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ANALYSIS OF DISCRETE-TIME QUEUES WITH CORRELATED ARRIVALS, NEGATIVE CUSTOMERS AND SERVER INTERRUPTION

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Abstract. This paper analysis a discrete time infinite capacity queueing system with correlated arrival and negative customers served by two state Markovian server. Positive customers are generated according to the first order Markovian arrival process with geometrically distributed lengths of *On* periods and *Off* periods. Further, the geometrically distributed arrival of negative customer removes the positive customers is any, and has no effect when the system is empty. The server state is a two state Markov chain which alternate between *Good* and *Bad* states with geometrically distributed service times. Closed-form expressions for mean queue length, unfinished work and sojourn time distributions are obtained. Numerical illustrations are also presented.

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1. Introduction

Discrete time queuing system is an efficient tool to model telecommunication systems, B-ISDN networks based on ATM and mobile networks. These networks apply the slotted transportation of fixed size packets. In recent years, great attention is paid in studying discrete time queueing systems with different variants. For comprehensive studies on discrete time queues one may refer to Takagi [15], Bruneel and Kim [7], Hunter [11], Woodward [17].

There are situations in which an arrival may harm the entire system and existing customers (like virus to a computer server). In queueing terminology such customers are called as negative customers. Queues with negative customers are also called as G-queues. Recent past years excellent study has been made with negative customers. A study on G-queues was first introduced by Gelenbe [8]. Artalejo [1] and Gelenbe [9, 10] provided excellent surveys on this topic. Although many continuous-time queueing models with negative arrivals have been studied, their discrete-time counterparts received very little attention in the literature. The early work about negative customers in discrete-time without retrials can be found in Atencia and Moreno [2–4]. They have considered single server discrete-time queue, with negative arrivals and various killing disciplines caused by the negative customers but not considering the arrival of *On*-*Off* sources.

Keywords. On-Off Source, negative customers, two state Markovian server, sojourn time distribution.

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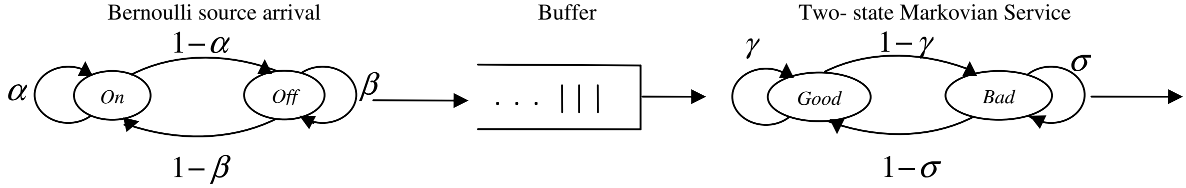

FIGURE 1. Bernoulli source arrival with two state Markovian service.

Viterbi [16] used the matrix-analytic technique to derive a closed-form expression for the mean queue length with heterogeneous *On-Off* sources with each source generating a single cell per slot in the *On* state. A Markov-modulated Poisson process (MMPP) has been used by Baiocchi *et al.* [5], to approximate the arrival process to a MMPP/G/1/K queue. Wang and Zhang [12] discusses a discrete-time single-server retrial queue with geometrical arrivals of both positive and negative customers in which the server is subject to breakdowns and repairs. Using generating function approach, a discrete-time queuing system with binary *On-Off* Markov model is analysed by Bruneel [6].

Wang *et al.* [13], discusses a discrete-time single-server *On-Off* source queueing system with negative customer, without server interruption. Whereas Mehmet Ali *et al.* [14], discusses a discrete time queueing system with server interruption for modelling wireless ATM multiplexer, without negative customers.

The proposed work is applicable to the Mobile banking system that consists of a base server and a number of mobile users. Customers who request for transactions will be connected to a base server through mobile and form a queue. We say a link to be in a *Bad* state when the transaction is very likely to fail due to errors and in a *Good* state when transaction is likely to be successful. There are situation at which the negative customers will affect the queue in different ways (individual removal, batch removal, disaster, triggered movement, survey paper by Artelejo [1]). In the present paper, killing methodology considered is removal of customers at the head (RCH killing methodology). Using Markovian processes approach, this paper investigates a discrete-time infinite queueing system with single *On-Off* source, as well as geometrically distributed service times with two states of a server, namely *Good* and *Bad* and negative customers.

The structure of the paper is organised as follows: Section 2 gives the description of the model. In Section 3, Markov chain and generating function analysis is presented. In Section 4, steady state buffer content is developed. Expressions for the modified service time, unfinished work, stationary sojourn time and special cases are derived in Sections 5, 6, 7 and 8, respectively. Performance measures, numerical illustration and conclusion are presented in Sections 9, 10 and 11 respectively.

2. Mathematical model

We consider a discrete time queues with correlated arrivals and negative customers. Positive customers are generated with geometrically distributed lengths of *On* periods and *Off* periods and are served by two state Markovian server which alternates between *Good* and *Bad* states (Fig. 1) with geometrically distributed service times. We consider a late arrival system, in which the time axis is divided into fixed length intervals called slots and are marked by $0, 1, \ldots, m, \ldots$ in order. It is always assumed that all the queueing activities such as arrivals, removals and departures occur at the slot boundaries. Potential arrivals of positive as well as negative customers are occur in (m^-, m) in order and the potential departure of a positive customer occurs in (m, m^+). Positive customer arrive in a stochastic way and wait in infinite capacity waiting room (buffer) on first-in, first-out (FIFO) basis and they are considered for service and leaves the system. Further, the geometrically distributed arrival of negative customer removes the positive customers if any, and has no effect when the system is empty.

2.1. The positive arrival processes

The arrival of positive customers is modelled as the first order Markovian arrival process $(A_n, n \geqslant 0)$ with the number of positive arrivals in the arbitrary slot to be a random variable which depends on the number of arrivals during immediately preceding time slot. A discrete time, single *On-Off* source arrival process which alternate between *On* and *Off* periods respectively is considered. That is exactly one positive customer is generated in each time slot when the Markov chain is in state *On* and no positive customer is generated when the Markov chain is in state *Off.* The two independent parameters α and β which denote the probabilities that the Markov chain remains in states *On* and *Off* respectively. It is assumed that, the lengths of the *On-Off* periods are geometrically distributed random variables with rate $1-\alpha$ and $1-\beta$ respectively, when $0<\alpha<1$, $0<\beta<1$. Thus, the parameters of the positive arrival process are defined as

$\alpha = P$ (a positive arrival occurs during a slot/a positive arrival occurs during previous slot),
$\beta = P$ (no positive arrival occurs during a slot/no positive arrival occurs during previous slot).

Thus, according to Hunter [11], the steady state probability of mean positive arrival during an arbitrary slot is given by$\lambda_1 = \frac{1-\beta}{2-\alpha-\beta}$, where $0<\lambda_1<1$, which is the steady state condition of the time the chain spends in *On* periods. Since the rate at which the positive customer enters into the slot in steady state and the total time spends in *On* periods is considered, λ_1 is also called the effective arrival rate.

2.2. The removal rule and arrival times of negative customers

Upon arrival the negative customer removes the positive customer under service and vanishes. But, if the negative customer arrives during idle period (on empty system) it has no effect and lost. The inter-arrival times of negative customers $(B_n, n \geqslant 1)$ are independent and geometrically distributed random variables with the distribution,

$$P(B_n = k) = \bar{\theta}^{k-1}\theta,\ k \geqslant 1,\ \bar{\theta} = 1-\theta \text{ and } E(B_n) = \theta^{-1}.$$

2.3. The service processes

We assume that the service is also modelled as two state Markov chain, which alternates between *Good* and *Bad* states. In *Bad* state the service is likely to fail due to some errors and in *Good* state the server is ready to serve. We assume that the during a slot, if the server is in *Good* state, the server is ready to serve the customers, while in *Bad* state, server will not render serve even when there are customers in the queue. Server's state is then characterized by two independent parameters γ and σ, which is defined as

$\gamma =$ P (server is in *Good* state during a slot/server is in *Good* state during the previous slot),
$\sigma =$ P (server is in *Bad* state during a slot/server is in *Bad* state during the previous slot).

