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A NEW HYBRID DECISION MAKING SYSTEM FOR SUPPLIER SELECTION

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Abstract. The objective of this paper is to develop a hybrid decision making system using Data Envelopment Analysis (DEA) and linguistic fuzzy models for selecting the best supplier in the presence of multiple decision makers. In this hybrid system, first the weights of selected criteria are obtained from each of the decision makers as linguistic fuzzy numbers within the framework of group decision making. Then, to select the best supplier, absolute weight restriction (AWR) model is incorporated into the DEA model. A real case study demonstrates the application of the model.

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1. Introduction

Suppliers are considered as part of the value chain in any organization. On average, in manufacturing factories, the costs of buying raw materials and services constitute up to 70% of the total cost of the products and in high-technology firms purchased materials and services represent up to 80% [28]. In addition, quality of services and time of supplier delivery create another level of decision-making complexity regarding out-sourcing and selecting appropriate supplier. Selection of an appropriate supplier clearly decreases purchase costs and improves the cooperation between the buyer and supplier. Thus, purposeful selection of an appropriate supplier is one of the most important decisions at the organization level because, aside from meeting operational needs of the organization, suppliers are considered as part of the executors of organization's strategic goals [20].

Most decision-makers and experts select their needed suppliers on the basis of personal experiences or intuition. However, these methods are completely subjective and many previous studies have mentioned their weaknesses [64]. Therefore, in order to systematize these decisions, many quantitative methods and mathematical models have been created for selection of appropriate supplier in the organizations among which Multi Criteria Decision Making (MCDM) and Data Envelopment Analysis (DEA) can be mentioned. At the same time, since new concepts like just in time (JIT) production in industries have been developed, the organizations' emphasis has been pointed more toward simultaneous use of quantitative and qualitative data in supplier selection process. Therefore, using fuzzy logic for solving the problem of supplier selection is rapidly expanding.

On the other hand, nowadays, considering the advantages of group decision making, supplier selection in different industries is regarded as a group decision making process. This means that various levels and groups

Keywords. Supplier selection, group decision making, linguistic fuzzy number, DEA, absolute weight restriction model.

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of the organization participate in this process. A decision making group can consist of the personnel, experts, or managers of units such as research and development, engineering, quality assurance, purchasing, *etc*. In this situation, each of the decision maker groups can independently express their opinion on the various features of the suppliers in the process of supplier evaluation and selection.

The objective of this paper is to develop a hybrid decision making system using DEA technique and fuzzy models to provide the possibility of selecting the best supplier in a situation where several decision makers are involved in selection process.

This paper proceeds as follows. In Section 2, literature review is presented. In Section 3, the theoretical basis and primary definitions are provided. The framework of proposed model is presented in Section 4. Numerical example is given in Section 5. Finally, in Section 6, concluding remarks are discussed.

2. Literature review

So far, various methodologies have been proposed for solving the problem of supplier selection. These quantitative approaches are designed on the basis of numerous models and techniques which can be roughly classified as below:

1. Multiple Attribute Decision Making (MADM) models.
2. Multiple Objective Decision Making (MODM) models.
3. DEA.
4. Artificial Intelligence (AI).
5. Hybrid or Integrated Approaches.

Each of these classifications includes various methods and techniques. Samples of these techniques together with concerned authors are briefly presented in Table 1.

Techniques used in the first category are generally used for selecting the best alternative [1]. Different techniques and approaches which are utilized in this category include Analytic Hierarchy Process (AHP), Analytic Network Process (ANP), Multiple Attribute Utility Theory (MAUT), and Technique for Order Preference by Similarity to Ideal Solution (TOPSIS). Nowadays, AHP is extensively used for solving the problem of supplier selection [59]. Ghodsypour and O'Brien [27] used AHP for supplier ranking. As it was mentioned before, the criteria of supplier selection are interrelated and dependent upon each other in real world. Therefore, traditional methods like AHP cannot be appropriately utilized from this perspective. In this regard, Saaty [60] proposed ANP method for solving this problem which is more advanced than AHP.

Ustun and Demirtas [68] integrated the ANP and multi-objective mixed ILP (integer linear programming). Lee and Kim [46] presented a methodology using ANP and zero one goal programming (ZOGP) for selecting information systems that have multiple criteria and interdependency among criteria. Lee and Kim [47] described an integrated approach of interdependent information system project selection using Delphi method, ANP, and goal programming (GP). Hajeeh and Al-Othman [31] used AHP to select the most appropriate technology for seawater desalination.

Bross and Zhao [6] proved that MAUT is an appropriate and useful method for formulating sustainable sourcing strategies. In addition, MAUT is a strong method for solving problems with multiple conflictions attributes. In any case, this method is employed for solving the problem of international supplier selection in environments associated with high complexities and risk [6]. TOPSIS technique is one of the well-known classical techniques for MADM problems. This technique selects the best choice on the basis of minimizing the distance of alternative with Positive Ideal Solution (PIS) and maximizing the distance with Negative Ideal Solution (NIS).

Techniques used in the second category are used for designing and optimizing global utility function for decision making. This utility function is objectively calculated and optimized in some evaluation methods, and implicitly investigated and optimized in some other [1]. Techniques such as Linear Programming (LP), GP, and Non-Linear Programming (NLP) can be placed in this category. Yurdakul [75] introduced a combined model of the AHP and GP, to consider multiple objectives and constraints, simultaneously. Çebi and Bayraktar [7] proposed an integrated model for supplier selection. In their model, the supplier selection problem was structured as an integrated Lexicographic Goal Programming (LGP) and AHP model including both quantitative and qualitative conflicting factors. Kumar *et al.* [43] applied a fuzzy GP approach. To incorporate the imprecise aspiration levels of the goals, they formulated a vendor selection problem as a fuzzy mixed integer goal programming that includes three primary goals: minimizing the net cost, minimizing the net rejections, and minimizing the net late deliveries subject to realistic constraints regarding buyer's demand, vendor's capacity, vendor's quota flexibility, purchasing value of items, budget allocation to individual vendor, *etc.*

The third category belongs to one of the popular supplier evaluation and selection methods, *i.e.* DEA. DEA is a non-parametric method invented by Charnes *et al.* [8] on the basis of LP and for evaluating the relative efficiency of Decision Making Units (DMU). DEA is used in three stages for evaluating efficiency. This first stage is related to identification and determination of appropriate DMUs. Then, the inputs and outputs should be selected for evaluating relative efficiency of DMUs. In the last stage, DEA model is used for analyzing data [29]. Narasimhan *et al.* [51] proposed a supplier evaluation method using DEA combined with a weighted model to categorize suppliers into four performance clusters: HE (high performance and efficient), HI (high performance and inefficient), LE (low performance and efficient), and LI (low performance and inefficient). Talluri and Narasimhan [65] proposed an objective framework for effective supplier sourcing using DEA, which considers multiple strategic and operational factors in the evaluation process. Suppliers are categorized into groups based on performance, which assists managers in identifying candidates for strategic long-term partnerships, supplier development programs, and pruning. On the other hand, in many real applications, the input and output variables cannot be exactly measured. Thus, several approaches have been proposed to deal with imprecise data. Guo and Tanaka [30] proposed fuzzy DEA model and an extension of fuzzy DEA model by considering relationship between DEA and regression analysis with fuzzy input and output data. Zerafat Angiz *et al.* [49] introduced concept of "local α-level" for measuring efficiency of DMUs under uncertainty. They proposed a model that can include some uncertainty information from the intervals within the α-cut approach. Hatami-Marbini *et al.* [33] reviewed fuzzy DEA papers over the past 20 years. They presented a classification scheme for fuzzy DEA methods. Azadi *et al.* [2] developed an integrated DEA model in fuzzy context to select the best sustainable suppliers.

