher efficient DMUs have more input resources relatively. Moreover, we can find that the efficiency perspective and revenue perspective have the same optimal solutions. If the comprehensive perspective is considered, the efficient *DMUs* 5, 6, and 10 will be selected as the best partners because they all have technical efficiency and scale efficiency.

5. Conclusions

As the business environment becomes more and more complicated, mergers and acquisitions (M&A) becomes more and more popular and important in practical management. M&A has been studied by many researchers, mainly focused on investigating the impacts of M&A. However, few research publications provide an approach to determine which companies should be merged with a given particular company.

In this paper, we introduce non-parametric DEA models based on 0-1 integer linear programming to find the best partners from different candidate *DMU*s. Firstly, we review traditional models. Then based on different perspectives of efficiency, revenue, and cost, the corresponding efficiency, revenue, and cost measures are introduced. By compositing these perspectives, a comprehensive DEA model is proposed. The proposed approaches are rather flexible, being able to select the best partners from different perspectives. For example, we can use the cost model when the objective enterprise has tight capital. On the other hand, we also can rely on the comprehensive model for the best partner selection when taking into account the comprehensive perspective of the enterprise. An illustration is presented to test our models.

As merger and acquisition become more significant and more common, future studies could attempt to give more consideration to location, political influences, and other factors. Another future study could be the availability of *DMU* data and statistics, which are normally proprietary for the different companies.

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