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A MULTI-RETAILER SUPPLY CHAIN MODEL WITH BACKORDER AND VARIABLE PRODUCTION COST

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Abstract. The modern marketing environment involves variability and randomness within the numerous parties of any supply chain network. Thus, formation of a supply chain model including multiple buyers and variable production rate is more acceptable than assuming a single-buyer with constant production rate model. This paper considers a supply chain network, where a single-vendor manufactures products in a batch production process and supplies them to a set of buyers over multiple times. Instead of assuming a fixed production rate, as commonly used in the literature, a variable production rate is introduced by the vendor and the production cost of the vendor is treated as a function of production rate. The continuous review inventory model is applied for multiple buyers to inspect inventory levels and a crashing cost is incurred by all buyers to reduce their lead times. The lead time demand follows a normal distribution. The unsatisfied demands at the buyers end are partially backordered. A model is formulated to minimize the joint expected cost of the vendor-buyers supply chain system. A classical optimization technique is utilized to solve the model. An improved algorithm is developed to obtain the numerical solution of the model. Finally, numerical examples are given to illustrate the model.

Mathematics Subject Classification. 90B05, 90B06.

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1. Introduction

In the modern marketing environment, it is more relevant to optimize the total system cost jointly for all parties involved in the supply chain system than to optimize the individual cost of each party [29]. Currently, a vendor or manufacturer typically delivers products to numerous buyers. Many vendors build their own retail outlet to deliver products to multiple buyers. Thus, a single-vendor multi-buyer model is applicable in many cases. Goyal [16] proposed the integrated inventory model with coordination between a single buyer and a single vendor as a pioneering approach. Banerjee [2] extended Goyal's [16] model by assuming a lot-for-lot policy, which was again extended by Goyal [18] with SSMD policy. Goyel [18] suggested a supply chain model, where the vendor's production quantity is an integer multiple of the buyer's order quantity. Ha and Kim [20] developed

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a lot splitting supply chain model with a single retailer and a single supplier. Ouyang *et al.* [27] investigated an integrated vendor-buyer cooperative model with controllable lead time and stochastic demand. Sarkar and Majumder [29] developed an integrated inventory model with vendor's setup cost reduction and solved by a distribution free-approach. Cárdenas-Barrón *et al.* [6] surveyed a number of research articles regarding economic order quantity model. Cárdenas-Barrón and Sana [6] investigated the channel coordination of a two-echelon supply chain, where the demand pattern is dependent on sales' teams initiatives. Moon *et al.* [25] introduced a service level constrain in a continuous review model with variable stochastic lead time. Regarding backorder rate, Sarkar *et al.* [31] introduced random defective production rate with variable backorder rate. Sarkar *et al.* [33] introduced fill rate in a continuous review inventory model to minimize the total system cost with setup cost reduction. Sarkar and Mahapatra [33] developed a periodic review inventory model with fuzzy demand to minimize the total cost by considering setup cost reduction of vendor. Sarkar *et al.* [33] considered backorder price discount in an integrated inventory model, where they developed two models with lead time demand as normally distributed and without having any distribution. Sarkar *et al.* [36] introduced product specific (products having fixed lifetime) backordering policy in a two-echelon supply chain model with coordination between the supply chain players. Based on the imperfect quality of products, Sarkar *et al.* (2017) discussed the way to improve the quality by additional investment.

Banerjee and Burton [4] discussed a comparison between coordinated and independent replenishment policies in a single-vendor multi-buyer supply chain model. Banerjee and Banerjee [3] developed a multi-buyer inventory model using electronic data interchange with an order-up-to inventory control policy. Sarmah *et al.* [38] considered a single-supplier multi-buyer coordinated supply chain model with a trade credit policy. Hoque [21] discussed three different single-vendor multi-buyer models by synchronizing the production flow with equal and unequal sized batch transfer for the first two models and the last model, respectively. Guan and Zhao [19] developed a multi-retailer inventory system with a continuous review policy, which optimizes the decisions of pricing and inventory management with the aim of maximizing profit. Jha and Shankar [22] developed a single-vendor multi-buyer constrained non-linear model under service level constraint and solved it using a Lagrangian multiplier method. Cárdenas-Barrón and Treviño-Garza [6] developed a three-echelon supply chain model with multiple products and multiple periods. Glock and Kim [15] studied the effect of forward integration in a multi-retailer supply chain under retailer competition. Cárdenas-Barrón and Sana [9] studied a two-layer supply chain model with multiple items and a promotional effort.

To improve customer service and to reduce stockout loss, it is important to reduce lead time. Liao and Shyu [24] first incorporated a probabilistic inventory model assuming lead time as a unique decision variable. Ben-Daya and Rauf [5] considered an inventory model as an extension of Liao and Shyu's [24] model, where lead time is one of the decision variables. Ben-Daya and Rauf's [5] model dealt with no shortage and continuous lead time. Ouyang *et al.* [26] extended Ben-Daya and Rauf's [5] model by assuming discrete lead time and shortages. Pan and Yang [28] analyzed an integrated inventory model with lead time in a controllable manner. Annadurai and Uthayakumar [1] developed a periodic review inventory model under controllable lead time and lost sales reduction. Gholami-Qadikolaei *et al.* [13] developed a probabilistic inventory model with lead time and ordering cost reduction under budget and space constraint. Shin *et al.* [39] studied an integrated inventory model with controllable lead time and a service level constraint. You [41] studied an inventory model with partial backorder under vertical shift demand. Chung [11] assumed an integrated production-inventory model with backorder and used the method of comprising cost difference rate. Sarkar and Moon [31] developed an inventory model with variable backorder and deduced the procedure to reduce setup cost and quality improvement.