During *Good* state, exactly the service of one positive customer is completed per slot and in *Bad* state, service of the positive customer will not be completed. Both the lengths of the *Good* and *Bad* states are geometrically distributed random variables with rate $1-\gamma$ and $1-\sigma$ respectively. For avoiding trivial cases, we assume $0<\gamma<1$, $0<\sigma<1$.

Similarly, the steady state probability μ_1 of having a *Good* service during an arbitrary slot is given by $\mu_1 = \frac{1-\sigma}{2-\gamma-\sigma}$, where $0<\mu_1<1$, which corresponds to the overall fraction of *Good* service.

Positive customers are served by a two state Markovian server on the FCFS basis. Service of the positive customer takes place if the server is in *Good* state and therefore the service of a positive customer cannot start before the beginning of the slot following its arrival slot. Service times $(S_n, n \geqslant 1)$ are independent and geometrically distributed random variable with parameter μ and is given by

$$P(S_n = k) = (1-\mu)^{k-1}\mu,\ \ k \geqslant 1,\ \ E(S_n) = \mu^{-1} \text{ and } S(z) = \frac{\mu z}{1-(1-\mu)z},$$

where μ defines the probability that a positive customer concludes his service in a slot. The input such as O*n-Off* source, negative arrivals and *Good* and *Bad* state of service process are assumed to be mutually independent.

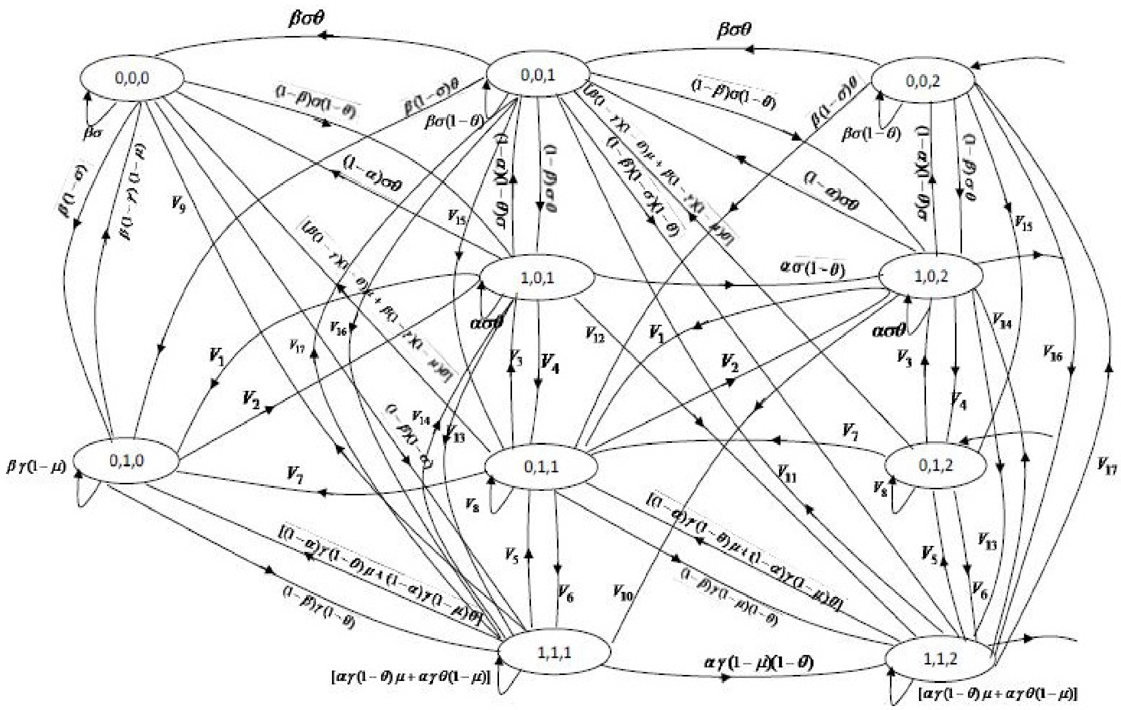

FIGURE 2. State transition diagram.

State transition diagram of the Markov chain is given below where

$V_1 = (1-\alpha)(1-\sigma)\theta;$

$V_2 = (1-\beta)(1-\gamma)(1-\theta)(1-\mu);$

$V_3 = [(1-\beta)\,(1-\gamma)\,(1-\theta)\,\mu + (1-\beta)\,(1-\gamma)\,(1-\mu)\,\theta];$

$V_4 = (1-\alpha)(1-\sigma)(1-\mu);$

$V_5 = (1-\alpha)\,\gamma\,(1-\theta)(1-\mu);$

$V_6 = [(1-\beta)\,\gamma\,(1-\theta)\,\mu + (1-\beta)\,\gamma\,(1-\mu)\,\theta];$

$\mathrm{V}_7 = [\beta\,\gamma\,\theta\,(1-\mu) + \beta\,\gamma\,\mu\,(1-\theta)];$

$V_8 = \beta\,\gamma\,(1-\theta)(1-\mu);$

$V_9 = [(1-\alpha)(1-\gamma)(1-\theta)\mu + (1-\alpha)(1-\gamma)\,\theta];$

$V_{10} = \alpha\,(1-\,\gamma)\,(1-\theta)\,(1-\mu);$

$V_{11} = [(1-\alpha)(1-\,\gamma)\,(1-\theta)\,\mu + (1-\alpha)(1-\,\gamma)\;\theta\,(1-\mu)];$

$V_{12} = \alpha\,(1-\sigma)\,(1-\theta);$

$V_{13} = \alpha\,(1-\sigma)\;\theta;$

$V_{14} = [\alpha\,(1-\,\gamma)\,(1-\theta)\,\mu + \alpha\,(1-\,\gamma)\;\theta\,(1-\mu)];$

$V_{15} = \beta\,(1-\sigma)(1-\theta);$

$V_{16} = (1-\beta)\,(1-\sigma)\,\theta;$

$V_{17} = (1-\alpha)\,(1-\gamma)\,(1-\theta)\,(1-\mu).$

3. GENERATING FUNCTION ANALYSIS

We define the system by a vector process $\{(A(m^+),\,S(m^+),\,N(m^+)),\quad m = 0,1,2\ldots,\}$ where:

$A(m^+)$ – number of positive customers arriving to the system during mth slot;
$S(m^+)$ – state of the server in the mth slot;
$N(m^+)$ – number of positive customers in the system at $m+$.

Also define,

$$Y_m(i,j,k) = \{A(m^+) = i,\;\; S(m^+) = j,\;\;\; N(m^+) = k\},\; i = 0,\,1\,;\;\; j = 0{,}1\;\; ;\, k = 0,\,1,\,2\ldots$$

whose state space is given by $S = \{(i,\,j,\,k) : \; i = 0{,}1\,;\;\; j = 0{,}1\;\; ;\, k = 0,\,1,\,2\ldots\}$.

The following generating function is defined to solve the system
$Q^m_{i,j}(z) = \sum_{k=0}^{\infty} P(A(m^+) = i,\ \ S(m^+) = j,\ \ N(m^+) = k) z^k$ with $i = 0{,}1;\ \ j = 0{,}1$ and if

$$j = \begin{cases} 0, \text{ if the server is in } \textit{Bad} \text{ state} \\ 1, \text{ if the server is in } \textit{Good} \text{ state.} \end{cases}$$

Also, the stationary distribution of the system is given by $\pi_{i,j,k} = \lim_{m\to\infty} P(Y_m(i,j,k))$ with

$Q_{0,0}(z) = \lim_{m\to\infty} Q^m_{0,0}(z);\quad Q_{1,0}(z) = \lim_{m\to\infty} Q^m_{1,0}(z)$ for server's *Bad* state and
$Q_{0,1}(z) = \lim_{m\to\infty} Q^m_{0,1}(z);\quad Q_{1,1}(z) = \lim_{m\to\infty} Q^m_{1,1}(z)$ for server's *Good* state.