Today, AI is used in different sections of science. One area in which AI is used is designing decision-making models. Thus, the fourth category is devoted to this issue. Techniques such as Artificial Neural Networks (ANN), Genetic Algorithm (GA), Expert Systems, and Fuzzy Set Theory (FST) are among the techniques included in this category. Kumar *et al.* [43] employed fuzzy goal programming for solving the problem of supplier selection with multiple objectives. Chen *et al.* [9] presented a hierarchical model on the basis of fuzzy sets theory for supplier selection problem. Lee [45] proposed an analytical approach to select suppliers under a fuzzy environment. This approach incorporates the benefits, opportunities, costs and risks (BOCR) to evaluate various aspects of suppliers.

In recent years, researchers have used hybrid methods for supplier selection and evaluation. These methods enjoy from the advantages of each of the integrated techniques and at the same time remove their weaknesses. These methods are placed in the fifth category. Integration of AHP and LP, AHP and GP, and AHP and DEA are instances of these methods.

However, selection of appropriate criteria for supplier evaluation is an issue which should be considered. Reviewing studies conducted in this regard reveals that the criterion of cost was used as the main criteria in the late 1970s and early 1980s. In the early 1990s, the criteria of production cycle time and customer responsiveness were added to the cost criteria and in the late 1990s, the criterion of flexibility was also taken into account. Finally, in recent years, environmental safety criteria were also considered as a key issue in the industry [36]. Dickson [14] proposed 23 different criteria for evaluation and selection of the appropriate suppliers. Weber *et al.* [74] reviewed 74 papers published since 1966 on the issues of supplier selection. They showed in their study that from among the criteria proposed in these papers as well as the study conducted by Dickson in 1996, 7 criteria are more important. The criteria are quality, cost, on time delivery, production facility,

TABLE 1. Classification of methods and techniques of supplier selection.

Authors	Technique	Category	No.
Kahraman *et al.* [41], Hou and Su [35], Jaganathan *et al.* [37]	AHP	MADM models	1
Talluri and Sarkis [67], Bayazit [4], Gencer and Gurpinar [26]	ANP		
Braglia and Petroni [5], Bross and Zhao [6]	MAUT		
Chen *et al.* [9], Wang *et al.* [72]	TOPSIS		
Talluri and Narasimhan [66], Ng [52]	LP	MODM models	2
Karpak *et al.* [42], Hajidimitriou and Georgiou [32]	GP		
Ghodsypour and O'Brien [28], Hong and Hayya [34]	NLP		
Weber [73], Liu *et al.* [48], Narasimhan *et al.* [51], Talluri and Narasimhan [65], Garfamy [25], Farzipoor Saen [18], Ross *et al.* [58], Farzipoor Saen and Zohrehbandian [23], Farzipoor Saen [21], Jassbi *et al.* [40]	DEA	DEA	3
Guo and Tanaka [30], Zerafat Angiz *et al.* [49], Hatami-Marbini *et al.* [33], Azadi *et al.* [2]	Fuzzy DEA		
Choy *et al.* [10], Bowersox *et al.* (2003)	ANN	Artificial intelligence	4
Ding *et al.* [15]	GA		
Vokurka *et al.* [70], Kwong *et al.* [44]	Expert System		
Chen *et al.* [9], Sarkar and Mohapatra [61], Florez-Lopez [24], Kumar *et al.* [43]	FST		
Ghodsypour and O'Brien [27]	AHP-LP	Integrated approach	5
Mendoza *et al.* [50], Wang *et al.* [71],	AHP-GP		
Sevkli *et al.* [63], Farzipoor Saen [19], Ramanathan [55]	AHP-DEA		

production capacity, technical capability, and geographical location. Dahel [12] introduces cost, quality, on time delivery, and supplier capacity as the criteria which are more practical for evaluation of suppliers. Demirates and Üstün [13] introduced 14 different criteria under the indices of Benefits, Opportunities, Costs, and Risks (BOCR). In summary, selection of supplier evaluation criteria depends upon the type of product and problem condition. Therefore, it is suggested to consider appropriate criteria regarding the above-mentioned points for each problem.

As it can be inferred from this brief review, so far various models have been designed and proposed for solving the problem of suppler selection. Also, to the best of knowledge of the authors, there is no model using the combination of fuzzy intersection and union in group decision making and DEA technique for solving the problem of supplier selection.

In summary, this paper has following contributions:

- The proposed model utilizes fuzzy union with fuzzy intersection for integrating the opinions of decision-makers.
- For the first time, the proposed model utilizes a new method for defining Quasi-Gaussian fuzzy number for describing fuzzy linguistic variables.
- The proposed model is a hybrid decision making system in which the intersection and union of the viewpoints of decision-makers as the acceptable range of each criterion is calculated. Then, using the obtained ranges, GP technique, and method proposed in the paper, absolute weight restriction (AWR) model of DEA technique is solved for evaluating suppliers.
- In this paper, a model for obviating the problem of non-relativity of efficiencies in the presence of absolute weight restriction is proposed[3].

[3] Non-relativity of efficiencies occurs when at least one of the DMUs has not relative efficiency equal to 1.

- In using GP technique for determination of the range of appropriate weights, proposed model is designed in a way that these ranges are always within the limits of decision-makers' viewpoints intersection and at the same time does not fall out of the boundaries of these viewpoints' union.

3. Theoretical foundations of DEA and primary definitions

Charnes *et al.* [8], for the first time, proposed CCR (Charnes-Cooper-Rhodes) model. Later, this model was developed by Banker *et al.* [3] and was called BCC (Banker–Charnes–Cooper) model.

3.1. Absolute weight restriction (AWR) model in DEA

As Farzipoor Saen [22] discussed, a crucial weakness of DEA is the lack of decision maker's opinion in DEA calculations. This causes DMUs to get artificially high efficiency scores by assigning unsuitable input and output weights.

One of the well-known methods for imposing weights in DEA is weight restriction method. The idea of incorporating weights in DEA calculations was first proposed in the context of bounds on factor weights in multiplier side. This led to the development of cone-ratio and assurance region models [11].

Several types of weight restrictions have been proposed in the DEA literature. In this research, we focus on the absolute weight restriction (AWR) in DEA models. To incorporate AWR into DEA, following restrictions should be added to the model [16,56,57].

$$\delta_i \leqslant v_i \leqslant \tau_i, \qquad \rho_r \leqslant u_r \leqslant \eta_r, \tag{3.1}$$

where Greek letters $(\delta_i, \tau_i, \rho_r, \eta_r)$ are upper and lower bounds determined by the decision makers.

Generally, the weight restrictions of (3.1) are changed in the following form:

$$A_t U \leqslant b_t, t = 1, 2, \ldots, l_1 \text{ and } C_h V \leqslant d_h, h = 1, 2, \ldots, l_2, \tag{3.2}$$

where A_t and C_h are respectively, $1 \times s$ and $1 \times m$ vectors. $b_t \in R$ and $d_h \in R$.