The production rate is assumed to be constant in the classical supply chain model, however, in many cases, the machine production rate may change [23]. Conard and McClamrock's [12] analysis stated that a 10% change in processing rate resulting a 50% change in machine tool cost. Moreover, the possibility of failure in the production process gradually increases with increasing production rate. As a result, the product quality may deteriorate at some rate. Thus, it is reasonable to consider the production rate as a decision variable not constant. Unit production cost also depends on the production rate and should be treated as one of the decision variables. Giri and Dohi [14] considered a generalized extended EMQ model with variable production rate by assuming stochastic machine breakdown and repair. Chang *et al.* [10] developed an EMQ model with variable production rate for a two-stage assembly system. Soni and Patel [40] studied an integrated single-supplier single-retailer inventory model with variable production rate and trade-credit policy.

This article develops a vendor-buyer supply chain model. Instead of considering a single-buyer, multiple buyers are assumed to construct the model [19, 22]. To improve customer service, variable lead time and lead time crashing cost are utilized [26]. Shortages are considered, which is partially backordered [11]. As a constant production rate is inappropriate in the recent several marketing situations, variable production rate and variable production cost [12, 23] are assumed to develop this proposed model. The paper is organized in the following way: Section 2 contains problem definition, notation, assumptions. Section 3 includes descriptions of the mathematical model and solution algorithm. Section 4 describes numerical experiments with optimal values of decision variables for different backorders and sensitivity analysis. Sections 5 and 6 consist of managerial insights and concluding remarks, respectively.

2. Problem definition, notation, and assumptions

This section includes the problem definition with discussion, notation, and assumptions to formulate the mathematical model.

2.1. Problem definition

This paper develops a supply chain model, where single vendor supplies products to multiple buyers. The production rate and production cost are considered as a variable quantities, as in many previous studies. Moreover, production cost is assumed as production rate and a special 'U'-shaped function is used to develop the model (Fig. 1). The vendor follows a single-setup-multi-delivery (SSMD) policy for delivering products to buyers. At the buyer's end, partial backorder is considered for shortages and the buyer minimizes lead time by utilizing a lead time crashing cost. The lead time demand is considered as stochastic and follows a normal distribution.

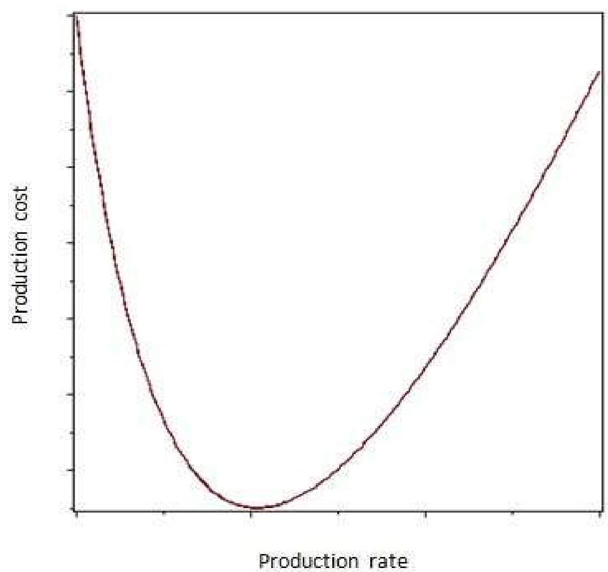

Figure 1. Graphical representation of production rate versus production cost.

2.2. Notation

For buyers
Decision variables

q_i order quantity for buyer i (units).
k_i safety factor for buyer i.
L_i length of lead time for buyer i (week/weeks).

Parameters

n number of buyers.
d_i average demand per unit time (units).
A_{bi} ordering cost of the buyer per order ($ / order).
h_{bi} holding cost rate per unit time ($ / unit / unit time).
σ_i standard deviation of the demand per unit time (units).
π_i stockout cost per unit of shortage($ / unit).
π_{0i} marginal profit per unit item for buyer i ($ / unit).
β_i fraction of the demand for buyer i that will be backordered during stockout, $0 \leq \beta \leq 1$.

For vendor
Decision variables

P production rate per unit time (units).
Q delivery lot size of vendor such that $Q = \sum_{i=1}^{n} q_i$.
m number of lots (same for all buyers) delivered to each buyer in one production cycle (positive integer).

Parameters

$C(P)$ production dependent unit production cost ($ / unit).
A_v setup cost for vendor ($ / setup).
h_v holding cost rate per unit time for vendor ($ / unit / unit time).

Other notation

X_i normally distributed lead time demand for buyer i with mean $d_i L_i$ and standard deviation $\sigma_i\sqrt{L_i}$.
$E(\cdot)$ mathematical expectation.
x^+ maximum value of x and 0.

2.3. Assumptions

1. A single vendor supplies single type of products to multiple buyers with equal lot size.
2. To satisfy the demand of each buyer, the vendor supplies a total of Q quantity such that $Q = \sum_{i=1}^{n} q_i$.
3. The vendor manufactures mQ quantity against the order of q_i quantity of buyer i, but the shipment should be in quantity Q over m times. The shipment procedure follows the relation $q_i = d_i \frac{Q}{D}$, *i.e.*, $\frac{q_i}{d_i} = \frac{Q}{D}$.
4. The inventory is continuously reviewed by each buyer. According to this policy, an order is placed whenever the level of inventory decreases to a particular inventory level (reorder point).
5. Production rate is a variable quantity that varies within the range P_{min} ($P_{min} > D = \sum_{i=1}^{n} d_i$) and P_{max}.
6. The unit production cost of the vendor is a function of P having the expression as $C(P) = \left(\frac{a_1}{P} + a_2 P\right)$ (Khouja and Mehrez, 1994), where a_1 and a_2 are constants producing the best fit for the function $C(P)$.
7. Partial backorder is considered with backorder ratio β_i for the ith retailer.
8. For the ith retailer, it is assumed $L_{i,0} \equiv \sum_{j=1}^{n_i} b_{i,j}$, where $L_{i,r}$ is the length of lead time with components $1, 2, \ldots, r$ crashed to their minimum duration. Thus, $L_{i,r}$ can be expressed as $L_{i,r} = L_{i,0} - \sum_{j=1}^{r}(b_{i,j} - a_{i,j})$, $r = 1, 2, \ldots, n$; and the lead time crashing cost per cycle $C_i(L_i)$ is expressed as $C_i(L_i) = c_{i,r}(L_{i,r-1} - L_i) + \sum_{j=1}^{r-1} c_{i,j}(b_{i,j} - a_{i,j})$, $L \in [L_{i,r}, L_{i,r-1}]$.