Sum of these four server's state gives the stationary distribution of the queue length of the system. It is assumed that the condition for the system to be stable when $\rho<1$ and is shown in Theorem 1. By definition $Y_m(1,0,0) = 0$ and $Y_m(1,1,0) = 0$, because it is impossible that the system is empty in the mth slot with $A(m^+) = 1$, which gives $\pi_{1,0,0} = 0$ and $\pi_{1,1,0,} = 0$.

Kolmogorov equations of the stationary distribution of the system are given by

$$\begin{aligned}\pi_{0,0,0} &= \beta\sigma\pi_{0,0,0} + \beta\sigma\theta\pi_{0,0,1} + [\beta(1-\gamma)(1-\theta)\mu + \beta(1-\gamma)(1-\mu)\theta]\pi_{0,1,1} \\ &+ [(1-\alpha)(1-\gamma)(1-\theta)\mu + (1-\alpha)(1-\gamma)\theta]\pi_{1,1,1} + (1-\alpha)\sigma\theta\pi_{1,0,1} \\ &+ \beta(1-\gamma)(1-\mu)\pi_{0,1,0},\end{aligned} \tag{3.1}$$

$$\begin{aligned}\pi_{0,0,k} &= \beta\sigma\theta\pi_{0,0,k+1} + [\beta(1-\gamma)(1-\theta)\mu + \beta(1-\gamma)(1-\mu)\theta]\pi_{0,1,k+1} + \beta(1-\gamma)(1-\mu)(1-\theta)\pi_{0,1,k} \\ &+ [(1-\alpha)(1-\gamma)(1-\theta)\mu + (1-\alpha)(1-\gamma)(1-\mu)\theta]\pi_{1,1,k+1} + (1-\alpha)\sigma\theta\pi_{1,0,k+1} \\ &+ \beta\sigma(1-\theta)\pi_{0,0,k} + (1-\alpha)\ (1-\gamma)(1-\theta)(1-\mu)\pi_{1,1,k} + (1-\alpha)(1-\theta)\sigma\pi_{1,0,k}, \qquad k \geqslant 1\end{aligned} \tag{3.2}$$

$$\begin{aligned}\pi_{1,0,1} &= \alpha\sigma\theta\pi_{1,0,1} + (1-\beta)\sigma(1-\theta)\pi_{0,0,0} + (1-\beta)\sigma\theta\pi_{0,0,1} + (1-\beta)(1-\gamma)(1-\theta)(1-\mu)\pi_{0,1,0} \\ &+ [(1-\beta)(1-\gamma)(1-\theta)\mu + (1-\beta)(1-\gamma)(1-\mu)\theta]\pi_{0,1,1} \\ &+ [\alpha(1-\gamma)(1-\theta)\mu + \alpha(1-\gamma)\theta(1-\mu)]\pi_{1,1,1},\end{aligned} \tag{3.3}$$

$$\begin{aligned}\pi_{1,0,k} &= \alpha\sigma\theta\pi_{1,0,k} + \alpha\sigma(1-\theta)\pi_{1,0,k-1} + (1-\beta)\sigma\theta\pi_{0,0,k} + (1-\beta)(1-\gamma)(1-\theta)(1-\mu)\pi_{0,1,k-1} \\ &+ (1-\beta)\sigma(1-\theta)\pi_{0,0,k-1} + [(1-\beta)(1-\gamma)(1-\theta)\mu + (1-\beta)(1-\gamma)(1-\mu)\theta]\pi_{0,1,k} \\ &+ [\alpha(1-\gamma(1-\theta)\mu + \alpha(1-\gamma)(1-\mu)\theta]\pi_{1,1,k} + \alpha(1-\gamma)(1-\theta)(1-\mu)\pi_{1,1,k-1} \\ &+ (1-\beta)\sigma(1-\theta)\pi_{0,0,k-1}, \qquad k \geqslant 2\end{aligned} \tag{3.4}$$

$$\begin{aligned}\pi_{0,1,0} &= \beta\gamma(1-\mu)\pi_{0,1,0} + \beta(1-\sigma)\pi_{0,0,0} + (1-\alpha)(1-\sigma)\theta\pi_{1,0,1} + \beta(1-\sigma)\theta\pi_{0,0,1} \\ &+ [\beta\gamma\theta(1-\mu) + \ \beta\gamma\mu(1-\theta)]\pi_{0,1,1} + [(1-\alpha)\gamma(1-\theta)\mu + (1-\alpha)\gamma\theta(1-\mu)]\pi_{1,1,1},\end{aligned} \tag{3.5}$$

$$\begin{aligned}\pi_{0,1,k} &= \beta(1-\sigma)(1-\theta)\pi_{0,0,k} + (1-\alpha)(1-\sigma)(1-\theta)\pi_{1,0,k} + (1-\alpha)(1-\sigma)\theta\pi_{1,0,k+1} \\ &+ \beta\gamma(1-\theta)(1-\mu)\pi_{0,1,k} + [(1-\alpha)\gamma(1-\theta)\mu + (1-\alpha)\gamma(1-\mu)\theta]\pi_{1,1,k+1} \\ &+ (1-\alpha)\gamma(1-\theta)(1-\mu)\pi_{1,1,k} \ + \ \beta(1-\sigma)\theta\pi_{0,0,k+1} \\ &+ [\beta\gamma\theta(1-\mu) + \beta\gamma\mu\ (1-\theta)]\pi_{0,1,k+1}, \qquad k \geqslant 1\end{aligned} \tag{3.6}$$

$$\begin{aligned}\pi_{1,1,1} &= (1-\beta)(1-\sigma)(1-\theta)\pi_{0,0,0} + (1-\beta)(1-\sigma)\theta\pi_{0,0,1} + \alpha(1-\sigma)\theta\pi_{1,0,1} \\ &+ (1-\beta)\gamma(1-\mu)(1-\theta)\ \pi_{0,1,0} + [\alpha\gamma(1-\theta)\mu + \alpha\gamma\theta(1-\mu)]\pi_{1,1,1} \\ &+ [\ (1-\beta)\gamma(1-\theta)\mu + (1-\beta)\gamma\theta(1-\mu)]\pi_{0,1,1},\end{aligned} \tag{3.7}$$

$$\begin{aligned}\pi_{1,1,k} &= (1-\beta)(1-\sigma)\theta\pi_{0,0,k} + \alpha(1-\sigma)(1-\theta)\pi_{1,0,k-1} + (1-\beta)(1-\sigma)(1-\theta)\pi_{0,0,k-1} \\ &+ [\alpha\gamma(1-\theta)\mu + \alpha\gamma\theta(1-\mu)]\pi_{1,1,k} + [(1-\beta)\gamma(1-\theta)\mu + (1-\beta)\gamma\theta(1-\mu)]\pi_{0,1,k} \\ &+ (1-\beta)\gamma(1-\mu)(1-\theta)\pi_{0,1,k-1} + \alpha\gamma(1-\mu)(1-\theta)\pi_{1,1,k-1} + \alpha(1-\sigma)\theta\pi_{1,0,k}, \quad k \geqslant 2.\end{aligned} \tag{3.8}$$

4. Steady state buffer content

This section is devoted to find the stationary distribution of the steady state buffer contents $Q(z)$ at the boundary of an arbitrary slot and the expected number of customers in the system.

