RAIRO-Oper. Res. 50 (2016) 491–501
DOI: 10.1051/ro/2015029

RAIRO Operations Research
www.rairo-ro.org

ON THE BERTH ALLOCATION PROBLEM

Shaikha Ahmed Jumaa Alzaabi[1] and Ali Diabat[1]

Abstract. The rapid growth of the maritime industry has created a need for improvement in container terminal operations, by effectively utilizing the available resources. One of the most important seaside planning problems that has received considerable attention in the literature is the assignment of quay space to vessels, commonly referred to as the Berth Allocation Problem (BAP). Despite the significant contributions to the BAP found in the literature, there are certain important requirements that have not been considered. These include vessels of different sizes, suitability of a berth to a vessel, known as service requirement, and the possibility for one vessel to be accommodated by more than one berth. Thus, we formulate a mixed integer program (MIP) that explicitly considers these factors, in order to produce more realistic results. The model assumes that the quay is partitioned into berths of the same size and that several berths can be assigned to one vessel, given that the vessel is too long to be accommodated by a single berth. Considering the possibility of occupation of several berths by one vessel implies that the sequence of berths occupied is valid and feasible. In addition, we consider two extensions; the first extension of the model accounts for the different service requirements of each vessel, while the second assumes different berth lengths. A preliminary computational analysis is conducted to test the effectiveness of the proposed models and provide useful insights to port operators.

Mathematics Subject Classification. 90B06, 90B35.

Received April 4, 2015. Accepted June 16, 2015.

1. Introduction

The immense growth witnessed by the maritime industry over the past decade has drawn a great deal of attention towards the research and analysis of maritime operations [7, 26]. In addition, the standardization of shipping containers has tremendously impacted the concept of shipping, permitting a smoother handling of material goods and improved shipping operations [10]. In combination with the globally increasing demand in goods, the world economy has experienced unprecedented growth from this sector [1]. In order to accommodate the increasing demand and sustain the growth of the sector, container terminals are required to enhance their operations towards more efficient utilization of resources, given the fact that this growth is understandably not supported by an equal growth in container terminal capacity [24]. Thus, in order to overcome the challenge, container terminal operators are aiming to optimize the utilization of existing resources [4].

Container terminal operations can be distinguished into quayside and landside operations; quayside operations involve receiving the vessel and handling its containers, while the landside operations focus on storing and

Keywords. Berth allocation problem, MIP formulation, container terminals, maritime logistics.

[1] Department of Engineering Systems and Management, Masdar Institute of Science and Technology, Abu Dhabi, United Arab Emirates. adiabat@masdar.ac.ae

Article published by EDP Sciences

transporting the containers [17]. The three major problems pertaining to quayside operations are the Berth Allocation Problem (BAP) [8, 13, 25], the Quay Crane Assignment Problem (QCAP) and the Quay Crane Scheduling Problem (QCSP) [2]. The BAP aims to assign berthing spaces to incoming vessels, while the QCAP assigns available cranes to handle the vessels, and the QCSP schedules the operation of these cranes [3]. The overall efficiency of the container terminal largely depends on the efficiency of each problem individually, and the way in which they interact. In the present work, we focus on the BAP, which is the first in the sequence of quayside operational problems [9].

Generally, the assumptions considered for the modeling of each problem play a crucial role in the outcome of the model and subsequently on the practicality of the solution [5]. There is a trade-off between accuracy and computational efficiency and selecting the appropriate level is often a difficult task. Relaxing too many simplifying assumptions will render the problem extremely difficult to solve, if not impossible, due to high complexity and thus, solving it would be very time consuming. On the other hand, not considering enough circumstances will render the result unrealistic, despite reducing complexity and solving time to an absolute minimum [21]. Several works have been developed that address the BAP, each with a varying level of complexity and different assumptions. However, there remain certain important gaps in the literature that this work aims to address.

The main contribution of the current paper is thus the incorporation of realistic circumstances that have not been previously taken into account simultaneously, into a novel formulation. The first consideration is the different length of vessels. By allocating vessels to berths, knowing their different lengths, it would be possible to exploit that information to better utilize the entire berthing space. Service requirements are the second important consideration which is usually omitted. For example, there may be service agreements between port and vessel operators, determining priorities, fixed departure times or other criteria for the suitability of a certain berth assigned to a vessel. This is commonly seen in practice, but rarely implemented in theoretical models. Another important consideration is allowing for a single vessel to occupy more than one berths. Finally, the consideration of different berth lengths is implemented to extend the model. This would allow for a discretized, rather than continuous approach as dictated by the first consideration. This could lead to an equally good solution but in significantly reduced computational time.