9. The lead time crashing cost entirely belongs to the buyer's cost component.
10. The time horizon is infinite.

3. Mathematical model

This section contains mathematical models for vendor and buyers along with solution procedures.

3.1. Mathematical model for buyers

The ordering cost for the ith buyer is $\frac{A_{bi}d_i}{q_i}$ as the expected cycle time for each buyer is $\frac{q_i}{d_i}$. The inventory level is continuously reviewed by each buyer. Thus, the ith buyer places an order (q_i) only when the level of inventory reaches to a specified indicator say, reorder point (r_i). The net inventory level for buyer i just before and after receipt of an order is $r_i - d_iL_i$ and $q_i + r_i - d_iL_i$, respectively. Therefore, the approximated average inventory for buyer i over the cycle is $\frac{q_i}{2} + r_i - d_iL_i$. Now, r_i can be expressed as $D_iL_i + k_i\sigma_i\sqrt{L_i}$ which results in the average inventory for the ith buyer being $\frac{q_i}{2} + k_i\sigma_i\sqrt{L_i}$. Again, $(1-\beta_i)$ is the fraction of demand that is not backordered. Hence, the holding cost for buyer i per unit time is $h_{bi}[\frac{q_i}{2} + r_i - d_iL_i + (1-\beta_i)E(X_i - r_i)^+]$. As π_{0i} and π_i are the marginal profit and stockout cost per unit item, respectively, for buyer i, $\{\pi_i + \pi_{0i}(1-\beta_i)\}\frac{d_i}{q_i}E(X_i - r_i)^+$ is the shortage cost per item per unit time. According to assumption 8, the lead time crashing cost per unit time can be expressed as $R(L_i)\frac{d_i}{q_i}$ for buyer i.

Total expected cost for buyer i is

$$TEC_{bi} = \text{Ordering cost+holding cost+shortage cost+lead time crashing cost}$$

Thus, TEC_{bi} leads to the following expression:

$$TEC_{bi}(q_i, k_i, L_i) = \left[\frac{A_{bi}d_i}{q_i} + h_{bi}\left\{\frac{q_i}{2} + k_i\sigma_i\sqrt{L_i} + (1-\beta_i)E(X_i - r_i)^+\right\}\right.$$
$$\left. + \{\pi_i + \pi_{0i}(1-\beta_i)\}\frac{d_i}{q_i}E(X_i - r_i)^+ + R(L_i)\frac{d_i}{q_i}\right], \tag{3.1}$$

where, X_i is the lead time demand for buyer i having a normal distribution with d_iL_i and $\sigma_i\sqrt{L_i}$ as mean and standard deviation, respectively. Shortages occur only when $X_i > r_i$ for each buyer. The expected shortage at the end of the cycle for the ith buyer is $E(X_i - r_i)^+ = \int_{r_i}^{\infty}(x_i - r_i)dF(x) = \sigma_i\sqrt{L_i}\psi(k_i)$, where, $\psi(k_i) = \phi(k_i) - k_i[1-\Phi(k_i)]$, ϕ is the standard normal probability density function, and Φ is the cumulative distribution function of a normal distribution.

According to assumption 3 and using $E(X_i - r_i)^+ = \sigma_i\sqrt{L_i}\psi(k_i)$, (3.1) becomes

$$TEC_{bi}(Q, k_i, L_i) = \left[\frac{A_{bi}D}{Q} + h_{bi}\left\{\frac{Q}{2D}d_i + k_i\sigma_i\sqrt{L_i} + (1-\beta_i)\sigma_i\sqrt{L_i}\psi(k_i)\right\}\right.$$
$$\left. + \{\pi_i + \pi_{0i}(1-\beta_i)\}\frac{D}{Q}\sigma_i\sqrt{L_i}\psi(k_i) + R(L_i)\frac{D}{Q}\right]. \tag{3.2}$$

3.2. Mathematical model for the vendor

The setup cost for the vendor per unit time is

$$\frac{A_vD}{mQ}.$$

The average inventory of the vendor is

$$\left[\left\{mQ\left(\frac{Q}{P} + (m-1)\frac{Q}{D}\right) - \frac{m^2Q^2}{2P}\right\} - \left\{\frac{Q^2}{D}(1+2+\ldots+(m-1))\right\}\right]\frac{D}{mQ}$$
$$= \frac{Q}{2}\left[m\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P}\right] \text{ (Fig. 2).}$$