Theorem 4.1. *In steady state, the stationary distribution of the buffer content at the boundary of an arbitrary slot is given by* $Q(z) = \frac{T_{0,0}(z)+T_{0,1}(z)+T_{1,0}(z)+T_{1,1}(z)}{T(z)}$, *under condition* ρ *<1, where* $T_{0,0}(z) + T_{0,1}(z) + T_{1,0}(z) + T_{1,1}(z)$ *and* $T(z)$ *are given Appendix I.*

Proof. Multiplying equations (3.1)−(3.8) by z^k and summing over k, these equations becomes

$$\begin{aligned}[z-\beta\sigma(\theta+z(1-\theta)]Q_{0,0}(z) = {} & z\beta\sigma\theta\pi_{0,0,0} + z\beta(1-\gamma)(1-\mu)\theta\pi_{0,1,0} \\ & + \beta(1-\gamma)h(z)Q_{0,1}(z) \\ & + (1-\alpha)\sigma[\theta+z(1-\theta)]Q_{1,0}(\mathrm{z}) + (1-\alpha)(1-\gamma)h(z)Q_{1,1}(z),\end{aligned} \tag{4.1}$$

$$\begin{aligned}[1-\alpha\sigma(\theta+z(1-\theta))]Q_{1,0}(z) = {} & z(1-\beta)\sigma\theta\pi_{0,0,0} + z(1-\beta)(1-\gamma)(1-\mu)\theta\pi_{0,1,0} \\ & + (1-\beta)\sigma[\theta+ z(1-\theta)]Q_{0,0}(z) \\ & + (1-\beta)(1-\gamma)h(z)Q_{0,1}(z) + \alpha(1-\gamma)h(z)Q_{1,1}(\mathrm{z}),\end{aligned} \tag{4.2}$$

$$\begin{aligned}[z-\beta\gamma\, h(z)]Q_{0,1}(z) = {} & z\beta(1-\sigma)\theta\pi_{0,0,0} + z\beta\gamma(1-\mu)\theta\pi_{0,1,0} \\ & + \beta(1-\sigma)[\theta+z(1-\theta)]Q_{0,0}(z) + (1-\alpha)(1-\sigma)[\theta+z(1-\theta)]Q_{1,0}(z) \\ & + (1-\alpha)\gamma\, h(z)Q_{1,1}(z),\end{aligned} \tag{4.3}$$

$$\begin{aligned}[1-\alpha\gamma\, h(z)]Q_{1,1}(z) = {} & z(1-\beta)(1-\sigma)\theta\pi_{0,0,0} + z(1-\beta)\gamma(1-\mu)\theta\pi_{0,1,0} \\ & + (1-\beta)(1-\sigma)[\theta+z(1-\theta)]Q_{0,0}(z) + \alpha(1-\sigma)[\theta+z(1-\theta)]Q_{1,0}(z) \\ & + (1-\beta)\gamma\, h(z)Q_{0,1}(\mathrm{z}),\end{aligned} \tag{4.4}$$

where $h(z) = (1-\theta)\mu + \theta(1-\mu) + z(1-\theta)(1-\mu)$.

Equations (4.1)−(4.4) can be written as the following system of equations

$$\begin{aligned}&[z-\beta\sigma(\theta+z(1-\theta)]Q_{0,0}(z) - (1-\alpha)\sigma[\theta+z(1-\theta)]Q_{1,0}(\mathrm{z}) - \beta(1-\gamma)h(z)Q_{0,1}(z) \\ &\qquad - (1-\alpha)(1-\gamma)h(z)Q_{1,1}(z) = z\beta\sigma\theta\pi_{0,0,0} + z\beta(1-\gamma)(1-\mu)\theta\pi_{0,1,0},\end{aligned} \tag{4.5}$$

$$\begin{aligned}&-(1-\beta)\sigma[\theta+ z(1-\theta)]Q_{0,0}(z) + [1-\alpha\sigma(\theta+ z(1-\theta))]Q_{1,0}(z) - (1-\beta)(1-\gamma)h(z)Q_{0,1}(z) \\ &\qquad - \alpha(1-\gamma)h(z)Q_{1,1}(\mathrm{z}) = z(1-\beta)\sigma\theta\pi_{0,0,0} + z(1-\beta)(1-\gamma)(1-\mu)\theta\pi_{0,1,0},\end{aligned} \tag{4.6}$$

$$\begin{aligned}&-\beta(1-\sigma)[\theta+z(1-\theta)]Q_{0,0}(z) - (1-\alpha)(1-\sigma)[\theta+z(1-\theta)]Q_{1,0}(z) + [z-\beta\gamma\, h(z)]Q_{0,1}(z) \\ &\qquad - (1-\alpha)\gamma\, h(z)Q_{1,1}(z) = z\beta(1-\sigma)\theta\pi_{0,0,0} + z\beta\gamma(1-\mu)\theta\pi_{0,1,0},\end{aligned} \tag{4.7}$$

$$\begin{aligned}&-(1-\beta)(1-\sigma)[\theta+z(1-\theta)]Q_{0,0}(z) - \alpha(1-\sigma)[\theta+z(1-\theta)]Q_{1,0}(z) - (1-\beta)\gamma\, h(z)Q_{0,1}(\mathrm{z}) \\ &\qquad + [1-\alpha\gamma\, h(z)]Q_{1,1}(z) = z(1-\beta)(1-\sigma)\theta\pi_{0,0,0} + z(1-\beta)\gamma(1-\mu)\theta\pi_{0,1,0}.\end{aligned} \tag{4.8}$$

From equations (4.5) to (4.8), set

$$\begin{aligned}
&a_{11} = z - \beta\sigma(\theta + z(1-\theta)), && a_{12} = -(1-\alpha)\sigma(\theta + z(1-\theta)),\\
&a_{13} = -\beta(1-\gamma)h(z), && a_{14} = -(1-\alpha)(1-\gamma)h(z),\\
&a_{21} = -(1-\beta)\sigma(\theta + z(1-\theta)), && a_{22} = 1 - \alpha\sigma(\theta + z(1-\theta)),\\
&a_{23} = -(1-\beta)(1-\gamma)h(z), && a_{24} = -\alpha(1-\gamma)h(z),\\
&a_{31} = -\beta(1-\sigma)(\theta + z(1-\theta)), && a_{32} = -(1-\alpha)(1-\sigma)(\theta + z(1-\theta)),\\
&a_{33} = z - \beta\gamma\, h(z), && a_{34} = -(1-\alpha)\gamma\, h(z),\\
&a_{41} = -(1-\beta)(1-\sigma)(\theta + z(1-\theta)), && a_{42} = -\alpha(1-\sigma)(\theta + z(1-\theta)),\\
&a_{43} = -(1-\beta)\gamma\, h(z), && a_{44} = 1 - \alpha\gamma\, h(z).
\end{aligned}$$

Using these assumptions, equations (4.5) to (4.8) can be written as the matrix form $AX = B$, where

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14}\\ a_{21} & a_{22} & a_{23} & a_{24}\\ a_{31} & a_{32} & a_{33} & a_{34}\\ a_{41} & a_{42} & a_{43} & a_{44}\end{bmatrix}, \quad X = \begin{bmatrix} Q_{0,0}(z)\\ Q_{1,0}(z)\\ Q_{0,1}(z)\\ Q_{1,1}(z)\end{bmatrix} \text{ and}$$

$$B = \begin{bmatrix} z\beta\sigma\theta\pi_{0,0,0} + z\beta(1-\gamma)(1-\mu)\theta\pi_{0,1,0}\\ z(1-\beta)\sigma\theta\pi_{0,0,0} + z(1-\beta)(1-\gamma)(1-\mu)\theta\pi_{0,1,0}\\ z\beta(1-\sigma)\theta\pi_{0,0,0} + z\beta\gamma(1-\mu)\theta\pi_{0,1,0}\\ z(1-\beta)(1-\sigma)\theta\pi_{0,0,0} + z(1-\beta)\gamma(1-\mu)\theta\pi_{0,1,0}\end{bmatrix}.$$

