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NOTATION AND CLASSIFICATION FOR LOGISTIC NETWORK DESIGN MODELS

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Abstract. This paper presents a notation and a classification system for the design models of logistic network. Our notation consists of three fields (analogous to Graham's $\alpha|\beta|\gamma$ notation for scheduling problems). The proposed notation is applied for several articles from the literature. We focus on multi-period models with deterministic and stochastic demands. The proposed notation is based on three criteria corresponding to the main characteristics of the logistic networks: the structure (field α), the management rules (field β) and the performance criteria (field γ). A description of solution methods, datasets and results is also provided. Most articles deal with deterministic, multi-level models and only few of them include the international aspect of logistics, lead-times or subcontracting. Datasets used to test the methods are randomly generated by the authors and have different sizes. The heuristic methods are most commonly used.

Keywords. Design, logistic network, classification, notation, facility location, dataset, performance criterion.

Mathematics Subject Classification. 90B10.

1. Introduction

A logistic network is a set of locations where a facility can be opened (supplier, production plant, warehouse or distribution center). All locations are linked by

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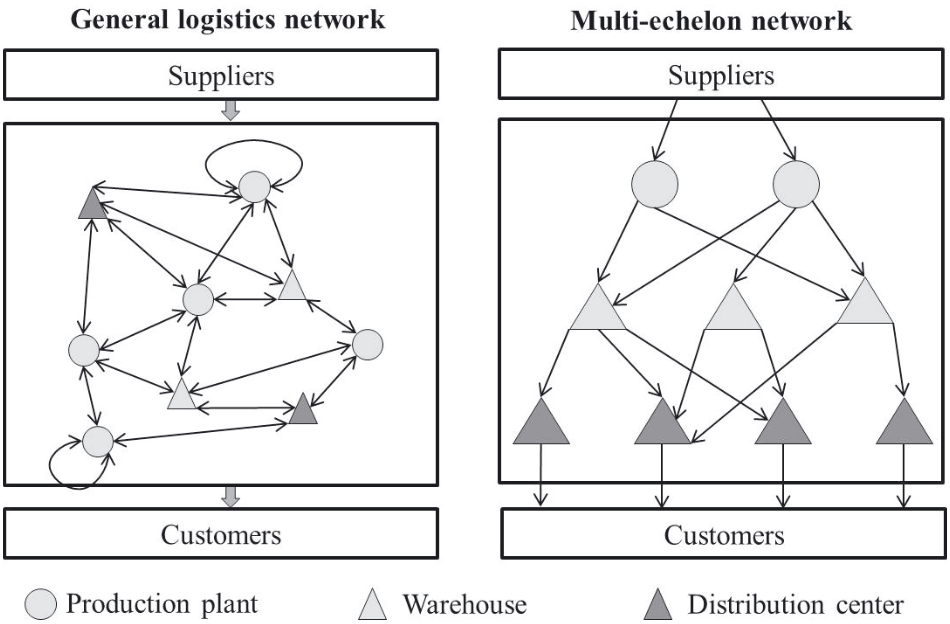

FIGURE 1. Modelling of logistics network.

roads and products are transferred through these roads. Martel [14] explained that there are two models for the logistic network in the literature (Fig. 1):

- a **general logistic network** is composed of different facilities (supplier, production plant, warehouse, distribution center) linked together by roads,
- a **multi-echelon network** prioritizes the facilities and defines existing roads between two successive facility levels.

In an increasing competitive industrial environment, the search for better costs is a constant concern. An efficient logistic network is a cost reduction source. According to Ambrosino and Scutellà [1], "these problems consist of determining the best way to transfer goods from the supply to the demand points (customers) by choosing the structure of the network while minimizing the overall costs".

Melo *et al.* provide a detailed review of reference [17], describing a possible model of network structure (nature of the planning horizon, single or multi period, the type of datas and the number of echelons). They proposed a classification of the literature according to typical logistic decisions such as capacity, inventory, procurement, production, routing and transport mode. But the authors do not give detailed information about the costs and the management rules of the network. In this paper, we will address multi-period models to better design and evolve a logistic network. We will focus on the characteristics such as costs, the structure of the network and management rules. To classify the different models, we propose a notation inspired by the Graham's notation $\alpha|\beta|\gamma$ which is used for scheduling problems [8]. α is the physical description of the model, β describes the management rules of the model and γ is both the performance criteria and the data used to compute them. For each model, we summarize the solution methods, the dataset size and the results. The papers have been chosen in the literature to enhance many constraints and management rules in the multi-period models. We illustrate our notation with twenty recent papers.