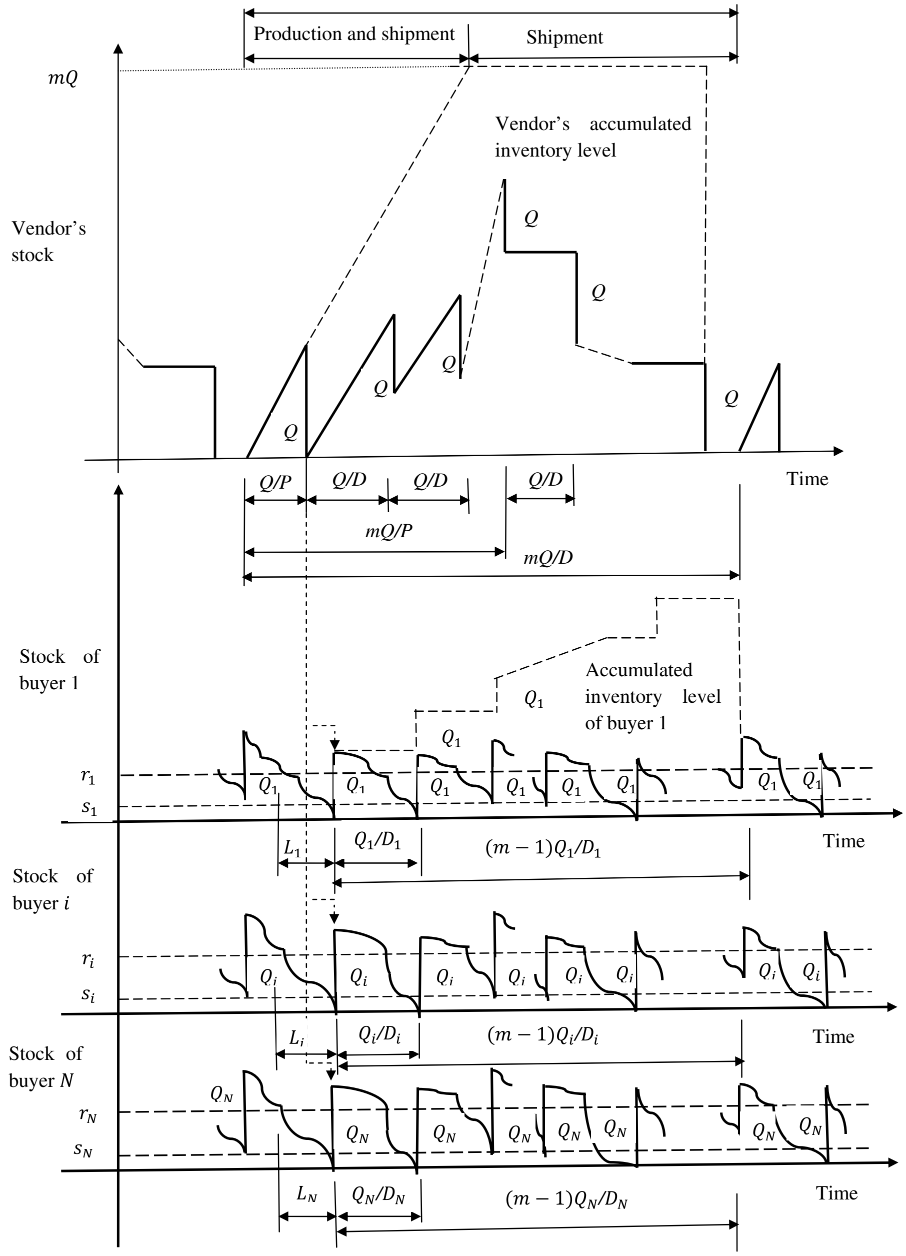

FIGURE 2. Inventory pattern for the vendor and buyers.

Therefore, the holding cost per unit time for vendor is

$$h_v \frac{Q}{2}\left[m\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right].$$

The production cost of the vendor is assumed to be a function of P. The expression of unit production cost is $C(P)=\left(\frac{a_1}{P}+a_2 P\right)$ [23]. The production rate that minimizes the unit production cost is $P^*=\sqrt{\frac{a_2}{a_1}}$. Therefore, the total expected cost of vendor is expressed as TEC_v= Setup cost+holding cost+material cost *i.e.*,

$$TEC_v(m,Q,P) = \frac{A_v D}{mQ} + \frac{Q}{2}h_v\left[m\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P}\right] + D\left(\frac{a_1}{P} + a_2 P\right). \tag{3.3}$$

In order to obtain the centralized decisions for both vendor and the buyers to minimize the entire supply chain cost, the total cost expression of both ends must be combined. Therefore, the joint total expected cost for both vendor and the buyers ($JTEC$) is obtained as follows:

$$\begin{aligned} JTEC(Q,k_i,L_i,P,m) &= \sum_{i=1}^{n}\frac{D}{Q}\left[A_{bi} + \{\pi_i + \pi_{oi}(1-\beta_i)\}\sigma_i\sqrt{L_i}\psi(k_i) + \frac{A_v}{m} + R(L_i)\right] \\ &+ \sum_{i=1}^{n} h_{bi}\left[\frac{Q}{2D}d_i + k_i\sigma_i\sqrt{L_i} + (1-\beta_i)\sigma_i\sqrt{L_i}\psi(k_i)\right] \\ &+ \frac{Q}{2}h_v\left[m\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P}\right] + D\left(\frac{a_1}{P} + a_2 P\right). \end{aligned} \tag{3.4}$$

Now, the aim is to obtain the optimal solution for all decision variables such that the joint total expected cost is minimized. The problem becomes an unconstrained minimization problem with five decision variables. Therefore, in order to obtain the optimal supply chain cost, the derivatives of the objective function are obtained with respect to all decision variables and equate them with zero. Now, according to the assumption, m is an integer and therefore, can be treated as a discrete decision variable. Thediscrete optimization technique is usedto obtain the optimal value for m [27,29]. After calculating derivatives with respect to Q, k_i, L_i, and P, we obtain