If the weight restrictions of (3.2) are added in the CCR model, the resulting model can be defined as below [39]:

$$\begin{array}{ll} \text{Max} \sum\limits_{r=1}^{s} u_r y_{rp} & \\ s.t.: \sum\limits_{i=1}^{m} v_i x_{ip} = 1, & \\ \quad \sum\limits_{r=1}^{s} u_r y_{rj} - \sum\limits_{i=1}^{m} v_i x_{ij} \leqslant 0 & j = 1, \ldots, n, \\ \quad A_t U \leqslant b_t & t = 1, 2, \ldots, l_1, \\ \quad C_h V \leqslant d_h & h = 1, 2, \ldots, l_2, \\ \quad u_r, v_i \geqslant 0 & r = 1, 2, \ldots, s, \quad i = 1, 2, \ldots, m. \end{array} \tag{3.3}$$

The important point is that assigning limits is not totally free and it should be noticed if the problem is feasible.

3.2. A feasible interval for weights in DEA

In AWR model, when the ranges of weights assigned by the decision-maker are added to the classical DEA model, the model, in some instances, has infeasible solution. In this case, a minor change in the range added to the model can make the problem feasible. To prevent dissatisfaction of decision makers, this change should be minimized. To this end, Jahanshahloo *et al.* [39] proposed a model which using GP technique and big-M method.

If b_t and d_h in Model (3.3) are respectively considered as the objectives of $A_tU \leqslant b_t$ and $C_hV \leqslant d_h$ restrictions, defining deviation variables, will be:

$$\begin{aligned} A_tU + n_t - p_t = b_t, \quad & t = 1,2,\ldots,l_1,\\ C_hV + n'_h - p'_h = d_h, \quad & h = 1,2,\ldots,l_2, \end{aligned} \tag{3.4}$$

where n_t, p_t $(t = 1,2,\ldots,l_1)$ and n'_h, p'_h $(h = 1,2,\ldots,l_2)$ are the deviation variables corresponding to weight restrictions in (3.3).

To have a feasible model, we may to alter b_t and d_h in the constraints (3.2). In other word, if imposing constraints (3.2) into the model do not destroy its feasibility, no alteration is necessary. But if model becomes infeasible, then a penalty should be imposed in order to have minimum deviation from the considered goals. To this end, p_t and p'_h in (3.4) should be minimized. Therefore, the Model (3.3) should be modified in the following form:

$$\begin{aligned} \text{Max} \quad & Z_p = \sum_{r=1}^{s} u_r y_{rp} - M\left(\sum_{t=1}^{l_1}(p_t) + \sum_{h=1}^{l_2}(p'_h)\right)\\ s.t. : & \sum_{i=1}^{m} v_i x_{ip} = 1,\\ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leqslant 0 \quad j = 1,\ldots,n,\\ & A_tU + n_t - p_t = b_t, \qquad t = 1,2,\ldots,l_1,\\ & C_hV + n'_h - p'_h = d_h, \qquad h = 1,2,\ldots,l_2,\\ & u_r, v_i \geqslant 0, \qquad r = 1,2,\ldots,s, \quad i = 1,2,\ldots,m,\\ & n_t, p_t, n'_h, p'_h \geqslant 0, \qquad t = 1,2,\ldots,l_1, \quad h = 1,2,\ldots,l_2, \end{aligned} \tag{3.5}$$

where M is a very big positive number. The Model (3.5) is always feasible [39].

3.3. l_1 norm method for ranking efficient units

The relative efficiency of DMUs is measured through DEA technique and in case the relative efficiency of a DMU is equal to 1, that DMU is selected as the most efficient unit. However, in some cases, there are more than one efficient DMU with relative efficiency of 1. In these conditions, there are different ranking methods that can be used for determining the efficient unit. Jahanshahloo *et al.* [38] proposed l_1-norm method for ranking efficient units. They discussed that their model is always feasible. Assuming constant return to scale, the model is given as follows [38]:

$$\begin{aligned} \text{Min} \quad & \Gamma_c^o(X,Y) = \sum_{i=1}^{m} x_i - \sum_{r=1}^{s} y_r + \alpha\\ s.t. \quad & \sum_{j=1, j\neq o}^{n} \lambda_j x_{ij} \leqslant x_i \quad i = 1,\ldots,m,\\ & \sum_{j=1, j\neq o}^{n} \lambda_j y_{rj} \geqslant y_r \quad r = 1,\ldots,s,\\ & x_i \geqslant x_{io} \qquad i = 1,\ldots,m,\\ & 0 \leqslant y_r \leqslant y_{ro} \qquad r = 1,\ldots,s,\\ & \lambda_j \geqslant 0 \qquad j = 1,\ldots,n, \quad j \neq 0, \end{aligned} \tag{3.6}$$

where $\alpha = \sum_{r=1}^{s} y_{ro} - \sum_{i=1}^{m} x_{io}$ and $\lambda = (\lambda_1, \ldots, \lambda_{o-1}, \lambda_{o+1}, \ldots, \lambda_n)$ are non-negative vectors of variables (envelopment form), α is the constant, and $\Gamma_c^o(X,Y)$ is the distance of (X_o, Y_o) from (X,Y) by using l_1-norm.

4. Proposed model

Based on previous discussions, the process of supplier selection is proposed as follows:

4.1. Determine important criteria for supplier selection.
4.2. Derive the weight of each selected criteria.
4.3. Calculate the relative efficiency of suppliers.
4.4. Re-determine the weights of criteria for obviating non-relativity of efficiencies among suppliers (if necessary).
4.5. Rank suppliers with relative efficiency of 1 (if necessary).
4.6. Review the weights of criteria and re-evaluate the suppliers (if necessary).

The first step is to determine the essential criteria for supplier evaluation. Identifying important and applicable criteria is necessary for a rational and impartial selection. In the second step, every decision maker determines the weights of the selected criteria through fuzzy linguistic variables. Then, the union and intersection interval of these viewpoints is obtained and Model (4.2) is solved in order to obtain the final interval of the weights of criteria. In the third step, after determination of weights interval, the relative efficiency of each supplier is calculated so that the best of them is selected on the basis of relative efficiency. If the efficiency obtained for the suppliers is not relative (none of the efficiencies is equal to 1) the fourth step is taken. In this step, the weights of the criteria are again calculated by Model (4.4) so that first, the weights obtained are within the acceptance limits of decision makers and second, at least one of the suppliers have the relative efficiency of 1. If more than one supplier with relative efficiency of 1 is selected as the results of steps 3 and 4, the fifth step is executed. In this case, l_1-norm method is utilized for ranking the efficient units in order to determine the superior supplier. Finally, if incorporating criteria weights into Model (4.2) does not make feasible solution, the sixth step is taken and through analyzing the data and determining the reason of their incidence, the weights will be reviewed by the decision makers. In summary, the above mentioned steps are shown in Figure 1. At this juncture, the steps are discussed in more details.

4.1. Determining important criteria

In order to make a rational and correct decision in the process of evaluating and selecting suppliers, many criteria should be considered carefully. Dickson [14] and Weber *et al.* [74] suggested some guidelines for choosing appropriate criteria in supplier selection problems.

In this study, four criteria are selected by experts:

- Price.
- Product quality.
- On-time delivery.
- Environmental factors.

4.2. Deriving the weight of selected criteria

The next step is to specify the weights of the criteria which were chosen in the first step. Therefore, appropriate weighting system should be designed to assign the weights to the criteria. These weights should be integrated into the model to conclude the final weight of each criterion.