This system can be solved using Cramer's rule. The unknowns $Q_{0,0}(z), Q_{1,0}(z), Q_{0,1}(z),\ Q_{1,1}(z)$ gives the generating functions of server's states, which are given by

$$Q_{0,0}(z) = \frac{T_{0,0}(z)}{T(z)}, Q_{1,0}(z) = \frac{T_{1,0}(z)}{T(z)}, Q_{0,1}(z) = \frac{T_{0,1}(z)}{T(z)},\ Q_{1,1}(z) = \frac{T_{1,1}(z)}{T(z)},$$

where

$$T_{0,0}(z) = \begin{vmatrix} z\beta\sigma\theta\pi_{0,0,0} + z\beta(1-\gamma)(1-\mu)\theta\pi_{0,1,0} & a_{12} & a_{13} & a_{14}\\ z(1-\beta)\sigma\theta\pi_{0,0,0} + z(1-\beta)(1-\gamma)(1-\mu)\theta\pi_{0,1,0} & a_{22} & a_{23} & a_{24}\\ z\beta(1-\sigma)\theta\pi_{0,0,0} + z\beta\gamma(1-\mu)\theta\pi_{0,1,0} & a_{32} & a_{33} & a_{34}\\ z(1-\beta)(1-\sigma)\theta\pi_{0,0,0} + z(1-\beta)\gamma(1-\mu)\theta\pi_{0,1,0} & a_{42} & a_{43} & a_{44}\end{vmatrix}, \tag{4.9}$$

$$T_{1,0}(z) = \begin{vmatrix} a_{11} & z\beta\sigma\theta\pi_{0,0,0} + z\beta(1-\gamma)(1-\mu)\theta\pi_{0,1,0} & a_{13} & a_{14}\\ a_{21} & z(1-\beta)\sigma\theta\pi_{0,0,0} + z(1-\beta)(1-\gamma)(1-\mu)\theta\pi_{0,1,0} & a_{23} & a_{24}\\ a_{31} & z\beta(1-\sigma)\theta\pi_{0,0,0} + z\beta\gamma(1-\mu)\theta\pi_{0,1,0} & a_{33} & a_{34}\\ a_{41} & z(1-\beta)(1-\sigma)\theta\pi_{0,0,0} + z(1-\beta)\gamma(1-\mu)\theta\pi_{0,1,0} & a_{43} & a_{44}\end{vmatrix}, \tag{4.10}$$

$$T_{0,1}(z) = \begin{vmatrix} a_{11} & a_{12} & z\beta\sigma\theta\pi_{0,0,0} + z\beta(1-\gamma)(1-\mu)\theta\pi_{0,1,0} & a_{14} \\ a_{21} & a_{22} & z(1-\beta)\sigma\theta\pi_{0,0,0} + z(1-\beta)(1-\gamma)(1-\mu)\theta\pi_{0,1,0} & a_{24} \\ a_{31} & a_{32} & z\beta(1-\sigma)\theta\pi_{0,0,0} + z\beta\gamma(1-\mu)\theta\pi_{0,1,0} & a_{34} \\ a_{41} & a_{42} & z(1-\beta)(1-\sigma)\theta\pi_{0,0,0} + z(1-\beta)\gamma(1-\mu)\theta\pi_{0,1,0} & a_{44} \end{vmatrix}, \tag{4.11}$$

$$T_{1,1}(z) = \begin{vmatrix} a_{11} & a_{12} & a_{13} & z\beta\sigma\theta\pi_{0,0,0} + z\beta(1-\gamma)(1-\mu)\theta\pi_{0,1,0} \\ a_{21} & a_{22} & a_{23} & z(1-\beta)\sigma\theta\pi_{0,0,0} + z(1-\beta)(1-\gamma)(1-\mu)\theta\pi_{0,1,0} \\ a_{31} & a_{32} & a_{33} & z\beta(1-\sigma)\theta\pi_{0,0,0} + z\beta\gamma(1-\mu)\theta\pi_{0,1,0} \\ a_{41} & a_{42} & a_{43} & z(1-\beta)(1-\sigma)\theta\pi_{0,0,0} + z(1-\beta)\gamma(1-\mu)\theta\pi_{0,1,0} \end{vmatrix}, \tag{4.12}$$

$$T(z) = |A|\,. \tag{4.13}$$

Then the stationary distribution of the steady state buffer contents at the boundary of an arbitrary slot is given by:

$$Q(z) = Q_{0,0}(z) + Q_{0,1}(z) + Q_{1,0}(z) + Q_{1,1}(z),$$

$$Q(z) = \frac{T_{0,0}(z) + T_{0,1}(z) + T_{1,0}(z) + T_{1,1}(z)}{T(z)}\cdot \tag{4.14}$$

By substituting equations (4.9)–(4.13) in (4.14), we get the stationary distribution of the steady state buffer content at the boundary of an arbitrary slot. Using the stability condition $\lim_{z\to 1} Q(z) = 1$ in equation (4.14), we get

$$T_{0,0}(1) + T_{0,1}(1) + T_{1,0}(1) + T_{1,1}(1) = T(1).$$

Simplifying this equation we get,

$$\pi_{0,0,0} + \pi_{0,1,0} = 1 - \frac{1-\beta}{(2-\alpha-\beta)(1-(1-\mu)(1-\theta)(1-\sigma))} = 1-\rho.$$

where $\rho < 1$ is to be the stability condition of the considered queueing system, which is given in Section 5.

Remark 4.2. Using the normalisation condition $\lim_{z\to 1} Q(z) = 1$ and the roots of $\nabla(z) = 0$, one can find the unknowns $\pi_{0,0,0}$ and $\pi_{0,1,0}$. Since the system contains too many parameters the expressions for the unknown probabilities $\pi_{0,0,0}$ and $\pi_{0,1,0}$ are not given explicitly, but the values are tabulated in Section 9 using MATLAB with the specific values for $\alpha, \beta, \gamma, \sigma, \theta, \mu$.

5. Modified service time

The actual service time that a positive customer receives before departing the system either by service completion or by a negative arrival can be defined as modified service time S^*. Its probability mass function is given by

$$P(S^* = k) = P(S_n = k)(1-\theta)^k(1-\sigma)^k + P(S \geqslant k)\theta(1-\theta)^{k-1}(1-\sigma)^k.$$

The probability generating function of the modified service time $S^*(z)$ is derived as

$$S^*(z) = \sum_{k=1}^{\infty} P(S^* = k)z^k = \frac{[1-(1-\mu)(1-\theta)(1-\sigma)]z}{1-(1-\mu)(1-\theta)(1-\sigma)z}.$$

That is the modified service time has geometric distribution with parameter $\tau = [1-(1-\mu)(1-\theta)(1-\sigma)]$, which means that the positive customer will leave the system with probability τ (effective service rate) and stay in the system with probability $1-\tau$ in each slot.

The average occupancy of the server is defined as the traffic intensity (ρ) of the system and is obtained as

$$\rho = \frac{\lambda_1}{\tau} = \frac{1-\beta}{(2-\alpha-\beta)(1-(1-\mu)(1-\theta)(1-\sigma))}. \tag{5.1}$$

6. Unfinished work

The remaining number of slots needed to serve all positive customers present in the system is known as the unfinished work (W) of the queueing system in steady state. According to Wang *et al.* [13], the unfinished work W is given by $W = (C-1)S + R$ where C is the stationary queue length including the customer being served if any, S is the service time of an arbitrary positive customer and R is the remainder service time of the positive customer which is being served in server. The probability generating function of W is obtained using the memory less property of geometrically distributed service time and is given by:

$$W(z) = \frac{Q_{0,0}(z,S^*(z)) + Q_{0,1}(z,S^*(z)) + Q_{1,0}(z,S^*(z)) + Q_{1,1}(z,S^*(z))}{S^*(z)}. \tag{6.1}$$

7. Stationary sojourn time distribution

The sojourn time of an arbitrary positive customer is defined as the total number of slots between the boundary of the arrival slot of a tagged positive customer and the departure instant of this positive customer. In the case of MMBP/G/1 queueing system without negative customers, the probability generating function (PGF) of the stationary sojourn time distribution of the positive customers $D(z)$ is known as (see Zhou and Wang [18]),

$$D(z) = \lim_{n\to\infty} \sum_{j=1}^{\infty} \frac{P(W_n = 1\,,\, A_n = 1)}{P(\,A_n = 1)}\, z^j,$$

where W_n is the unfinished work in the nth slot. In steady state, $\lim\limits_{n\to\infty} W_n = CS + R$ and $\lim\limits_{n\to\infty} P(A_n = 1) = \lambda_1$.