$$\begin{aligned} \frac{\partial JTEC(Q,k_i,L_i,P,m)}{\partial Q} &= \sum_{i=1}^{n}\frac{h_{bi}}{2D}d_i + \frac{h_v}{2}\left[m(1-\frac{D}{P}) - 1 + \frac{2D}{P}\right] \\ &- \sum_{i=1}^{n}\frac{D}{Q^2}\left[A_{bi} + \{\pi_i + \pi_{0i}(1-\beta_i)\}\sigma_i\sqrt{L_i}\psi(k_i)\right. \\ &\left. + A_v/m + R(L_i)\right] \\ \frac{\partial JTEC(Q,k_i,L_i,P,m)}{\partial k_i} &= \frac{D}{Q}\{\pi_i + \pi_{0i}(1-\beta_i)\}\sigma_i\sqrt{L_i}(\Phi(k_i)-1) + h_{bi}\sigma_i\sqrt{L_i} \\ &+ (1-\beta_i)\sigma_i\sqrt{L_i}(\Phi(k_i)-1) \\ \frac{\partial JTEC(Q,k_i,L_i,P,m)}{\partial L_i} &= \frac{D}{2Q}\{\pi_i + \pi_{0i}(1-\beta_i)\}\sigma_i\psi(k_i)L_i^{-1/2} - \frac{Dc_{i,r}}{Q} \\ &+ (k_i\sigma_i + (1-\beta_i)\sigma_i\psi(k_i))\frac{h_{bi}L_i^{-1/2}}{2} \\ \frac{\partial JTEC(Q,k_i,L_i,P,m)}{\partial P} &= \frac{D}{P^2}\left[\frac{Q}{2}h_v(m-2) - a_1\right] + a_2 D. \end{aligned} \tag{3.5}$$

Again, it is shown that the second-order partial derivative of the joint total cost function with respect to L_i is

$$\begin{aligned} \frac{\partial^2 JTEC(Q,k_i,L_i,P,m)}{\partial L_i^2} &= -\frac{D}{4Q}\{\pi_i + \pi_{0i}(1-\beta_i)\}\sigma_i\psi(k_i)L_i^{-3/2} \\ &- (k_i\sigma_i + (1-\beta_i)\sigma_i\psi(k_i))\frac{h_{bi}L_i^{-3/2}}{4}, \end{aligned} \tag{3.6}$$

which is a negative term for $0 < \beta_i < 1$ and positive values of all parameters and decision variables. Therefore, for fixed $Q,\ k_i,\ P$, and m, the function $JTEC(Q,k_i,L_i,P,m)$ is concave in L_i. Thus, for fixed $Q,\ k_i,\ P$, and

m, the minimum value of $JTEC(Q, k_i, L_i, P, m)$ is attained at the end point of the interval $[L_{i,j}, L_{i,j-1}]$. For a fixed positive integer m and for any fixed value of L_i, the values of Q, $\Phi(k_i)$, and P can be obtained by equating every individual equation of the system (3.5) to zero.

$$Q = \left\{ \frac{2D\{A_v/m + \sum_{i=1}^{n}(A_{bi} + [\pi_i + \pi_{0i}(1-\beta_i)]\sigma_i\sqrt{L_i}\psi(k_i) + R(L_i))\}}{\sum_{i=1}^{n} \frac{h_{bi}}{D} d_i + h_v \left[m\left(1 - \frac{D}{P}\right) - 1 + \frac{2D}{P}\right]} \right\}^{1/2} \tag{3.7}$$

$$\Phi(k_i) = 1 - \frac{h_{bi}}{\frac{D}{Q}(\pi_i + \pi_{0i}(1-\beta_i)) + (1-\beta_i)} \tag{3.8}$$

$$P = \left\{ \frac{2a_1 - Qh_v(m-2)}{2a_2} \right\}^{1/2}. \tag{3.9}$$

3.3. Solution algorithm

A closed form solution of the mathematical model is very difficult to obtain. The fixed point iteration technique is required to create a suitable algorithm in order to solve the model.

Step 1 Set $m = 1$ and input all parametric values.
Step 2 For all buyers $i = 1, 2, \ldots, n$ assign the values of all parameters and perform the following steps.
Step 3 For every combination of $L_{i,r}, r = 1, 2, \ldots, N_i$, $i = 1, 2, \ldots, n$ perform Steps 3a–3e.
Step 3a Set $k_i^{j1} = 0$ for each buyer i.
Step 3b Substitute k_i^{j1}, (i=1,2,. . . ,n) into (3.7) and evaluate Q^{j1}.
Step 3c Utilize Q^{j1} to determine the value of $\Phi(k_i^{j2})$ for each i from (3.8).
Step 3d Using the value of $\Phi(k_i^{j2})$, obtain the value of k_i^{j2} from the normal table.
Step 3e Repeat 3b to 3d until no changes occur in the values of Q^j and k_i^j and denote these values as Q^{j*} and k_i^{j*}, respectively.
Step 4 Evaluate the value of P^{j*} from (3.9) using the value of Q^{j*}.
Step 5 Denote the latest updated values of Q^j, k_i^j, and P^j as Q^{j**}, k_i^{j**}, and P^{j**}, respectively.
Step 6 Obtain $JTEC(Q^{j**}, k_i^{j**}, P^{j**}, L_{i,r}, m)$ and $Min_{j=1,2,\ldots,N_i} JATC(Q^{j**}, k_i^{j**}, P^{j**}, L_{i,r}, m)$ for all i.
Step 5 Set $m = m + 1$.
If $JTEC(Q_m^{**}, k_{im}^{**}, P_m^{**}, L_{i,m}, m) \leq JATC(Q_{m-1}^{**}, k_{m-1}^{**}, S_{m-1}^{**}, \theta_{m-1}^{**}, L_{m-1}, m-1)$, repeat Step 2, Step 3, and Step 4. Otherwise, go to Step 6.
Step 6 Set $JTEC(Q_m^{**}, k_m^{**}, S_m^{**}, \theta_m^{**}, L_m, m) = JTEC(Q_{m-1}^{**}, k_{m-1}^{**}, S_{m-1}^{**}, \theta_{m-1}^{**}, L_{m-1}, m-1)$.
Then, $(Q^{**}, k^{**}, L^{**}, S^{**}, \theta^{**}, m^{**})$ is the optimal solution and the optimal reorder point can be obtained from $R^{**} = DL^{**} + k^{**}\sigma\sqrt{L^{**}}$, where R^{**} denotes the optimal solution for R, the reorder point.