The remainder of this paper is organized as follows. A literature review of the BAP is presented in Section 2. Following that, Section 3 details the assumptions, the problem definition, and the formulation approach of the BAP, gradually introducing the above mentioned considerations. In Section 4, a set of experiments is performed to evaluate the effectiveness of the proposed model. Finally, Section 5 concludes the study's findings and provides certain directions for future research.

2. Literature review

The berth allocation problem aims to assign incoming vessels to their respective berths in order to handle their freight efficiently. This problem is recognized to be one of the most important processes for any container or marine terminal and is receiving considerable attention, as researchers and port operators aim to improve operations and maximize throughput. A very systematic classification of problems pertaining to berth allocation can be found in the recent work of [6]. In this section, we will describe the assumptions of the current formulation, before presenting notable works in the field and highlight the contribution of our work.

According to the classification by Bierwirth and Meisel [6] the current work is of the type $hybr\,|dyn|\,fix|\sum compl$. We will explain each individual attribute. The first attribute, *hybr*, has to do with the berth layout. A continuous berth layout assumes vessels can be moored anywhere along the quay, while in a discrete layout they are assigned to specific slots. In our case, since we assume berthing positions of different lengths, it is considered to be a hybrid layout, which combines the continuous and discrete layout. The next factor refers to the arrival of vessels. There is the static arrival, in which it is assumed that vessels have arrived and wait to be berthed, contrary to the dynamic case where the port operators do not know in advance which vessels to accommodate, but rather as they arrive. In the current work, we have assumed the latter case. The handling time required by each vessel is a given parameter, therefore is fixed, which explains the third attribute. Finally, our objective aims to minimize the sum of the total completion times for all vessels.

At this point, we will begin reviewing existing works, from oldest to most recent. In the early work of [15], which is considered the first paper to have proposed the berth allocation problem with dynamic vessel arrival times, the authors produce a heuristic based on the Lagrangian relaxation of the original developed BAP to obtain a linear problem. This enabled the model to be solved with smaller effort, while producing a near-optimal dynamic berth plan. In this problem, the processing time of the vessel is assumed to be dependent on the berth it has been assigned to. The model that was developed by [15] was an extension of the static berth allocation problem by [14], in which arrival times are known in advance.

On another note of classification, [20] study both the discrete and continuous BAP. In the discrete version of the problem, the quay is partitioned into berths, each able to accommodate a single vessel. In the continuous layout, vessels are assigned along the quay, not in specific discretized places but anywhere, as long as total vessel length does not exceed berth length at any time. The authors propose a new formulation of the dynamic scheduling problem for the discrete case, which is proven to be more compact.

Regarding the solution techniques developed, [16] used a heuristic approach to solve a continuous berth allocation problem of a multi-vessel container terminal in comparison with the authors' previous work on a solution method for a discrete berth allocation scheme. Considering that the objective of the model is to minimize the makespan, the method that was used is the first-fit-decreasing method, which is considered an efficient heuristic for the bin-packing problem. In the paper of [18], the authors present a two-phased solution method for determining the berthing time of a vessel as well as the positioning of the vessel along the berth by coming up with a genetic algorithm. In addition, their method determines the number of quay cranes to be assigned to the berthed vessels using a multi-objective genetic algorithm to solve this integrated problem. The reason is that genetic algorithms provide flexibility in solving optimization problems since the solution is near optimal and not exact. In [23] the authors study the static case of the BAP and develop a Lagrangian relaxation heuristic with the application of cutting planes, given that it is a non-deterministic polynomial-time (NP) problem. The authors report reaching optimal solutions in most instances. Later work of [22] formulates a non-linear mixed integer program for the dynamic case of the BAP. The authors implement techniques to reformulate the problem into a mixed integer program, while this time they develop a GA to solve it.

Recent developments in the formulation of the BAP include the consideration of fuel consumption and tidal restrictions. The former is found in the work by [12], in which the authors assume vessel arrival time to be a decision variable in order to regulate fuel consumption. Du *et al.* [8] additionally consider emissions, by converting consumption to gas emissions. Hu *et al.* [13] go even further by considering the joint berth allocation and quay crane assignment problem to obtain a better balance between fuel consumption, hadling times and emissions. Xu *et al.* [25] study the problem by imposing tidal restricitons, in the sense that vessels can be moored at a berth only when the tide allows for enough depth of water. The authors implement a parallel machine scheudling problem for both the static and dynamic BAP.