Using this, PGF of the stationary sojourn time distribution of the positive and negative customers, when the server alternate between *Good* and *Bad* state is given by

$$D(z) = \frac{Q_{1,0}(z,S^*(z)) + Q_{1,1}(z,S^*(z))}{\lambda_1} = \frac{1}{\lambda_1}\frac{T_{1,0}(z,S^*(z)) + T_{1,1}(z,S^*(z))}{T(z,S^*(z))}. \tag{7.1}$$

8. Special cases

1. When $\sigma = 0$, the server is always in *Good* state. Then, the model becomes a discrete-time single-server infinite-capacity queueing system with correlated arrivals, geometrically distributed service times and negative customers. Substituting $\sigma = 0$ in equation (5.1) it reduces to

$$\rho = \frac{1-\beta}{(2-\alpha-\beta)(1-(1-\mu)(1-\theta))}. \tag{8.1}$$

   Equation (8.1) coincides with equation of Wang *et al.* [13].

TABLE 1. Mean queue length and mean packet delay.

μ	$\pi_{0,0,0}$	$\pi_{0,1,0}$	Mean queue lengh	Mean packet delay
0.1	0.0262	0.0316	28.1457	50.1813
0.2	0.0554	0.0674	12.6088	21.0517
0.3	0.0883	0.1084	7.4339	11.7396
0.4	0.1256	0.1558	4.8413	7.3095
0.5	0.1686	0.2114	3.2861	4.8046
0.6	0.2186	0.2777	2.2461	3.2347
0.7	0.2778	0.3585	1.5006	2.1852
0.8	0.3495	0.4596	0.9385	1.4500
0.9	0.4390	0.5910	0.4975	0.9164

Also, when the server is always in *Good* state, that is when $\sigma = 0$, we get $Q_{1,0}(z) + Q_{1,1}(z) = Q_1(z)$ and $Q_{0,0}(z) + Q_{0,1}(z) = Q_0(z)$. Now equation (6.1) reduces to

$$W(z) = \frac{Q_1(z, S^*(z)) + Q_0(z, S^*(z))}{S^*(z)}, \tag{8.2}$$

which coincides with the PGF of the unfinished work of Wang *et al.* [13].
Similarly, when $\sigma = 0$ equation (7.1) reduces to

$$D(z) = \frac{Q_1(z, S^*(z))}{\lambda_1} = \frac{1}{\lambda_1} \frac{S^*(z)(1-\beta)(S^*(z) - h(S^*(z)))\pi_{0,0,0}}{S^*(z) - h(S^*(z))(\beta + \bar{\beta} S^*(z)) - (1-\alpha-\beta)h^2(S^*(z))}. \tag{8.3}$$

which coincides with the equation of Wang *et al.* [13].

2. When there is no negative arrival, that is if $\theta = 0$, and the server is in always *Good* state. Then the model becomes a discrete time queueing model with first order Markovian arrival process and geometrically distributed service times. Substituting $\theta = 0$ and $\sigma = 0$ in equation (5.1), we get

$$\rho = \frac{1-\beta}{(2-\alpha-\beta)\mu}. \tag{8.4}$$

Equation (8.4) is consistent with corresponding traffic intensity of Zhou and Wang [18].

9. PERFORMANCE MEASURES

1. Server's idle period $= \pi_{0,0,0} + \pi_{0,1,0}$.
2. Mean Queue length:
   Expected number of customers in the system is obtained by $E(Q) = Q'(1)$.
3. Average Packet delay:

The average packet delay of a positive customer can be obtained using sojourn time distribution, given by $E(D) = \lim_{z\to 1} \frac{\mathrm{d}(D(z))}{\mathrm{d}z}$.

10. NUMERICAL ILLUSTRATION

The impact on the main parameters of the performance measures are presented with numerical examples.

Using MATLAB, the values of the unknown constants $\pi_{0,0,0}$, $\pi_{0,1,0}$ are obtained. Using equation (4.14), the mean queue length and mean packet delay for different values of μ are obtained for the parameters $\alpha = 0.4$, $\beta = 0.3$, $\gamma = 0.5$, $\sigma = 0.4$, $\theta = 0.2$ and are tabulated in Table 1.

TABLE 2. Mean queue length *vs.* α, β.

β \ α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	10.5081	11.8524	13.3567	15.0584	17.0067	19.2415	21.8515	24.9159	28.5793
0.2	9.1337	10.4756	11.9887	13.7205	15.7076	18.0195	20.7523	24.0095	27.9725
0.3	7.5934	8.9179	10.4286	12.1727	14.1959	16.5796	19.4381	22.9066	27.2182
0.4	5.8459	7.1411	8.6319	10.3710	12.4148	14.8581	17.8389	21.5355	26.2551
0.5	3.8517	5.0943	6.5416	8.2485	10.2854	12.7634	15.8507	19.7848	24.9824
0.6	1.5577	2.7148	4.0795	5.7093	7.6941	10.1637	13.3120	17.4714	23.2224
0.7	0	0	1.1343	2.6222	4.4726	6.8369	9.9576	14.2722	20.6289
0.8	0	0	0	0	0.3591	2.4410	5.3187	9.5581	16.4264
0.9	0	0	0	0	0	0	0	1.9190	8.4474

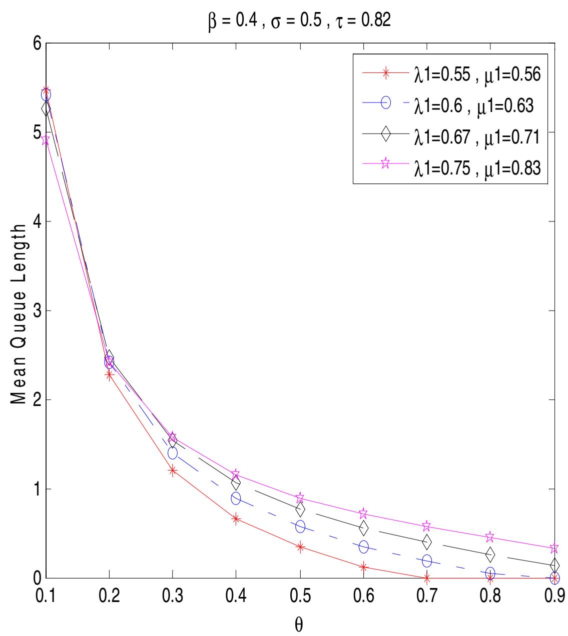

FIGURE 3. Mean Queue length *versus* theta.

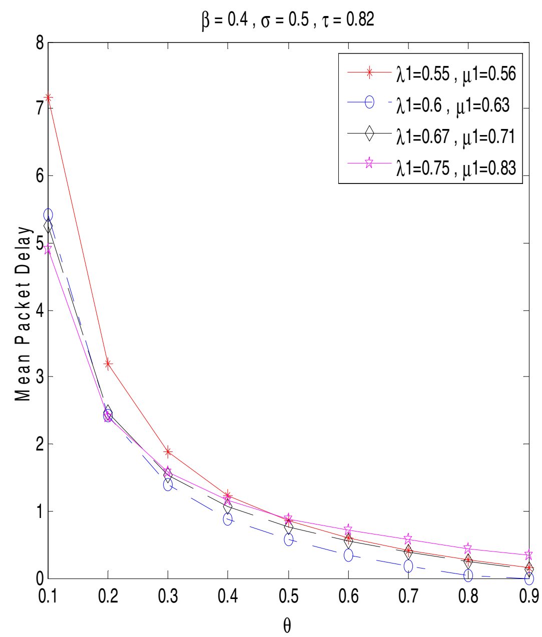

FIGURE 4. Mean packet delay *versus* theta.

From Table 1, one can assume that, when the service rate increases, mean queue length and mean packet delay decrease.