4.2.1. Definition of the linguistic variables

In proposed model, linguistic variables with 5 fuzzy linguistic terms having quasi-Gaussian membership function are used for determination of the selected criteria because:

1. Gaussian and quasi-Gaussian membership functions are closer to human behavior.

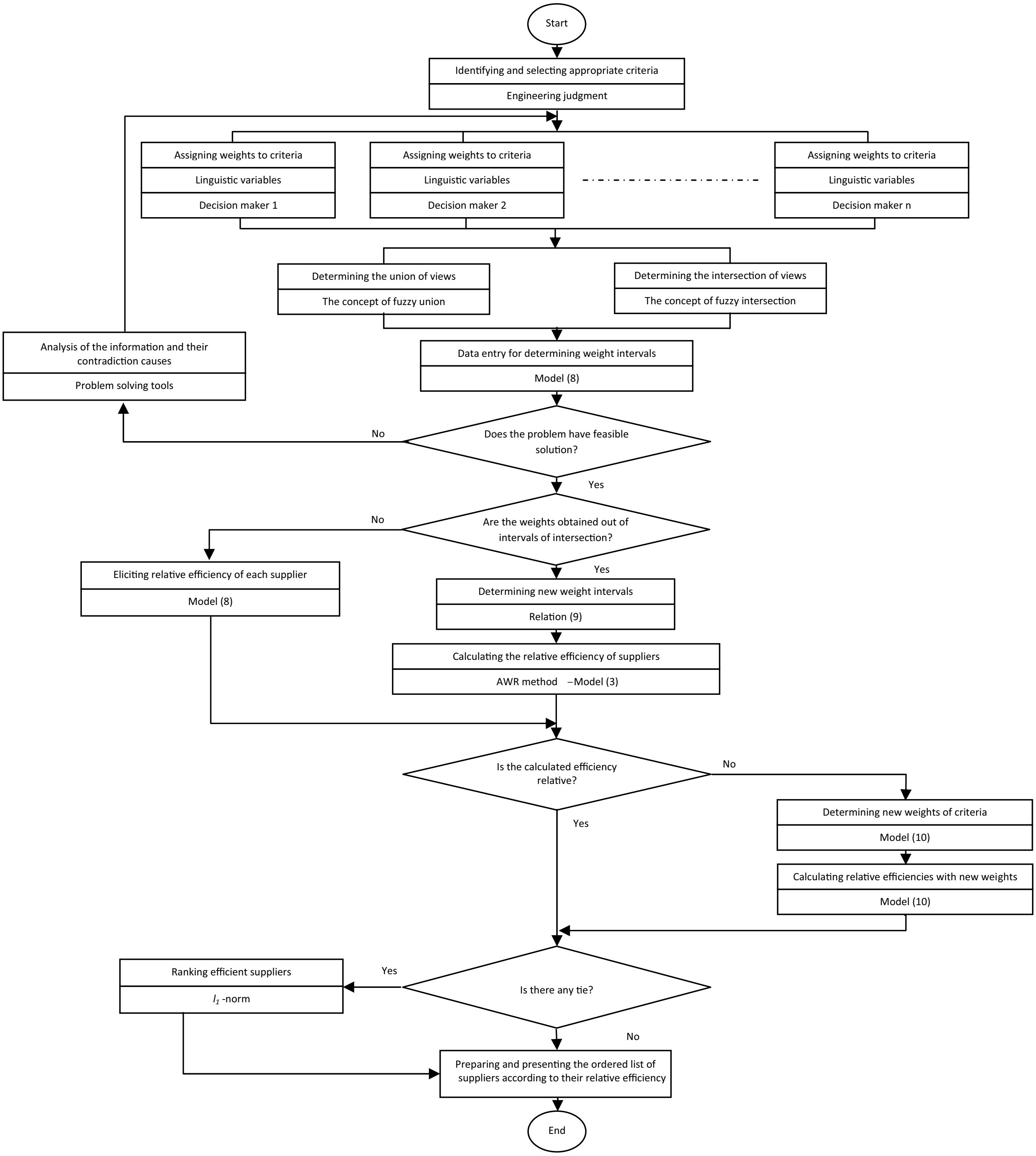

Figure 1. Proposed hybrid model.

TABLE 2. Collecting the viewpoints of the experts regarding linguistic terms.

Experts	Criterion	Numerical range of importance										Linguistic terms of the selected range				
		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	VL	L	M	H	VH
Expert 1	Price															✓
Expert 2	Price															✓
Expert 3	Price															✓

TABLE 3. Frequency of viewpoints collected from the experts and their membership degrees.

	Frequencies					Membership degree				
Importance range	VH	H	M	L	VL	VH	H	M	L	VL
0	0	0	0	1	5	0.00	0.00	0.00	0.03	1.00
0.1	0	0	0	3	5	0.00	0.00	0.00	0.10	1.00
0.2	0	0	0	22	4	0.00	0.00	0.04	0.73	0.80
0.3	0	0	5	30	2	0.00	0.00	0.17	1.00	0.40
0.4	0	0	38	25	0	0.00	0.00	0.69	0.83	0.00
0.5	0	20	48	7	0	0.00	0.26	1.00	0.23	0.00
0.6	4	56	46	0	0	0.10	0.73	0.85	0.00	0.00
0.7	25	77	22	0	0	0.63	1.00	0.58	0.00	0.00
0.8	38	57	3	0	0	0.95	0.74	0.06	0.00	0.00
0.9	40	3	0	0	0	1.00	0.04	0.00	0.00	0.00
1	40	1	0	0	0	1.00	0.01	0.00	0.00	0.00

2. Triangular or trapezoidal membership functions reflect only 3 and 4 points of given interval, respectively, and other points are not considered.
3. Adjusting Gaussian and quasi-Gaussian membership functions with reality are easily attained by changing mean and variance of membership function [54].

The linguistic terms used in this study for stating the importance of the selected criteria are as below:

Very low	low	Middle	High	Very high
VL	L	M	H	VH

To determine the shape and range of each linguistic term, a questionnaire was designed. Then the opinion of each expert regarding the importance of selected criteria was collected in the interval of [0–1]. Then, they were asked to mark the equivalent of the linguistic term selected. In judgmental sampling method, opinions of experts and decision makers are obtained to determine the importance and priority of different criteria. For more details on judgmental sampling method (see Sekaran [62]). Table 2 presents the viewpoints of 3 experts regarding the criterion of price as an example.

After obtaining the viewpoints of 100 experts and directors, the data of the questionnaires were derived and the frequency table of each defined linguistic term was prepared. Then, the frequencies should be normalized by linear normalization method (4.1). This method can be used in all linear equations.

$$n_{ij} = \frac{c_{ij}}{c_j^*} \quad \text{with} \quad c_j^* = \underset{j}{\text{Max}}\ c_{ij} \tag{4.1}$$

where c_{ij} is the frequency of the ith importance range related to jth term.

By normalizing the frequencies, the membership degree of each element is obtained. The results are presented in Table 3.

For determining the shape of the membership function of each linguistic variable, the Gaussian membership function is fitted to these data using Matlab 7.5 software. Figure 2 shows the membership function of each linguistic variable.

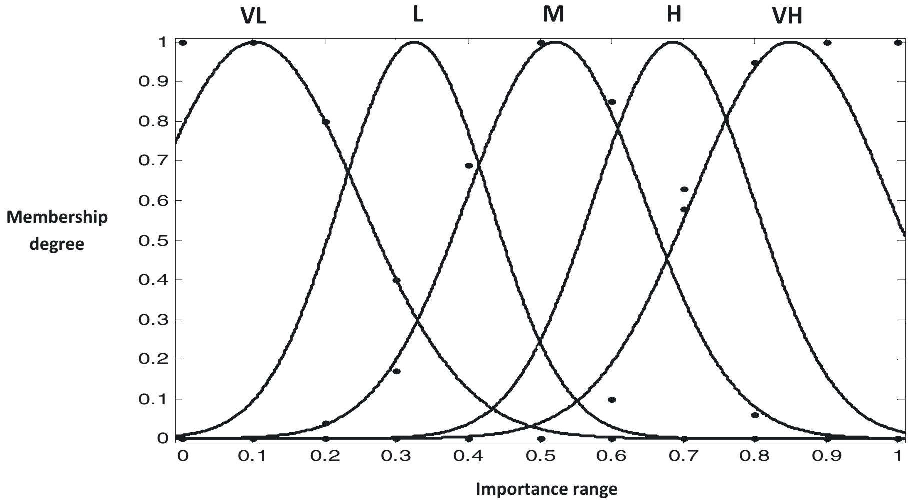

FIGURE 2. Fitted function to each linguistic variable.