Recently, interest has also been focused on the integration of two or more of the quayside operational planning problems. Proponents of integration support that it allows for full exploitation of problem information, as the simultaneous decisions can potentially be superior than the traditional sequential approach. In [19] the authors integrate the berth allocation and crane assignment problems. They suggest the decrease of marginal productivity of quay cranes assigned to vessels, as well as the increase in handling time if the vessels are not positioned at their desired berth. This is an example of the rare consideration of service requirements, as implemented in the current work. Also in [11] the authors integrate the BAP and QCAP. The developed formulations include a mixed integer quadratic program as well as a linearization which transforms it into a mixed integer linear program. The authors aim to minimize the cost associated with the transshipment of containers, while at the same time maximizing the total value of the assigned quay profiles.

Finally, it is evident that several studies have addressed the BAP and many advancements have been made to the formulation, especially over the recent years. However, very few papers have focused on vessel length characteristics and respective berth service requirements. We aim to address both aspects in the formulation developed in the current paper, thus contributing to the pool of research with new assumptions. In the following sections we will present the outcome of this work.

3. Problem description and formulation

The purpose of the current section is to introduce the general problem, along with its assumptions and notation. Moving on, the formulation is described in terms of its objective function and constraints. Upon presenting the base model, the extensions are then gradually added to create the enhanced version.

3.1. Problem description

In general, the model deals with the assignment of an arriving vessel, j, to a berth, i, along the quay given that a single quay contains multiple berths. It is assumed in this model that each berth is of the same length. The elapsed times of arrival of the vessels are deterministic and are known one day, to several days, ahead of time. The vessels that arrive to the terminal are of different types, sizes, and lengths. Some of these vessels can be berthed at a single berth, where their length does not exceed the length of the berth, while other vessels will be larger and can occupy more than one berth. Conventionally, the terminal will require a certain space between each berthed vessel as a safety margin.

3.2. Problem formulation

In this section, the basic problem is initially presented, upon which the extension will subsequently be built.

3.2.1. Vessel length consideration

To formulate the problem, the following notation is used.

Sets

I the set of all berths indexed by i
J the set of all vessels indexed by j

Parameters

n $\triangleq$ Total number of berths
m $\triangleq$ Total number of vessels
b_j $\triangleq$ Number of berths required to serve vessel j
M $\triangleq$ Sufficiently large constant
p_j $\triangleq$] Processing time of vessel j
a_j $\triangleq$ Arriving time of vessel j
w_j $\triangleq$ Weight of priority of vessel j

Decision Variables

$$X_{ij} = \begin{cases} 1 \text{ if vessel } j \text{ is moored at berth } i \\ 0 \text{ otherwise} \end{cases}$$

$$Z_{ijj'} = \begin{cases} 1 \text{ if vessels } j \text{ and } j^{'} \text{ are moored at berth } i, j \text{ is processed before } j' \\ 0 \text{ otherwise} \end{cases}$$

$$S_j \triangleq \text{Start time of processing of vessel } j$$

$$Y_i = \begin{cases} 1 \text{ if berth } i \text{ is the first berth assigned to vessel } j \\ 0 \text{ otherwise} \end{cases}$$

The model can then be stated as follows.

$$\min \sum_{j=1}^{m} w_j \left(s_j + p_j - a_j\right) \tag{3.1}$$

Subject to

$$\sum_{i=1}^{n} X_{i,j} = b_j \qquad j \in J \tag{3.2}$$

$$S_j \geqslant a_j \qquad j \in J \tag{3.3}$$

$$S_{j'} \geqslant S_j + p_j - M(1 - Z_{ijj'}) \qquad jj^{'} \in J, j \neq j^{'}, i \in I \tag{3.4}$$

$$Z_{ijj'} + Z_{ij'j} \leqslant \frac{1}{2}\left(X_{ij} + X_{ij'}\right) \qquad jj^{'} \in J, j < j^{'}, i \in I \tag{3.5}$$

$$Z_{ijj'} + Z_{ij'j} \geqslant X_{ij} + X_{ij'} - 1 \qquad jj^{'} \in J j \neq j^{'}, i \in I \tag{3.6}$$

$$\sum_{k=1}^{i-1} X_{kj} \leqslant b_j \left(1 - Y_{ij}\right) \qquad j \in J, i \in I \tag{3.7}$$

$$\sum_{k=1+b_j}^{n} X_{kj} \geqslant b_j \left(1 - Y_{ij}\right) \qquad j \in J, i \in I \tag{3.8}$$

$$\sum_{i=1}^{N} Y_{ij} = 1 \qquad j \in J \tag{3.9}$$

$$X_{ij} \left\{0, 1\right\} \qquad j \in J, i \in I \tag{3.10}$$

$$Z_{ijj'} \left\{0, 1\right\} \qquad jj^{'} \in J j \neq j^{'}, i \in I. \tag{3.11}$$