Table 2 gives the effect of β on the mean queue length for different values of α. The remaining parameters chosen here is $\gamma = 0.5, \sigma = 0.4, \theta = 0.1, \mu = 0.4$.

From Table 2 we can say that increase of *On* source will increase the mean queue length and increase of *Off* source will decrease the mean queue length. Figures 3 and 4 gives the impact the negative arrival θ with mean queue length and mean packet delay for different values of effective arrival rate λ_1.

The relationship between the mean queue length (mean packet delay) and the effective service rate τ for different values of the effective arrival rate λ_1 are illustrated in Figures 5 and 6.

Figures 7 and 8 give the impact of effective arrival rate λ_1 and β on mean queue length and mean packet delay.

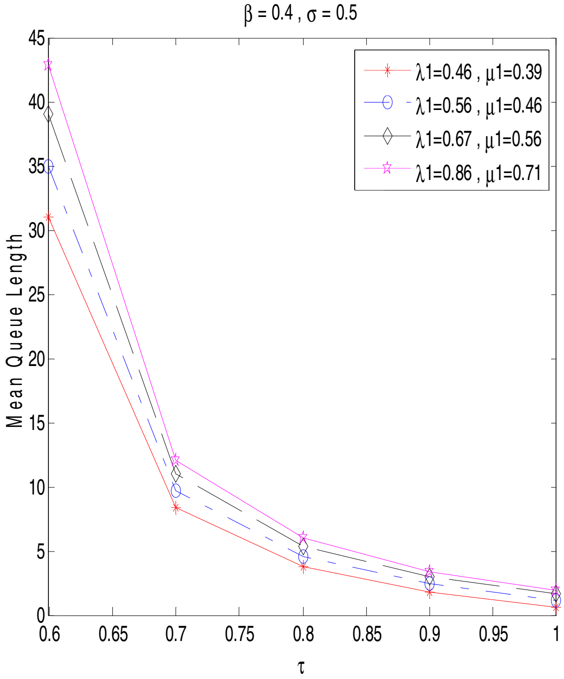

FIGURE 5. Mean queue length *versus* effective service rate.

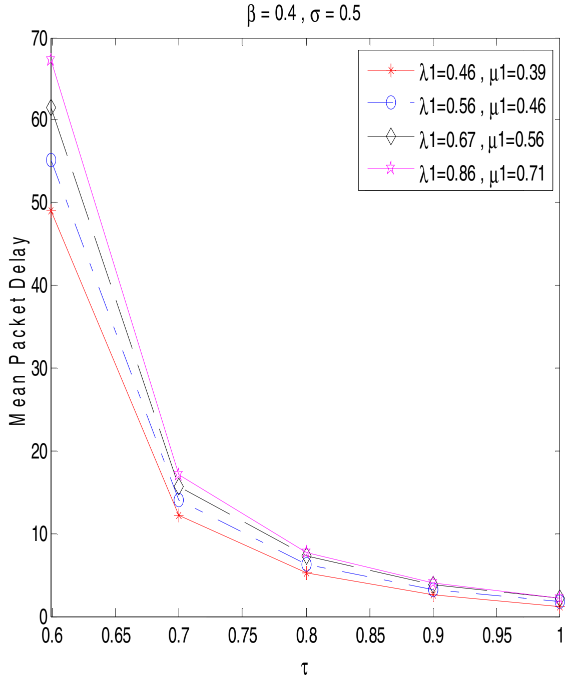

FIGURE 6. Mean packet delay *versus* effective service rate.

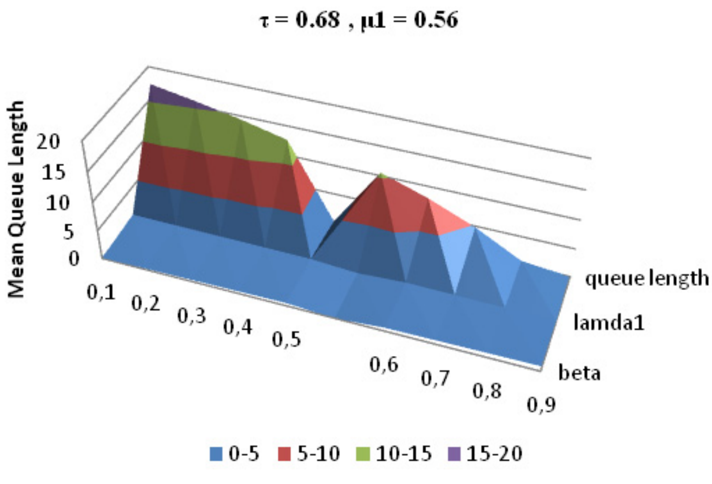

FIGURE 7. Mean queue length *versus* effective arrival rate and β.

Figures 9 and 10 give the relationship between the mean queue length(mean packet delay) and effective arrival rate for different values of effective service rate.

One can observe that, from all the figures, the mean queue length and mean packet delay are increasing with the increasing values of effective arrival rate λ_1.

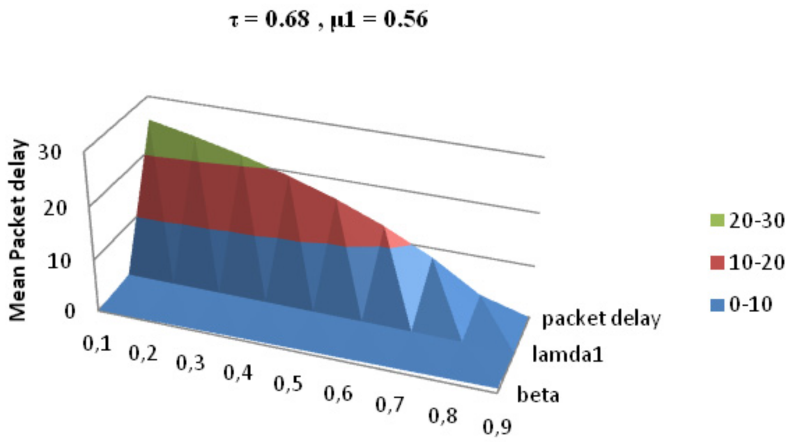

FIGURE 8. Mean packet delay *versus* effective arrival rate and β.

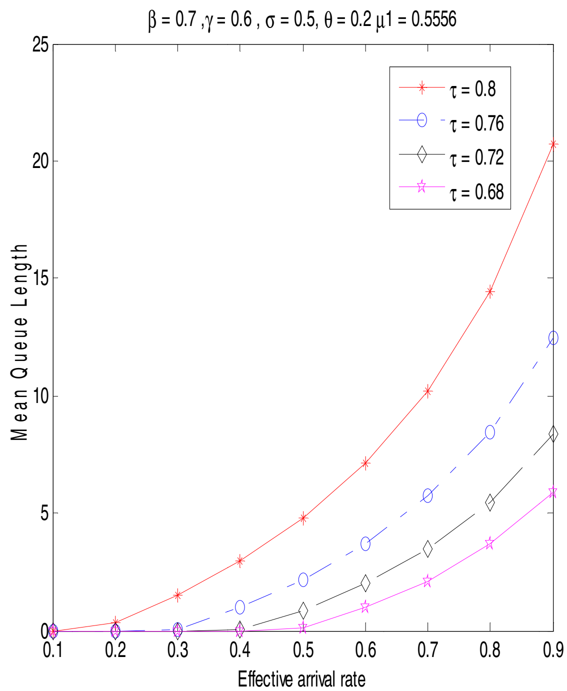

FIGURE 9. Mean queue length *versus* effective arrival rate.