4. Numerical examples

The following parameter values are used to interpret the model numerically: $d_1 = 200$ units/week, $d_2 = 300$ units/week, $d_3 = 200$ units/week, $A_v = \$4000$/setup, $A_{b1} = \$100$/setup, $A_{b2} = \$150$/setup, $A_{b3} = \$100$/setup, $h_v = \$10$/unit/week, $h_{b1} = \$11$/unit/week, $h_{b2} = \$11$/unit/week, $h_{b3} = \$12$/unit/week, $\sigma_1 = 9$, $\sigma_2 = 10$, $\sigma_3 = 15$, $\pi_{01} = \$150$/unit, $\pi_{02} = \$140$/unit, $\pi_{03} = \$152$/unit, $\pi_1 = \$50$/unit, $\pi_2 = \$50$/unit, $\pi_3 = \$51$/unit, $a_1 = 2 \times 10^4$, and $a_2 = 0.01$. The lead time data are given below in Table 1.

From Tables 2–4, the optimal values of the decision variables are obtained at different backorder ratios. Tables 2–4 represent optimal results for β=0.0, 0.5, and 0.8, respectively. Many solutions in each table are shown for the different lead times. The decisions regarding optimal lead time are displayed in Table 5.

Therefore, it is found that minimum cost is attained at 4 weeks of lead time for every buyer and the optimal shipment is 3 for each of the three backorder ratio values.

TABLE 1. Lead time data.

Buyer i	Lead time component	Normal duration ($b_{i,r}$)	Minimum duration ($a_{i,r}$)	Unit crashing cost ($c_{i,r}$)
1	1	20	6	0.1
	2	20	6	1.2
	3	16	9	5.0
2	1	20	6	0.5
	2	16	9	1.3
	3	13	6	5.1
3	1	25	11	0.4
	2	20	6	2.5
	3	18	11	5.0

TABLE 2. Total optimal cost for $\beta_i = 0$, $i = 1, 2, 3$.

m	L_1	L_2	L_3	Q	k_1	k_2	k_3	P	$C(P)$	$JTEC$	r_1	r_2	r_3
3	3	4	4	469.943	1.789	1.765	1.755	1328.543	28.339	30 287.838	39	58	68
3	4	4	4	466.458	1.792	1.769	1.759	1329.199	28.338	30 270.744	47	58	68
3	4	3	4	469.925	1.789	1.765	1.755	1328.547	28.339	30 282.853	47	48	68
3	4	4	3	471.373	1.787	1.764	1.754	1328.274	28.339	30 276.958	47	58	57
3	4	3	3	474.778	1.784	1.760	1.751	1327.633	28.340	30 288.206	47	48	57
3	3	4	3	474.797	1.784	1.760	1.751	1327.630	28.340	30 293.196	39	58	57
3	3	3	4	473.367	1.785	1.762	1.752	1327.899	28.340	30 299.351	39	48	68
3	3	3	3	478.160	1.781	1.757	1.747	1326.996	28.341	30 303.874	39	48	57

TABLE 3. Total optimal cost for $\beta_i = 0.5$, $i = 1, 2, 3$.

m	L_1	L_2	L_3	Q	k_1	k_2	k_3	P	$C(P)$	TEC	r_1	r_2	r_3
3	3	4	4	471.905	1.561	1.541	1.525	1328.174	28.340	30153.094	36	54	61
3	4	4	4	468.487	1.565	1.544	1.529	1328.817	28.339	30131.424	43	54	61
3	4	3	4	471.879	1.561	1.541	1.525	1328.179	28.340	30148.556	43	44	61
3	4	4	3	473.274	1.560	1.539	1.524	1327.916	28.340	30146.095	43	54	51
3	4	3	3	476.604	1.556	1.536	1.520	1327.289	28.341	30162.393	43	44	51
3	3	4	3	476.631	1.556	1.536	1.520	1327.284	28.341	30166.934	36	54	51
3	3	3	4	475.255	1.558	1.537	1.522	1327.543	28.340	30169.646	36	44	61
3	3	3	3	479.920	1.553	1.532	1.517	1326.664	28.342	30182.676	36	44	51

TABLE 4. Total optimal cost for $\beta_i = 0.8$, $i = 1, 2, 3$.

m	L_1	L_2	L_3	Q	k_1	k_2	k_3	P	$C(P)$	TEC	r_1	r_2	r_3
3	3	4	4	474.268	1.322	1.308	1.283	1327.729	28.340	30016.004	32	49	54
3	4	4	4	470.930	1.326	1.312	1.287	1328.357	28.339	29989.695	39	49	54
3	4	3	4	474.234	1.322	1.308	1.283	1327.736	28.340	30011.853	39	40	54
3	4	4	3	475.563	1.320	1.306	1.281	1327.485	28.340	30012.957	39	49	45
3	4	3	3	478.804	1.317	1.302	1.277	1326.875	28.341	30034.312	39	40	45
3	3	4	3	478.840	1.317	1.302	1.277	1326.868	28.341	30038.465	32	49	45
3	3	3	4	477.529	1.318	1.304	1.279	1327.115	28.341	30037.601	32	40	54
3	3	3	3	482.040	1.313	1.299	1.273	1326.265	28.342	30059.281	32	40	45

TABLE 5. Summarization of optimal values.