The objective function (3.1) aims to minimize the weighted handling time required for all vessels. Note that the completion time, which we can define as $C_j = s_j + p_j - a_j$ is the time when vessel j is berthed at berth i, and the processing of the vessel begins. This is dependent on the nature of the process performed on the vessel. C_j is the summation of the starting time, s_j when the vessel is berthed and begins its handling time, and the processing time, p_j the duration it takes to handle vessel j. Further, let us take some time to explain the purpose of the weighted cost of each vessel. This is a parameter that acts as a weight in favor of certain vessels. This means that the higher the weighted cost, the higher the handling cost of the vessel per time unit. Thus, the vessels with the highest weight will be handled faster than the others, in an effort to minimize the total incurred cost. This parameter serves an additional purpose; aside from the cost, the vessel with the highest weight will also automatically have the highest priority, because our formulation will try to decide on the least amount of handling time in order to minimize once again the cost. Thus, in a way this accounts for other factors such as service agreements between vessel and port operators, regarding the priority of a certain vessel.

Constraints (3.2) are the multi-berth occupancy constraints. In this model, we are considering that a large vessel, which is larger in terms of length, can occupy several berths positioned side by side. To introduce the possibility of this situation during the berth planning process, this constraint requires that vessel j be berthed at berth(s) $i + b_j - 1$ if the length of the vessel requires it to occupy more than one berth.

Constraints (3.3) ensure that a vessel will not commence handling before its arrival time. In order to ensure that a vessel will commence handling after the previous vessel has been handled at the same berth, constraints (3.4) are in place. Vessels that have been assigned to the same berth will require a certain sequence and one vessel's processing time will begin after its predecessor leaves the berth. This constraint states that if vessels j and j' are both assigned to berth i and vessel j is processed before vessel j' (*i.e.*, $Z_{ijj'} = 1$), then the start time of vessel j' must be no earlier than $s_j + p_j$ (*i.e.* C_j).

Constraints (3.5) and (3.6) help to determine which berths will accommodate multiple vessels, and which precisely vessels. Under this requirement, we have two constraints that ensure that it is satisfied. The following constraint requires that one of $Z_{ijj'}$ and $Z_{ij'j}$ equals 1 if vessels j and j' are both assigned to berth i. They also ensure that $Z_{ijj'} = Z_{ij'j} = 0$ if one of vessels j and j' is not assigned to berth i.

Constraints (3.7) and (3.8) are the neighboring berth constraints. Given the possibility of a single vessel occupying multiple berths, the assigned berths will logically have to be positioned immediately next to each other. This is achieved with the use of the sufficiently small parameter a. We have two constraints that will ensure that if vessel j requires more than one berth, the feasible berths i must be consecutive. Constraint (3.9) defines the first berth that vessel j will occupy. Finally, constraints (3.10) and (3.11) are the integrality constraints which define the nature of the decision variables.

3.2.2. Service requirements extension

In order to account for this extension, a new parameter is defined:

$$d_{ij} = \begin{cases} 1 \text{ if berth } i \text{ can be assigned to vessel } j \text{ (based on draft and others)} \\ 0 \text{ if otherwise} \end{cases}$$

This essentially dictates whether or not a certain vessel can be allocated to a berth, based on the ship's draft and service requirements. The way in which this is subsequently implemented in the formulation is through the following constraints:

$$X_{ij} \leqslant d_{ij} \quad \forall i = 1, \ldots, n, j = 1, \ldots, m. \tag{3.12}$$

This implies that vessel j will be assigned to berth i only if this is possible based on physical limitations, provided through the newly introduced decision variable.

3.2.3. Berth length extension

To further consider the length of the berth, the following parameter was defined:

$$l_i \triangleq \text{Length of berth } i$$
$$L_j \triangleq \text{Length of vessel } j.$$

This was incorporated into the model through the following constraint:

$$L_j \leqslant \sum_{i=1}^{n} l_i X_{ij} \quad \forall j = 1, \ldots, m \tag{3.13}$$

This constraint ensures that a vessel will be assigned to a berth or berths only if its length satisfies the total length of the berth(s) it is assigned to.