TABLE 4. Statistical information of the fitted functions.

Linguistic variable	Function	SSE	R-square	RMSE	Adj. R-sq.	Mean	Sigma
Very high	Gaussian	0.046	0.978	0.071	0.976	0.851	0.137
High	Gaussian	0.017	0.988	0.043	0.987	0.687	0.111
Middle	Gaussian	0.018	0.988	0.045	0.987	0.522	0.124
Low	Gaussian	0.017	0.988	0.043	0.988	0.326	0.104
Very low	Gaussian	0.014	0.992	0.039	0.991	0.103	0.146

The statistical information of the fitted functions to each derived data is presented in Table 4.

In Table 4, SSE is the sum of squares due to error. R-square is determination coefficient. RMSE is the Root Mean Squared Errors. Adj. R-square is adjusted determination coefficient, and Sigma is the standard deviation of the fitted function to the data. Considering the results and the Adj. R-square which is more than 0.9 in all functions, it can be concluded that the fitted functions are appropriate. Therefore, it can be used as a foundation to define the linguistic variables. If the Adj. R-square is less than 0.9, viewpoints of experts should be re-collected and the above-mentioned steps should be repeated.

Since the interval $\pm 3\sigma$ from the mean is considered for investigation of function behavior in the fuzzy sets with Gaussian membership function [69], it is possible to draw the diagram of membership function of each linguistic term on the basis of the information (see Tab. 4) and using quasi-Gaussian fuzzy number so that the linguistic variables are defined as Figure 3. Due to position in the upper and lower limits of importance range, the membership degree of $Mean + 3\sigma$ and $Mean - 3\sigma$ for the terms very high and very low will be equal to 1.

4.2.2. Determining the weight interval of each criterion

Fuzzy linguistic variables are used for determining the weights of criteria in the model. Hence, the concept of union and intersection in fuzzy numbers can be used for integrating the viewpoints of experts and deriving the range accepted by them. In fact, intersection among the viewpoints of decision makers which is usually used as a range in fuzzy numbers can be regarded as the common and agreed upon viewpoint of all decision makers and used as the output of group decision making for specifying the weight of each criterion. However, in most cases, incorporating absolute weight restriction in DEA model would make the problem infeasible. Also, in some other cases, there is no common range among the viewpoints of decision makers. Since the weight assigned to each criterion by every decision maker is based on their personal knowledge and experience, the union range of

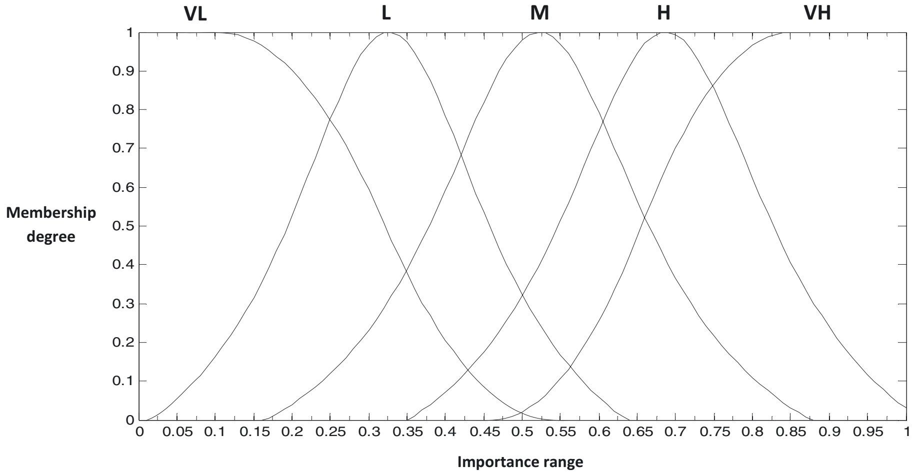

FIGURE 3. Linguistic variables for determining the importance of criteria.

TABLE 5. The upper and lower limits of linguistic variables.

Linguistic variable	Very low	Low	Middle	Low	Very high
Upper limit	0.54	0.64	0.88	1.00	1.00
Lower limit	0	0.01	0.16	0.35	0.44

TABLE 6. The example of the weights interval based on the decision makers' viewpoints.

Decision maker's viewpoints combination					Union of viewpoints	Intersection of viewpoints
Very low	Low	Middle	High	Very high		
✓					[0.00, 0.54]	[0.00, 0.54]
✓	✓				[0.00, 0.64]	[0.01, 0.54]
✓	✓	✓			[0.00, 0.88]	[0.16, 0.54]
	✓				[0.01, 0.64]	[0.01, 0.64]
	✓	✓			[0.01, 0.88]	[0.16, 0.64]
	✓	✓	✓		[0.01, 1.00]	[0.35, 0.64]
		✓			[0.16, 0.88]	[0.16, 0.88]
		✓	✓		[0.16, 1.00]	[0.35, 0.88]
		✓	✓	✓	[0.16, 1.00]	[0.44, 0.88]
			✓		[0.35, 1.00]	[0.35, 1.00]
			✓	✓	[0.35, 1.00]	[0.44, 1.00]
				✓	[0.44, 1.00]	[0.44, 1.00]

decision makers' viewpoints can be used as the accepted range of criteria weights for integrating their viewpoints. The following steps are taken for integrating the viewpoints and deriving the accepted range:

Step 1:

For determining the fuzzy union and intersection of each linguistic variable defined in Figure 3, the upper and lower limits of each linguistic variable, as mentioned in Section 4.2.1, can be employed. The upper and lower limits of linguistic variables are presented in Table 5.

The union and intersection of some combinations of the viewpoints of decision makers are presented in Table 6 as an example.

Step 2:
As mentioned before, incorporating absolute weight restriction into DEA model may makes the problem infeasible in most cases. In order to prevent this drawback, Model (3.5) and GP technique in DEA is utilized in this study. This means that intersection range of viewpoints are considered as the weight interval of criteria in the model, and if the problem becomes infeasible, the weight interval considered is widened through GP technique to make the problem feasible. However, the degree of this widening should be controlled and should not fall out of the interval considered by the decision makers.

In this study, in order to control the degree of widening the range of weights, the union of viewpoints is used as the accepted range of integrating viewpoints of decision makers. In this case, the restrictions of union interval are added to Model (3.5) and accordingly, the model proposed in the study is offered as below:

$$\begin{aligned}
\text{Max} \quad & Z_p = \sum_{r=1}^{s} u_r y_{rp} - M\left(\sum_{t=1}^{l_1}(p_t) + \sum_{h=1}^{l_2}(p'_h)\right) \\
s.t.: & \sum_{i=1}^{m} v_i x_{ip} = 1, \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leqslant 0 \quad j = 1, \ldots, n, \\
& A_t U + n_t - p_t = b_t, \qquad t = 1, 2, \ldots, l_1, \\
& C_h V + n'_h - p'_h = d_h, \quad h = 1, 2, \ldots, l_2, \\
& A_t U + n_t \leqslant k_t, \qquad t = 1, 2, \ldots, l_1, \\
& C_h V + n'_h \leqslant f_h, \qquad h = 1, 2, \ldots, l_2, \\
& u_r, v_i \geqslant 0, \qquad r = 1, 2, \ldots, s, \quad i = 1, 2, \ldots, m, \\
& n_t, p_t, n'_h, p'_h \geqslant 0, \qquad t = 1, 2, \ldots, l_1, \quad h = 1, 2, \ldots, l_2,
\end{aligned} \tag{4.2}$$

where k_t and f_h are the union of problem outputs and inputs, respectively.