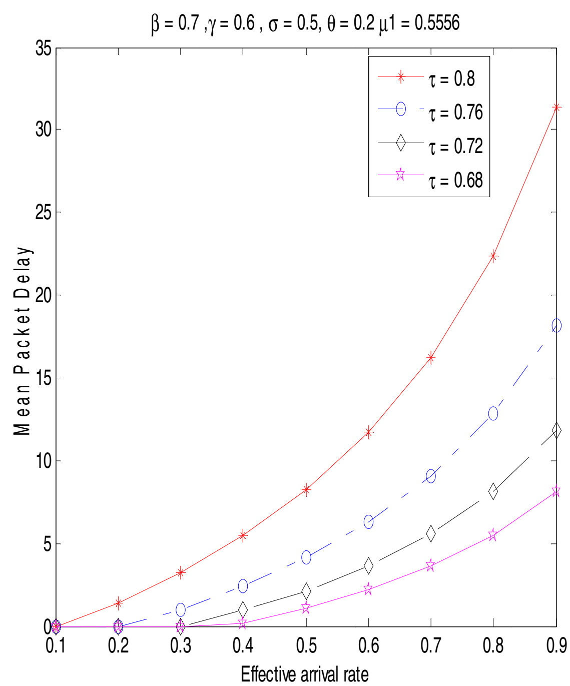

FIGURE 10. Mean packet delay *versus* effective arrival rate.

11. CONCLUSION

In this paper, a discrete time infinite queueing system with *On-Off* source, as well as geometrically distributed service times with two states of a server namely *Good* and *Bad* and negative customers are analysed using Markovian processes approach. Closed-form expressions for stationary distribution of the steady state buffer contents at the boundary of an arbitrary slot, unfinished work and stationary sojourn time distribution are obtained. Thereafter the measures of interest are also evaluated with numerical illustration. Future work can be carried out with this model for multi-servers with 'm' homogeneous *On-Off* sources.

Appendix I

Applying Cramer's rule and simplifying we get:

$$
\begin{aligned}
&T_{0,0}(z)+T_{0,1}(z)+T_{1,0}(z)+T_{1,1}(z)=s1\times[1+\alpha^2+\beta^2-2\alpha-2\beta+2\alpha\beta]\\
&\times\Big\{[\theta+z(1-\theta)]\,h^2(z)\left(\gamma-\gamma^2-\sigma\gamma\right)+[\theta+z(1-\theta)]^2\,h(z)\left(-1+\gamma-\sigma^2-\sigma\gamma+2\sigma\right)\Big\}\\
&+s2\times[1+\alpha^2+\beta^2-2\alpha-2\beta+2\alpha\beta]\\
&\times\Big\{[\theta+z(1-\theta)]\,h^2(z)\left(-1+\sigma+2\gamma-\gamma^2-\sigma\gamma\right)+[\theta+z(1-\theta)]^2\,h(z)\left(\sigma-\sigma^2-\sigma\gamma\right)\Big\}\\
&+s3\times[\theta+z(1-\theta)]\,h(z)\\
&\times\Big\{\left(\beta-\beta^2-\alpha\beta\right)(1-\sigma-\gamma)-\left(1-2\alpha-\beta+\alpha^2+\alpha\beta\right)(z-\sigma z)-2\sigma\,\gamma\,z(1-\alpha-\beta)\\
&+\gamma^2 z\left(2-3\alpha-2\beta+\alpha^2+\alpha\beta\right)\Big\}+s4\times[\theta+z(1-\theta)]\,h(z)\\
&\times\left\{\left(\alpha-\alpha^2-\alpha\beta\right)(z-\sigma z-\gamma z)-\left(1-\alpha-2\beta+\beta^2+\alpha\beta\right)+\sigma\left(2-2\alpha-3\beta+\beta^2+\alpha\beta\right)\right\}+(s4+z\times s3)\\
&\times\Big\{z-\gamma^2h^2(z)\,(1-\alpha-\beta)-[\theta+z(1-\theta)]^2\,\sigma^2\,(1-\alpha-\beta)-[\theta+z(1-\theta)]\,(\alpha z+\beta)\,\sigma\Big\}\\
&-(\mathrm{s}1+z\times s2)\times\gamma\,\beta\times h^2(z)\ (\alpha\,z+\beta)-s2\times(\beta+z(1-\beta))\times(\alpha\,z+\beta)\\
&+(1-\alpha-\beta)\Big\{[\theta+z(1-\theta)]^2\,\sigma\,(\beta+z(1-\beta))\times s1+h^2(z)\,\gamma\Big\}\\
&+\{s1\times[\theta+z(1-\theta)]+s2\times\ h(z)\}\times\left[(\beta+z-\beta z)\times\beta+\left(z+az^2-az\right)\times(1-\beta)\right],
\end{aligned}
\tag{A.1}
$$

where

$$
s1=(z\sigma\theta\pi_{0,0,0}+z(1-\gamma)(1-\mu)\theta\pi_{0,1,0}),\quad s2=(z(1-\sigma)\theta\pi_{0,0,0}+z\gamma(1-\mu)\theta\pi_{0,1,0}),
$$
$$
s3=\ (z(1-\beta)\theta\pi_{0,0,0}+z(1-\beta)(1-\mu)\theta\pi_{0,1,0}),\quad s4=(z\beta\theta\pi_{0,0,0}+z\beta(1-\mu)\theta\pi_{0,1,0}).
$$

$$
\begin{aligned}
T(z)=|A|=&\Big([z-\beta\sigma[\theta+z(1-\theta)]]\ \ [z-\beta\gamma\,h(z)]\\
&\times\left\{[1-\alpha\sigma[\theta+z(1-\theta)]]\,[1-\alpha\gamma\,h(z)]-\alpha^2(1-\alpha)(1-\gamma)h(z)[\theta+z(1-\theta)]\right\}\Big)\\
&-(1-\alpha)(1-\beta)\gamma^2h^2(z)\,[z-\beta\sigma[\theta+z(1-\theta)]]\ [1-\alpha\sigma[\theta+z(1-\theta)]]\\
&-(1-\alpha)(1-\beta)(1-\sigma)(1-\gamma)h(z)[\theta+z(1-\theta)]\ [z-\beta\sigma[\theta+z(1-\theta)]]\ [1-\alpha\gamma\,h(z)]\\
&-\alpha(1-\alpha)(1-\sigma)\gamma\,h(z)[\theta+z(1-\theta)]\,[z-\beta\sigma[\theta+z(1-\theta)]]\,\{1+(1-\beta)(1-\gamma)h(z)\}\\
&-(1-\alpha)(1-\beta)\sigma^2[\theta+z(1-\theta)]^2\,[z-\beta\gamma\,h(z)]\ [1-\alpha\gamma\,h(z)]\\
&+(1-\alpha)^2(1-\beta)^2\sigma^2\gamma^2[\theta+z(1-\theta)]^2h^2(z)\\
&-2(1-\alpha)(1-\beta)\sigma(1-\sigma)(1-\gamma)h(z)[\theta+z(1-\theta)]^2\,\{\beta+\gamma(1-\alpha-\beta)h(z)\}\\
&+2(1-\alpha)(1-\beta)(1-\sigma)\gamma(1-\gamma)h^2(z)[\theta+z(1-\theta)]\,\{1-2\alpha\sigma[\theta+z(1-\theta)]\}\\
&-\alpha(1-\alpha)(1-\beta)\sigma(1-\sigma)(1-\gamma)h(z)[\theta+z(1-\theta)]^2\,[z-\beta\gamma\,h(z)]\\
&+\beta^2(1-\sigma)(1-\gamma)h(z)[\theta+z(1-\theta)]\,[1-\alpha\gamma\,h(z)]\,[1-\alpha\sigma[\theta+z(1-\theta)]]\\
&+\alpha^2\beta^2(1-\sigma)^2(1-\gamma)^2h^2(z)[\theta+z(1-\theta)]\\
&-(1-\alpha)(1-\beta)(1-\sigma)^2(1-\gamma)h(z)[\theta+z(1-\theta)]^2\,\{1-\alpha-\beta+\alpha\beta(1-\gamma)h(z)\}\\
&+2\alpha(1-\alpha)(1-\sigma)(1-\gamma)h(z)[\theta+z(1-\theta)]\,[z-\beta\gamma\,h(z)]\,.
\end{aligned}
$$

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