4. Computational analysis

The data used as input to our models refers to the case of 10 berths and 10 vessels. The size of the berths remains the same throughout the various cases, yet the vessel lengths increases. The reasoning behind this is that realistically a port will not change the size of its berths, unless it expands to include new ones. What does change is the size of incoming vessels each time. By testing a wide range of vessel sizes we can draw useful conclusions regarding the impact of vessel size on total handling time and container terminal efficiency. The input data for each case can be seen in Table 1–7. Note that some values for the vessel lengths, especially for the first cases are unrealistically small. However, it is essential to test these for the purposes of the current analysis.

The experimental analysis was conducted with the help of the commercial software GAMS (General Algebraic Modeling System). The code was run on a Dell workstation with Intel(R)Core(TM)i5 2540M CPU @2.60GHz

Table 1. Case 1 data.

Vessel	Length (m)	Berth	Length (m)
1	8	1	150
2	17	2	50
3	26	3	200
4	9	4	150
5	20	5	125
6	15	6	250
7	19	7	250
8	24	8	75
9	13	9	150
10	18	10	200

Table 2. Case 2 data.

Vessel	Length (m)	Berth	Length (m)
1	16	1	150
2	34	2	50
3	52	3	200
4	18	4	150
5	40	5	125
6	30	6	250
7	38	7	250
8	48	8	75
9	26	9	150
10	36	10	200

Table 3. Case 3 data.

Vessel	Length (m)	Berth	Length (m)
1	28	1	150
2	59.5	2	50
3	91	3	200
4	31.5	4	150
5	70	5	125
6	52.5	6	250
7	66.5	7	250
8	84	8	75
9	45.5	9	150
10	63	10	200

Table 4. Case 4 data.

Vessel	Length (m)	Berth	Length (m)
1	32	1	150
2	68	2	50
3	104	3	200
4	36	4	150
5	80	5	125
6	60	6	250
7	76	7	250
8	96	8	75
9	52	9	150
10	72	10	200

Table 5. Case 5 data.

Vessel	Length (m)	Berth	Length (m)
1	40	1	150
2	85	2	50
3	130	3	200
4	45	4	150
5	100	5	125
6	75	6	250
7	95	7	250
8	120	8	75
9	65	9	150
10	90	10	200

Table 6. Case 6 data.

Vessel	Length (m)	Berth	Length (m)
1	56	1	150
2	119	2	50
3	182	3	200
4	63	4	150
5	140	5	125
6	105	6	250
7	133	7	250
8	168	8	75
9	91	9	150
10	126	10	200

Table 7. Case 7 data.

Vessel	Length (m)	Berth	Length (m)	Vessel	Length (m)	Berth	Length (m)
1	80	1	150	6	150	6	250
2	170	2	50	7	190	7	250
3	260	3	200	8	240	8	75
4	90	4	150	9	130	9	150
5	200	5	125	10	180	10	200

Table 8.

Case	Model 1		Model 2		Model 3	
	Time	Cost	Time	Cost	Time	Cost
1	2.16	AED 16 458.90	2.16	AED 16 458.90	4.13	AED 20 824.90
2	4.32	AED 32 917.80	4.32	AED 32 917.80	4.80	AED 37 283.80
3	7.55	AED 57 606.15	11.44	AED 63 864.15	11.44	AED 63 864.15
4	8.63	AED 65 835.60	12.44	AED 72 093.60	12.44	AED 72 737.60
5	10.79	AED 82 294.50	14.43	AED 88 552.50	15.75	AED 89 744.50
6	15.11	AED 115 210.00	18.41	AED 121 470.00	19.07	AED 122 660.00
7	21.58	AED 164 590.00	24.39	AED 170 850.00	27.90	AED 182 520.00

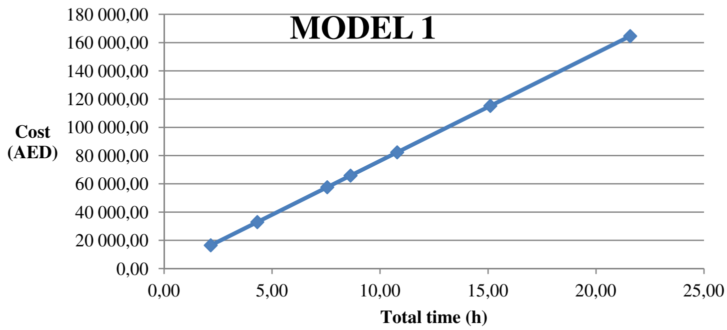

Figure 1. Results for model 1.