β	m	L_1	L_2	L_3	Q	k_1	k_2	k_3	p	$C(p)$	TEC	r_1	r_2	r_3
0.0	3	4	4	4	470.930	1.326	1.312	1.287	1328.357	28.339	29989.695	47	58	68
0.5	3	4	4	4	468.487	1.565	1.544	1.529	1328.817	28.339	30131.424	43	54	61
0.8	3	4	4	4	470.930	1.326	1.312	1.287	1328.357	28.339	29989.695	39	49	54

TABLE 6. Sensitivity analysis for different key parameters.

Parameters	Changes(in %)	TEC^N	Parameters	Changes(in %)	TEC^N
	−50%	−0.41		−50%	−5.01
	−25%	−0.22		−25%	−2.65
A_{b1}	+25%	+0.19	A_v	+25%	+2.47
	+50%	+0.38		+50%	+4.75
	−50%	−1.23		−50%	−5.81
	−25%	−0.49		−25%	−3.00
h_{b1}	+25%	+0.57	h_v	+25%	+2.86
	+50%	+1.14		+50%	+5.70

4.1. Sensitivity analysis

Some key parameters are changed at the percentage values −50%, −25%, +25%, and +50%. Each parameter is changed one at a time while keeping the other parameters fixed. The effects of this changes of the key parameters are illustrated in Table 6.

Variations of key parameters A_{bi}, h_{b1}, A_v, and h_v are considered. For the sake of simplicity, the cost parameters of buyer 1 are taken into consideration. Observations from sensitivity analysis are described as follows:

- Vendor's cost components are more sensitive than buyer's cost components.
- Holding cost of buyer is more sensitive that ordering cost, which is true for all buyers.
- Vendor's holding cost is also more sensitive than vendor's setup cost, but the sensitivity is less than that of the buyer.

5. MANAGERIAL INSIGHTS

This article provides a two-echelon single-vendor multi-buyer supply chain model with variable production rate. A constant entity under which the managerial decisions are obtained is irrelevant in the modern marketing environment. Thus, decisions are made on the basis of many variable quantities such as order quantity, lead time, reorder point, production rate, production cost, and number of shipments. Moreover, a number of retailers are considered to imitate a real life scenario for obtaining realistic managerial decisions. The managerial insights of this chapter are furnished point-wise as follows:

- The managerial decisions are made under a variable production rate. This assumption is more realistic than a fixed production rate.
- Production cost is also considered as variable and a special type of cost function is assumed to obtain more generalized decisions than with a fixed production cost.
- Manager can reduce lead time and enhance customer service by incurring a lead time crashing cost.

6. CONCLUDING REMARKS

This study proposed a single-vendor multi-buyer supply chain model. Variable lead time was considered at the buyer's end. The lead time demand was assumed to follow a normal distribution. The vendor's production rate was considered as a variable rather than as a fixed entity. Moreover, the unit production cost was also considered as a variable [23] that was dependent on the production rate, and a special type of function was considered to establish the relation between the production rate and the unit production cost. At the end of production, the finished goods were delivered to a number of buyers through a multiple delivery policy. The optimal decision variables were obtained for different backorder ratios combined with various lead times. It was observed that the value of the chain increased with increasing backorder ratio and the lowest cost was attained at four weeks of lead time. There are many possible extensions and further research directions in this model. One suitable direction for extension is to incorporate a multi-echelon and multi-product supply chain network. Another avenue of extension is to reduce setup and ordering costs for vendor and buyers, respectively. To maintain the quality of products, the model can be extended by considering by quality improvement by continuous investment as in Sarkar *et al.* (2017) or as in Huang *et al.* (2011).

References

[1] K. Annadurai and R. Uthayakumar, Reducing lost-sales rate in *(T,R,L)* inventory model with controllable lead time. *Appl. Math. Modell.* **34** (2010) 3465−3477.

[2] A.Banerjee, A joint economic-lot-size model for purchaser and vendor. *Decision Sci.* **17** (1986) 292−311.

[3] A. Banerjee and S. Banerjee, A coordinated order-up-to inventory control policy for a single supplier and multiple buyers using electronic data interchange. *Int. J. Prod. Econom.* **35** (1994) 85−91.

[4] A. Banerjee and J.S. Burton, Coordinated versus independent inventory replenishment policies for a vendor and multiple buyers. *Int. J. Prod. Econom.* **35** (1994) 215−222.

[5] M. Ben-Daya and A. Raouf, Inventory models involving lead time as a decision variable. *J. Oper. Res. Soc.* **45** (1994) 579−582.

[6] L.E. Cárdenas-Barrón, K.J. Chung and G. Treviño-Garza, Celebrating a century of the economic order quantity model in honor of Ford Whitman Harris. *Int. J. Prod. Econom.* **155** (2014) 1−7.

[7] L.E. Cárdenas-Barrón and S.S. Sana, A production-inventory model for a two-echelon supply chain when demand is dependent on sales teams' initiatives. *Int. J. Prod. Econom.* **155** (2014) 249−258.

[8] L.E. Cárdenas-Barrón and G. Treviño-Garza, An optimal solution to a three echelon supply chain network with multi-product and multi-period. *Appl. Math. Modell.* **38** (2014) 1911−1918.

[9] L.E. Cárdenas-Barrón and S.S. Sana, Multi-item EOQ inventory model in a two-layer supply chain while demand varies with a promotional effort. *Appl. Math. Modell.* **39** (2015) 6725−6737.

[10] H.J. Chang, R.H. Su, C.T. Yang and M.W. Weng, An economic manufacturing quantity model for a two-stage assembly system with imperfect processes and variable production rate. *Comput. Industrial Eng.* **63** (2012) 285−293.

[11] C.J. Chung, An easy method to derive the integrated vendor-buyer production-inventory model with backordering using cost-difference rate comparison approach. *Math. Comput. Modell.* **57** (2013) 632−640.