Step 3:
After solving Model (4.2), 3 situations may arise:

1. The problem may become infeasible due to applying absolute restrictions of union range.
2. The weights derived from model are within the weight interval of the criteria (interval considered by decision makers (DMs)).
3. The weights derived from model are not within the weight interval of the criteria (interval considered by DMs).

For situations 1 and 2, Sections 4.3 and 4.6 of the proposed model are run, respectively. Situation 3 happens when the problem is infeasible and the model changes the weight interval within the framework of problem restriction in order to eliminate this problem. In this situation, the final and acceptable interval of criteria weights should again be derived using relation (4.3) to be the basis for calculating the relative efficiency of suppliers in AWR model in Section 4.3.

$$\begin{aligned}
\min(a_r^*, w_{r1}, w_{r2}, \ldots, w_{rp}) \leqslant u_r \leqslant \text{Max}(b_r^*, w_{r1}, w_{r2}, \ldots, w_{rp}) \\
\min(c_i^*, w'_{i1}, w'_{i2}, \ldots, w'_{ip}) \leqslant v_i \leqslant \text{Max}(d_i^*, w'_{i1}, w'_{i2}, \ldots, w'_{ip})
\end{aligned} \tag{4.3}$$

where $(a_r^*, b_r^*, c_i^*, d_i^*)$ are respectively the lower and upper limits of the criteria determined by decision makers and $(w_{r1}, \ldots, w_{rp})$ and $(w'_{i1}, \ldots, w'_{ip})$ are the weights of outputs and inputs obtained from solving Model (4.2) for each DMU, respectively.

4.3. Evaluating suppliers by calculating their relative efficiency

After determining the weight interval of each selected criteria, they should be incorporated into the DEA model. In this study, constant return to scale is assumed. Due to the efficiency improvement of inefficient suppliers by decreasing inputs, the Model (3.3) is utilized. In the third step of Section 4.2.2, if the weights extracted from Model (4.2) are in the range of criteria weights, the relative efficiency of DMUs can easily be obtained through the extracted weights and calculating $\sum_{r=1}^{s} u_r y_{rp}$ in Model (4.2). If these weights are not within the range accepted by DMs, the new weights are added to DEA model after being calculated and the problem is again solved on the basis of Model (3.3) and the relative efficiency of DMUs are obtained.

4.4. Re-determining the criteria weights for obviating non-relativity of efficiencies among suppliers

Incorporating absolute weight restriction into DEA model, in addition to possibility of making the problem infeasible, in some cases makes the efficiency among DMUs non-relative [53]. In this case, the weight interval of the criteria should be changed in a way so that they are placed within the interval accepted by DMs and at least one DMU has the relative efficiency of 1. In this paper, the following model is proposed when this problem arises.

$$
\begin{aligned}
&\mathrm{Min} Z = \sum_{t=1}^{l_1} (p_t) + \sum_{h=1}^{l_2} (p'_h) \\
&s.t.: \sum_{j=1}^{n} \sum_{i=1}^{m} v_i x_{ij} = 1, \\
&\quad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leqslant 0 \quad j = 1, \ldots, n, \\
&\quad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \geqslant -M\omega_j \rightarrow j = 1, \ldots, n, \\
&\quad \sum_{j=1}^{n} \omega_j \leqslant n - 1 \\
&\quad A_t U + n_t - p_t = b_t, \qquad t = 1, 2, \ldots, l_1, \\
&\quad C_h V + n'_h - p'_h = d_h, \quad h = 1, 2, \ldots, l_2, \\
&\quad \omega_j \in \{0, 1\} \qquad j = 1, \ldots, n, \\
&\quad u_r, v_i \geqslant 0, \qquad r = 1, 2, \ldots, s, \quad i = 1, 2, \ldots, m, \\
&\quad n_t, p_t, n'_h, p'_h \geqslant 0, \qquad t = 1, 2, \ldots, l_1, \quad h = 1, 2, \ldots, l_2,
\end{aligned} \tag{4.4}
$$

where M is a very big number and ω_j is a binary variable.

After obtaining the weight of each criterion by Model (4.4), the relative efficiency of DMUs can easily be obtained through the extracted weights by equation (4.5).

$$
\frac{\sum_{r=1}^{s} u_r y_r}{\sum_{i=1}^{m} v_i x_i}. \tag{4.5}
$$

Example: suppose that 3 suppliers with 2 inputs and 2 outputs, as presented in Table 7, are considered for supplying one goods. The intervals of weights are also determined by DMs and are presented in the Table 7.

TABLE 7. Data of supplier selection problem.

Criteria	C_1 (U_1)	C_2 (U_2)	C_3 (V_1)	C_4 (V_2)
Weights interval of criteria	[0.1, 0.6]	[0.2, 0.5]	[0.2, 0.8]	[0.4, 0.7]
Data	y_1	y_2	x_1	x_2
DMU1	1	0.3	1	0.7
DMU2	0.5	1	0.6	1
DMU3	1	0.3	0.9	1

TABLE 8. Results of solving AWR model.

CCR (AWR)	Outputs				Inputs				Results		Efficiency
	Elicited weights		Data		Elicited weights		Data		$\sum_{r=1}^{2} u_r y_r$	$\sum_{i=1}^{2} v_i x_i$	$\frac{\sum_{r=1}^{2} u_r y_r}{\sum_{i=1}^{2} v_i x_i}$
	u_1	u_2	y_1	y_2	v_1	v_2	x_1	x_2			
DMU1	0.6	0.5	1	0.3	0.51	0.7	1	0.7	0.75	1	0.75
DMU2	0.6	0.5	0.5	1	0.50	0.7	0.6	1	0.80	1	0.80
DMU3	0.6	0.5	1	0.3	0.33	0.7	0.9	1	0.75	1	0.75

TABLE 9. Result of calculating relative efficiency using Model (4.4).

Model (4.4)	Outputs				Inputs				Results		Efficiency
	Elicited weights		Data		Elicited weights		Data		$\sum_{r=1}^{2} u_r y_r$	$\sum_{i=1}^{2} v_i x_i$	$\frac{\sum_{r=1}^{2} u_r y_r}{\sum_{i=1}^{2} v_i x_i}$
	u_1	u_2	y_1	y_2	v_1	v_2	x_1	x_2			
DMU1	0.1	0.32	1	0.3	0.2	0.25	1	0.7	0.196	0.375	0.522
DMU2	0.1	0.32	0.5	1	0.2	0.25	0.6	1	0.37	0.37	1.000
DMU3	0.1	0.32	1	0.3	0.2	0.25	0.9	1	0.196	0.43	0.456

If efficiency of these two suppliers are investigated through general AWR model, a proper decision regarding selection of the best supplier cannot be made because their efficiency is not relative. Table 8 presents these results.

Now, if the efficiency of these two suppliers is calculated *via* Model (4.4), the most efficient supplier can be selected by deriving new weights of the criteria. As it is observed in Table 9, supplier 2 can be selected.