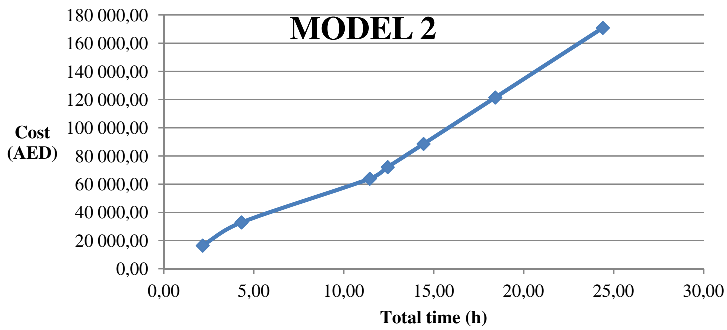

Figure 2. Results for model 2.

processors with Windows 7 operating system. Computational times were very low (in the scale of seconds) and thus were not reported.

The results of interest are the total required handling times for all vessels, depicted in the tables as Time, as well as the total cost. These are summarized in Table 8. Note that model 1 refers to the Berth Allocation Problem (BAP) with vessel length consideration, model 2 refers to the BAP with vessel length consideration and service requirements, while model 3 refers to the BAP with vessel length consideration, service requirements consideration and different berth lengths extension. Note that for the first model, the berth length is considered to be the average of the berths with different lengths of the following model.

In Figures 1–3, we can see the cost with respect to time required to handle all vessels. It is interesting to note that the relation is almost linear in all of the cases. We can see that as the size of the vessels increases in terms of length both the time and the cost increase at a greater rate for all three cases. This has to do with

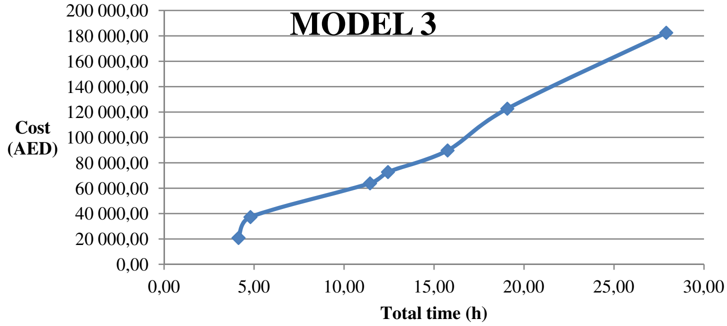

Figure 3. Results for model 3.

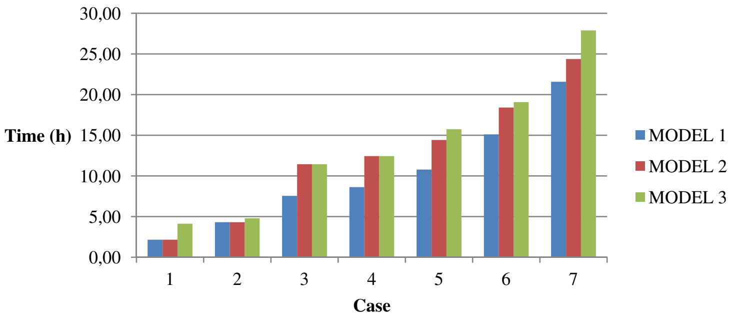

Figure 4. Comparison of total time between all cases.

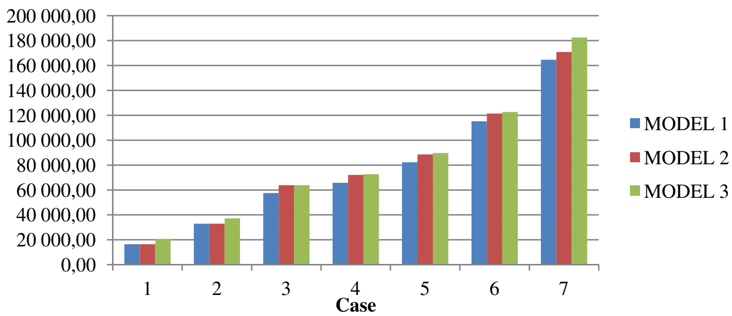

Figure 5. Comparison of total cost between all cases.

the fact that the larger the vessel, the fewer the options of being berthed, given that berths have fixed length capacities. Therefore, a large vessel may have to wait for a berth to become available.