[12] C.J. Conrad and H. McClamrock, The drilling problem: a stochastic modeling and control example in manufacturing. *IEEE Tran. Auto. Cont.* **32** (1987) 947−958.

[13] A. Gholami-Qadikolaei, A. Mirzazadeh and R. Tavakkoli-Moghaddam, Lead time and ordering cost reductions in budget and storage space restricted probabilistic inventory models with imperfect items. *RAIRO: OR* **49** (2015) 215−242.

[14] B.C. Giri and T. Dohi, Computational aspects of an extended EMQ model with variable production rate. *Comput. Oper. Res.* **32** (2005) 3143−3161.

[15] C.H. Glock and T. Kim, The effect of forward integration on a single-vendor multi-retailer supply chain under retailer competition. *Int. J. Prod. Econom.* **164** (2015) 179−192.

[16] S.K. Goyal, An integrated inventory model for a single supplier-single customer problem. *Int. J. Prod. Res.* **15** (1976) 107−111.

[17] S.K. Goyal, Economic ordering policy for deteriorating items over an infinite time horizon. *Eur. J. Oper. Res.* **28** (1987) 298−301.

[18] S.K. Goyal, A joint economic-lot-size model for purchaser and vendor: a comment. *Deci. Sci.* **19** (1988), 236−241.

[19] R. Guan and X. Zhao, Pricing and inventory management in a system with multiple competing retailers under (r, Q) policies. *Comput. Oper. Res.* **38** (2011) 1294–1304.

[20] D. Ha and S.L. Kim, Implementation of JIT purchasing: an integrated approach. *Prod. Plan. Cont.* **8** (1997) 152−157.

[21] M.A. Hoque, Synchronization in the single-manufacturer multi-buyer integrated inventory supply chain. *Eur. J. Oper. Res.* **188** (2008) 811−825.

[22] J.K. Jha and K. Shanker, Two-echelon supply chain inventory model with controllable lead time and service level constraint. *Comput. Industrial Eng.* **57** (2009) 1096−1104.

[23] M. Khouja and A. Mehrez, Economic production lot size model with variable production rate and imperfect quality. *J. Oper. Res. Soc.* **45** (1994) 1405−1417.

[24] C.J. Liao and C.H. Shyu, An analytical determination of lead time with normal demand. *Int. J. Oper. Prod. Manag.* **11** (1991) 72−78.

[25] I. Moon, E. Sin and B. Sarkar, Min-max distribution free continuous review model with a service level constraint and variable lead time, *Appl. Math. Comput.* **229** (2014) 310−315.

[26] L.Y. Ouyang, N.C. Yeh and K.S. Wu, Mixture inventory model with backorders and lost sales for variable lead time. *J. Oper. Res. Soc.* **47** (1996) 829−832.

[27] L.Y. Ouyang, K.S. Wu and C.H. Ho, Integrated vendor-buyer cooperative models with stochastic demand in controllable lead time. *Int. J. Prod. Econom.* **92** (2004) 255−266.

[28] J.C.H. Pan and J.S. Yang, A study of an integrated inventory with controllable lead time. *Int. J. Prod. Res.* **40** (2002) 1263−1273.

[29] B. Sarkar, A production-inventory model with probabilistic deterioration in two-echelon supply chain management. *Appl. Math. Modell.* **37** (2013) 3138−3151.

[30] B. Sarkar and A. Majumder, Integrated vendor-buyer supply chain model with vendor's setup cost reduction. *Appl. Math. Comput.* **224** (2013) 362−371.

[31] B. Sarkar and I. Moon, Improved quality, setup cost reduction, and variable backorder costs in an imperfect production process. *Int. J. Prod. Econom.* **155** (2014) 2014−2013.

[32] B. Sarkar, L.E. Cárdenas-Barrón, M. Sarkar and M.L. Singgih, An economic production quantity model with random defective rate, rework process and backorders for a single stage production system. *J. Manufact. Syst.* **33** (2014) 423−435.

[33] B. Sarkar and A.S. Mahapatra, Periodic review fuzzy inventory models with variable lead time and fuzzy demand. *Int. Tran. Oper. Res.* **24** (2017) 1197-1227.

[34] B. Sarkar, K.S. Chaudhuri and I. Moon, Manufacturing setup cost reduction and quality improvement for the distribution free continuous-review inventory model with a service level. *J. Manufact. Syst.* **34** (2015) 74−82.

[35] B. Sarkar, B. Mondal and S. Sarkar, Quality improvement and backorder price discount under controllable lead time in an inventory model. *J. Manufact. Syst.* **35** (2015) 26−36.

[36] B. Sarkar, Supply chain coordination with variable backorder, inspections, and discount policy for fixed lifetime products. *Math. Prob. Eng.* **2016** (2016) 6318737.

[37] B. Sarkar, A. Majumder, M. Sarkar, B.K. Dey and G. Roy, Two-echelon supply chain model with manufacturing quality improvement and setup cost reduction. *J. Indus. Manag. Optim.* **13** (2017) 1085−1104.

[38] S.P. Sarmah, D. Acharya and S.K. Goyal, Coordination of a single-manufacturer/multi-buyer supply chain with credit option. *Int. J. Prod. Econom.* **111** (2008) 676−685.

[39] D. Shin, R. Guchhait, B. Sarkar and M. Mittal, Controllable lead time, service level constraint, and transportation discounts in a continuous review inventory model. *RAIRO-Oper. Res.* **50** (2015) 921−934.

[40] H.N. Soni and K.A. Patel, Optimal strategy for an integrated inventory system involving variable production and defective items under retailer partial trade credit policy. *Deci. Supp. Syst.* **54** (2012) 235−247.

[41] P.S. You, Optimal times of price reductions for an inventory model with partial backorder and vertical shift demand. *RAIRO: OR* **41** (2007) 35−47.