4.5. Suppliers ranking

After specifying the weight interval of each selected criteria, these intervals are incorporated into DEA model as restriction. In this study, assuming constant returns to scale and due to the improvement of efficiency of inefficient suppliers by decreasing inputs (*e.g.* reducing prices and reducing percentage of rejected items supplied by the suppliers), Model (3.3) is utilized.

4.6. Reviewing the weights of criteria and reevaluating the suppliers

This step is taken if, in the third step of Section 4.2.2, incorporating absolute weight restrictions belonging to the union range of viewpoints into the model makes it infeasible. At this step, factors contributing to this problem are systematically investigated, analyzed, and obviated. The used mechanism in this process is to analyze the information obtained and redefining the range of decision makers' viewpoints. In other words, after identifying the factor leading to the problem, the issue is investigated with the decision makers and after correcting their viewpoints, the weight interval of each criterion is again determined to make the problem feasible at the end.

TABLE 10. Data set of supplier selection problem.

Criteria	Suppliers (DMUs)															
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
UP	2190	2200	2195	2198	2205	2200	2185	2190	2190	2198	1966	2205	2182	2208	2201	2195
NOS	65 800	38 600	60 200	58 200	55 600	65 350	93 251	75 300	88 500	93 300	95 600	41 560	87 450	49 520	52 320	85 900
CO_2	4083	4656	4385	3584	4003	3525	3001	4958	4385	4083	3574	2998	3360	4311	4730	3951
NWN	61 200	38 561	57 792	57 502	52 820	60 122	93 200	74 100	88 394	92 150	93 100	41 526	84 827	49 500	52 310	85 700
NOT	60 112	35 405	59 500	51 352	48 105	62 312	91 560	74 900	88 153	90 129	94 562	38 956	84 652	49 510	52 153	83 288

TABLE 11. Normalized data set.

Criteria	Suppliers (DMUs)															
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
UP	0.992	0.996	0.994	0.995	0.999	0.996	0.990	0.992	0.992	0.995	0.890	0.999	0.988	1.000	0.997	0.994
NOS	0.688	0.404	0.630	0.609	0.582	0.684	0.975	0.788	0.926	0.976	1.000	0.435	0.915	0.518	0.547	0.899
CO_2	0.824	0.939	0.884	0.723	0.807	0.711	0.605	1.000	0.884	0.824	0.721	0.605	0.678	0.870	0.954	0.797
NWN	0.657	0.414	0.620	0.617	0.567	0.645	1.000	0.795	0.948	0.989	0.999	0.446	0.910	0.531	0.561	0.920
NOT	0.636	0.374	0.629	0.543	0.509	0.659	0.968	0.792	0.932	0.953	1.000	0.412	0.895	0.524	0.552	0.881

5. CASE STUDY

Data used in this case study are related to the problem of supplier selection in Sazeh Gostar Co. which is a company supplying automobile parts and modules to SAIPA Automotive Co. Sazeh Gostar Co. has played an outstanding role in developing automotive parts industry of Iran and currently it covers more than 500 automotive part suppliers in its supply network. As scheduled, the company enjoys the capacity to produce the parts and modules required for over 2000 cars per day. Sazeh Gostar Co. wishes to select the best supplier from among 16 suppliers for supplying windshield wiper.

5.1. Determining criteria of supplier selection problem

In this study, the four criteria mentioned in Section 4.1 are used for evaluating and selecting the best supplier. The indices of unit price, percent of shipments received from the supplier without nonconforming units, percent of shipments to arrive on time, and CO_2 emission are used for the criteria of price, quality, on-time delivery, and environmental factors, respectively. One of the pitfalls of using DEA is to incorporate indices, ratios or percentages into input/output set. To avoid this pitfall, in this study, indices are considered as percentages separated as numerator and denominator [17]. For instance, the index of percentages of shipments delivered on time is written as the number of shipments delivered on time in the last year divided by the total number of shipments delivered in the last year. Then the numerator of this fraction is considered as the output and its denominator as the input of DEA model. Therefore, the inputs are unit price (UP), number of shipments (NOS), and amount of CO_2 emission (ton/year) originated from suppliers' electricity consumption (CO_2). The outputs are number of shipments received from the supplier without inconsistent units (NWN) and number of shipments to arrive on time (NOT). The data set for these 16 suppliers are presented in Table 10.

Now, the data set are normalized through equation (4.1). The results of these calculations are presented in Table 11.

5.2. Deriving the criteria weights

To determine the importance of criteria, four decision makers (DMs) are involved. Each of these DMs assigns the weights to each criterion on the basis of linguistic weighing variables presented in Figure 3. Then, using Tables 5 and 6, the range of union and intersection of the viewpoints of DMs are derived as group viewpoint. The weight importance and intervals obtained on their basis are indicated in Table 12.

Table 12. Union and intersection interval of decision makers' viewpoints.

Criteria	Decision makers DM_1	DM_2	DM_3	DM_4	Final intersection interval	Final union interval
UP	H	M	H	H	[0.35, 0.88]	[0.16, 1.00]
NOS	M	L	M	H	[0.35, 0.64]	[0.01, 1.00]
CO_2	VH	VH	H	VH	[0.44, 1.00]	[0.35, 1.00]
NWN	VH	VH	VH	H	[0.44, 1.00]	[0.35, 1.00]
NOT	H	H	VH	VH	[0.44, 1.00]	[0.35, 1.00]

Table 13. Weights obtained by running Model (4.2).

Criteria	Suppliers (DMUs) 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
UP	0.350	0.350	0.350	0.350	0.350	0.350	0.350	0.350	0.350	0.350	0.373	0.350	0.350	0.350	0.350	0.336
NOS	0.421	0.589	0.417	0.547	0.507	0.494	0.397	0.350	0.284	0.296	0.350	0.640	0.388	0.515	0.422	0.350
CO_2	0.440	0.440	0.440	0.440	0.440	0.440	0.440	0.377	0.440	0.440	0.440	0.614	0.440	0.440	0.440	0.440
NWN	0.440	0.761	0.440	0.720	0.681	0.440	0.574	0.440	0.440	0.475	0.440	0.916	0.440	0.440	0.440	0.514
NOT	0.603	0.440	0.599	0.440	0.440	0.676	0.440	0.491	0.465	0.440	0.555	0.440	0.570	0.697	0.604	0.440

Table 14. Results of running Model (3.3).

	Suppliers (DMUs) 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Relative efficiency	0.699	0.500	0.685	0.709	0.642	0.751	1.000	0.741	0.871	0.903	1.000	0.590	0.918	0.639	0.626	0.876
UP	0.336	0.388	0.336	0.339	0.336	0.336	0.456	0.336	0.336	0.336	0.447	0.336	0.336	0.340	0.336	0.336
NOS	0.517	0.640	0.528	0.640	0.618	0.580	0.284	0.367	0.360	0.363	0.284	0.640	0.450	0.640	0.558	0.406
CO_2	0.377	0.377	0.377	0.377	0.377	0.377	0.448	0.377	0.377	0.377	0.440	0.637	0.377	0.377	0.377	0.377
NWN	0.440	0.811	0.440	0.762	0.738	0.440	0.574	0.440	0.440	0.489	0.560	0.916	0.440	0.440	0.440	0.531
NOT	0.645	0.440	0.656	0.440	0.440	0.709	0.440	0.494	0.487	0.440	0.440	0.440	0.578	0.773	0.686	0.440

After determining the union and intersection range of decision makers' viewpoints, these intervals are set according to Model (4.2) and the model is solved using Lingo 8.0 software. The values of the obtained weights along with the input and output variables are presented in Table 13.