Figures 4 and 5 compare the results between the three models in terms of time and cost. As we add requirements and restrictions, from one model to another, we see the effect of this on the time and cost. In the first model, we only consider the vessel length, which will only impact the time required to handle that vessel. However, in the second model we consider additionally the service requirements. This immediately increases the handling time and cost, even though nothing else has changed. Finally, when we consider different berth lengths, this tightens even more our solution space, leading the optimal solution to be 'worse' than in the previous two cases. These observations also demonstrate the validity of our models, given that this was an expected outcome.

5. Conclusions

The current work aims to address the Berth Allocation Problem (BAP), which constitutes one of the most important problems in quayside operations planning. The motivation behind this work lies in the considerable growth of the maritime industry over the past decades. Increasing container terminal efficiency is of vital importance in order for contemporary ports to meet the growing demand and challenges. This is why we chose to investigate the first in the series of quayside operations, which is a strong determinant of the efficiency and subsequent success of a container terminal. The next in the series of quayside operations is the Quay Crane Assignment Problem (QCAP), followed by the Quay Crane Scheduling Problem (QCSP).

In order to successfully address the selected problem, we first conducted an extensive literature review on existing practices and methods around the BAP. We identified common assumptions, which allowed us to spot others that were not commonly considered. One of these included the consideration of vessel length in the formulation. Given the fact that a quay has certain available berths which may accommodate one, two or half a vessel, depending on its length, it is important to consider this parameter in the formulation.

Therefore, in order to construct a strong formulation that would provide realistic and applicable results, we incorporated these assumptions that are usually ignored. This constitutes the major contribution of our work. Furthermore, we extended the developed formulation to account for two additional considerations; first, we took into account service requirements. This means that the berthing position assigned to a vessel will depend on the reason it is being berthed. Therefore, if a vessel stops at the container terminal to discharge its containers, it would be positioned closer to the storage area than had it stopped for fuel. The second extension considers different berth lengths. In the initial formulation we assume that all berths are equal. However, by varying the length it is possible to take better advantage of the quay space.

Upon developing the models, we conducted the experimental analysis on the commercial software GAMS (General Algebraic Modeling System). Tests were run for all three formulations and subsequently compared. Results demonstrated that in all three developed models, the increase in vessel size leads to an almost linear increase in handling time, which exhibits an almost linear relation with total cost. As the size of the vessels increases in terms of length, both the time and the cost increase at a greater rate for all three cases. This has to do with the fact that the larger the vessel, the fewer the options of being berthed, given that berths have fixed length capacities. Therefore, a large vessel may have to wait for a berth to become available.

Furthermore, the three models were compared with respect to one another. As we add requirements and restrictions, from one model to another, we see the effect of this on the time and cost. In model 1, we only consider the vessel length, which will only impact the time required to handle that vessel. However, in model 2 we consider additionally the service requirements. This immediately increases the handling time and cost, even though nothing else has changed. Finally, when we consider different berth lengths, this tightens even more our solution space, leading the optimal solution to be 'worse' than in the previous two cases. These observations also demonstrate the validity of our models, given that this was an expected outcome.

Overall, the current work managed to effectively address the BAP for container terminals. Ultimately, the conclusions of this work can be used by port operators to improve their services and maintain a large customer base.

Acknowledgements. This work was supported by Grant No. EX2014-000003 provided by Abu Dhabi Ports, Abu Dhabi, United Arab Emirates. Thereon, we would like to acknowledge their invaluable contribution and extend our warm appreciations to the CEO of Abu Dhabi Ports, Capt. Mohamed Al Shamisi, and the Chairman, H.E. Minister of State Dr. Sultan Al Jaber.

References

[1] N. Al-Dhaheri and A. Diabat, The Quay Crane Scheduling Problem. *J. Manuf. Syst.* **36** (2015) 87–94.

[2] N. Al-Dhaheri and A. Diabat, A Lagrangian-relaxation-based heuristic for the multiship quay crane scheduling problem with ship stability constraints. To appear in *Ann. Oper. Res.* (2016).

[3] N. Al-Dhaheri, A. Jebali and A. Diabat, The quay crane scheduling problem with nonzero crane repositioning time and vessel stability constraints. *Comput. Ind. Eng.* **94** (2016) 230–244.

[4] N. Al-Dhaheri, A. Jebali and A. Diabat, A simulation based Genetic Algorithm approach for the Quay Crane Scheduling under uncertainty. *Simul. Model. Pract. Theory* **66** (2016) 122–138.

[5] J. Al Hammadi and A. Diabat, An Integrated Berth Allocation and Yard Assignment Problem for Bulk Ports: Formulation and Case Study. To appear in *RAIRO: RO* (2016). DOI:10.1051/ro/2015048

[6] C. Bierwirth and F. Meisel, A follow-up survey of berth allocation and quay crane scheduling problems in container terminals. *Eur. J. Oper. Res.* **244** (2015) 675–689.

[7] A. Diabat and E. Theodorou, An Integrated Quay Crane Assignment and Scheduling Problem. *Comput. Ind. Eng.* **73** (2014) 115–123.

[8] Y. Du, Q. Chen, X. Quan, L. Long and R.Y.K. Fung, Berth allocation considering fuel consumption and vessel emissions. *Transp. Res. Part E Logist. Transp. Rev.* **47** (2011) 1021–1037.

[9] Y.-M. Fu and A. Diabat, A Lagrangian relaxation approach for solving the integrated quay crane assignment and scheduling problem. *Appl. Math. Model.* **39** (2015) 1194–1201.

[10] Y.-M. Fu, A. Diabat and I.-T. Tsai, A multi-vessel quay crane assignment and scheduling problem: Formulation and heuristic solution approach. *Expert Syst. Appl.* **41** (2014) 6959–6965.

[11] G. Giallombardo, L. Moccia, M. Salani and I. Vacca, Modeling and solving the Tactical Berth Allocation Problem. *Transp. Res. Part B* **44** (2010) 232–245.

[12] M.M. Golias, M. Boile and S. Theofanis, Berth scheduling by customer service differentiation: A multi-objective approach. *Transp. Res. Part E* **45** (2009) 878–892.

[13] Q.-M. Hu, Z.-H. Hu and Y. Du, Berth and quay-crane allocation problem considering fuel consumption and emissions from vessels. *Comput. Ind. Eng.* **70** (2014) 1–10.

[14] A. Imai, K. Nagaiwa and C.W. Tat, Efficient planning of berth allocation for container terminals in Asia. *J. Adv. Transp.* **31** (1997) 75–94.

[15] A. Imai, E. Nishimura and S. Papadimitriou, The dynamic berth allocation problem for a container port **35** (2001) 401–417.

[16] A. Imai, E. Nishimura and S. Papadimitriou, Berth allocation with service priority. *Transp. Res. Part B* **37** (2003) 437–457.

[17] N. Kenan and A. Diabat, A Branch-and-Price Algorithm to Solve a Quay Crane Scheduling Problem. *Proc. Comput. Sci.* **61** (2015) 527–532.

[18] M. Krčum, A. Gudelj and S. Vlahinic, Genetic Algorithm for Solving Berth and Quay Cranes Assignment problems, in *2nd International Conference on Ports and Waterways* (2007) 165–177.

[19] F. Meisel and C. Bierwirth, Heuristics for the integration of crane productivity in the berth allocation problem. *Transp. Res. Part E* **45** (2009) 196–209.

[20] M.F. Monaco and M. Sammarra, The berth allocation problem: a strong formulation solved by a Lagrangean approach. *Transp. Sci.* **41** (2007) 25–280.

[21] W. Schoonenberg, J. Hols and A. Diabat, A Cost Based Approach for a Crane Assignment and Scheduling Problem, in *International Conference on Industrial Engineering and Systems Management (IESM)*, October 21–23 (2015).

[22] A. Simrin and A. Diabat, The dynamic berth allocation problem: A linearized formulation. *RAIRO: RO* **49** (2015) 473–494.

[23] A.S. Simrin, N.N. Alkawaleet and A.H. Diabat, A Lagrangian Relaxation based Heuristic for the Static Berth Allocation Problem using the Cutting Plane Method, in *Proceedings of the 15th International Conference on Enterprise Information Systems* (2013) 565–569.

[24] E. Theodorou and A. Diabat, A Joint Quay Crane Assignment and Scheduling Problem: Formulation, Solution Algorithm and Computational Results. *Optim. Lett.* **9** (2015) 799–817.

[25] D. Xu, C.-L. Li and J.Y.-T. Leung, Berth allocation with time-dependent physical limitations on vessels. *Eur. J. Oper. Res.* **216** (2012) 47–56.

[26] Q. Zeng, A. Diabat and Q. Zhang, A simulation optimization approach for solving the dual-cycling problem in container terminals. *Marit. Policy Manag.* **42** (2015) 87–94.