As is seen, the weights of input 1 for supplier 16, input 2 for suppliers 9, 10, and input 3 for supplier 8 are not within the intersection interval of decision makers and show that the model has changed these values to solve infeasibility problem. However, these values are still within the union interval of decision makers. Thus, using equation (4.3) the new interval of input and output weights of the problem should be derived. The resulting intervals are shown in the following expressions.

$$\text{Min } (0.350, 0.350, 0.350, 0.350, \ldots, 0.336) \leqslant v_1 \leqslant \text{Max}\,(0.880, 0.350, 0.350, 0.350, \ldots, 0.336) \rightarrow 0.336 \leqslant v_1 \leqslant 0.880$$
$$\text{Min } (0.350, 0.421, 0.589, 0.417, \ldots, 0.350) \leqslant v_2 \leqslant \text{Max}\,(0.640, 0.421, 0.589, 0.417, \ldots, 0.350) \rightarrow 0.284 \leqslant v_2 \leqslant 0.640$$
$$\text{Min } (0.350, 0.440, 0.440, 0.440, \ldots, 0.440) \leqslant v_3 \leqslant \text{Max}\,(1.000, 0.440, 0.440, 0.440, \ldots, 0.440) \rightarrow 0.377 \leqslant v_2 \leqslant 1.000$$
$$\text{Min } (0.440, 0.440, 0.761, 0.440, \ldots, 0.514) \leqslant u_1 \leqslant \text{Max}\,(1.000, 0.440, 0.761, 0.440, \ldots, 0.514) \rightarrow 0.440 \leqslant u_1 \leqslant 1.000$$
$$\text{Min } (0.440, 0.603, 0.440, 0.599, \ldots, 0.440) \leqslant u_2 \leqslant \text{Max}\,(1.000, 0.603, 0.440, 0.599, \ldots, 0.440) \rightarrow 0.440 \leqslant u_2 \leqslant 1.000$$

In next step, the Model (3.3) should be solved via the new weights interval. The results of Model (3.3) are presented in Table 14.

As addressed in Table 14, the relative efficiency scores of suppliers 7 and 11 are equal to 1. Therefore, to determine the best supplier, Section 4.5 is taken. After solving the Model (3.6) and ranking two above-mentioned

TABLE 15. Results of running Model (3.6).

	Relative efficiency	Best supplier
Supplier 7	0.1239	Supplier 11
Supplier 11	0.1419	

TABLE 16. Results of running CCR model.

	Results of CCR model					
Suppliers (DMUs)	Relative efficiency	Input 1	Input 2	Input 3	Output 1	Output 2
1	0.931	0	1.45	0	1.41	0.004
2	0.999	0	2.475	0	2.413	0
3	0.989	0.0093	1.572	0	0	1.572
4	0.987	0	1.642	0	1.600	0
5	0.949	0	1.718	0	1.675	0
6	0.956	0	1.451	0.010	0	1.451
7	1	0.665	0	0.564	1	0
8	0.997	0.007	1.259	0	0	1.259
9	1	0.0008	1.077	0.001	0.887	0.170
10	0.987	0	1.024	0	0.998	0
11	1	0.035	0.968	0	0	1
12	0.999	0	2.298	0	2.241	0
13	0.979	0	1.063	0.039	0.115	0.977
14	1	0	1.924	0.003	1.643	0.243
15	1	0	1.828	0	1.777	0.005
16	0.997	0	1.112	0	1.084	0

TABLE 17. Results of running AWR model.

	Results of AWR model					
Suppliers (DMUs)	Relative efficiency	Input 1	Input 2	Input 3	Output 1	Output 2
1	0.627	0.350	0.421	0.440	0.440	0.603
2	0.479	0.350	0.589	0.440	0.761	0.440
3	0.649	0.350	0.417	0.440	0.440	0.599
4	0.683	0.350	0.547	0.440	0.720	0.440
5	0.610	0.350	0.507	0.440	0.681	0.440
6	0.729	0.350	0.494	0.440	0.440	0.676
7	1	0.396	0.350	0.440	0.440	0.578
8			Infeasible			
9			Infeasible			
10			Infeasible			
11	0.994	0.373	0.350	0.440	0.440	0.555
12	0.590	0.350	0.640	0.614	0.916	0.440
13	0.910	0.350	0.388	0.440	0.440	0.570
14	0.699	0.350	0.515	0.440	0.440	0.697
15	0.580	0.350	0.422	0.440	0.440	0.604
16			Infeasible			

suppliers, the supplier 11 is selected as the superior one. The results of running Model (3.6) are presented in Table 15.

If the model proposed in this paper is not used, the 5 out of 16 suppliers will have relative efficiency score of 1 in which DMs cannot determine the best supplier. Meanwhile, the weights of some criteria might be zero or more than 1 which are illogical and are not acceptable for DMs. Also, if AWR model is used for calculating relative efficiency of suppliers, the problem becomes infeasible for suppliers 8, 9, 10, and 16. Tables 16 and 17 depict the results.

5.3. Managerial implications

As discussed in Section 1, the costs of buying raw materials constitute up to 70% of the total cost of the products [28]. On the other hand, the fierce competition among firms presses them to offer lower prices and higher quality. Therefore, nowadays selecting a suitable supplier is a strategic problem. In this paper, a new hybrid decision making system for ranking suppliers was proposed. As well, the viewpoints of managers are one of the decision making bases in firms. This paper proposed a decision making approach incorporating the viewpoints of managers. By presenting the viewpoints in the form of fuzzy linguistic variables, this paper tried to consider the expertise of managers in supplier selection problem. Also, the proposed approach takes into account the viewpoints of multiple decision makers. Using the viewpoints of a group of DMs, help the firm to make a systematic decision and consider all aspects of decision making problem. In this paper, a real world case study was presented. The results validate the proposed model.

6. Concluding remarks

As Chen *et al*. [9] addressed, coordination between a manufacturer and suppliers is typically a difficult and important link in the channel of distribution. Most of developed models for supplier selection problems are based upon simplistic perceptions of decision-making process and they do not address the complex and unstructured nature of purchasing decisions. However, several influence factors are often not taken into account in the decision making process, such as fuzzy linguistic variables and viewpoints of multiple decision-makers. The most important challenge in supplier selection problem is to develop a suitable method to select the right supplier. The supplier selection problem is a group decision-making problem in the presence of multiple criteria. Supplier selection problems deal with uncertain and imprecise situations and FST is a suitable tool to solve this sort of problems. In supplier selection problems, the use of linguistic variables is highly beneficial when performance values cannot be expressed by means of numerical values.

In this paper, to solve supplier selection problem fuzzy group decision making and DEA were used. Then a hybrid model was proposed. To this end, to consider the opinions of multiple decision makers a group decision making technique using fuzzy linguistic data and the concept of intersection and union in fuzzy numbers was employed so that the weight of each criterion is determined within an interval. Then, to determine the relative efficiency of suppliers this interval was incorporated into the DEA model by absolute restrictions. The case study validated the proposed model for selecting suitable suppliers.

Further researches can be done based on the results of this paper. Some of them are as follows:

- Similar studies can be done in the presence of both cardinal and ordinal data.
- If there are dual-role factors, some changes should be incorporated into the proposed hybrid model which can be an interesting topic for the future studies.
- In this paper, a model is proposed for selecting the appropriate suppliers. It seems that this model can be used in other areas such as choosing the best technology, international market selection, *etc*.

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