In Section 2 the context of the study will be specified as well as the proposed notation. Then, Section 3 is dedicated to single-echelon and multi-echelon deterministic models. The models with stochastic demands are described in Section 4. In these sections, initially, the description of a basic model is given. Then the main variants, solving methods and datasets are described.

2. Context and notation

The first models proposed in the literature are single-period models. They design an optimal logistic network with a known demand. These models are:

- *k-median problem* proposed by Kuehn and Hamburger in 1963 [13],
- *Uncapacitated Plant Location Problem (UPLP)* defined by Erlenkotter in 1978 [6],
- *Capacitated Plant Location Problem (CPLP)* defined by Sá in 1969 [21].

The *k-median problem* consists in opening a given number k of facilities selected from a set of available locations. The goal is to minimize the sum of the distances between the facilities and the customers. The *UPLP* is obtained from the *k-median problem* by releasing the constraint that fixes the number of facilities. The facilities are uncapacitated. The *CPLP* is similar to the *UPLP* but the facilities have a capacity constraint. The main drawback of these models is that they cannot adjust or anticipate developments of the economic situation. That is why some multi-period models have been developed. These models introduce periods (for instance years) and facilities can open or close during a period.

We propose a classification of multi-period models which differs in the considered structure of networks, management rules and performance criteria. We have summarized all these characteristics in Tables 1–3. The proposed notation is labeled in brackets. The three fields of the notation are:

- The network structure (α field)
  - **General Logistic Network** (GLN) or **Multi-Echelon Network** (MEN) (*cf.* Fig. 1),
  - **single-echelon** ($1E$) or **multi-echelon** (nE). The customer demands, and the location of a single-level of the logistic network, are considered in a single-echelon model. Generally the level is composed of warehouses. When several levels in the network are considered, we refer to a multi-echelon

TABLE 1. Network structure.

	E	GLN	MEN	P	$F.FC$	$Inter$	BOM	LT	$Tr.FC$	$Tr.M$	SA
Dias *et al.* [3]	1		X	1	X						
Saldanha-da-Gama *et al.* [22]	1		X	1							X
Melo *et al.* [16]	1		X	n	X						
Ghaderi *et al.* [7]	1		X	1							
Hinojosa *et al.* [11]	n		X	n	X						
Canel *et al.* [2]	n		X	1	X				X		
Melachrinoudis *et al.* [15]	n	X		n	X						
Syam [26]	n		X	n	X				X		
Ambrosino *et al.* [1]	n	X		1	X				X	X	
Martel [14]	n	X		1	X	X	X				
Vila *et al.* [32]	n	X		n	X	X	X		X		
Pirard [19]	n	X		n	X	X	X				
Hinojosa *et al.* [12]	n		X	n	X		X				X
Thanh *et al.* [27,28]	n	X		n	X		X				
Suon [24]	n		X	n	X	X	X		X	X	
Pan *et al.* [18]	n		X	1	X		X	X	X		
Shankar *et al.* [23]	n		X	1	X		X				
Tsao *et al.* [29]	n		X	1				X	X	X	X
Hameur-Lavoie *et al.* [10]	n		X	n	X			X	X	X	
Ramezani *et al.* [20]	n		X	n	X				X		

model. The goal of a two-echelon model is to locate production plants and warehouses for example.
  - **single-product** ($1P$) or **multi-product** (nP) model,
  - **finite capacity facility** ($F.FC$),
  - **international network** ($Inter$),
  - **Bill Of Materials** (BOM): list of the raw materials and sub-components to manufacture a product,
  - **lead-time** (LT): transport time between two facilities,
  - **transport capacity** ($Tr.FC$): transport capacities are considered,
  - **transport mode** ($Tr.M$): selection of the transport mode,
  - **Single-Allocation** (SA): when a customer is delivered by only one supplier.
- Management rules (β field)
  - **deterministic** (DD) or **stochastic** (SD) demands,
  - **type of inventory** (Inv): possibility of taking into account seasonal stocks (to smooth the production of each period), safety stocks (to manage a fluctuating demand) or order cycle stocks (in relation to batch production),

TABLE 2. Management rules.

	Demand	*Inv*	*OC*	*Tech*	*MC*	*SC*	*MP*	*BC*
Dias *et al.* [3]	D		X		X			
Saldanha-da-Gama *et al.* [22]	D		X					
Melo *et al.* [16]	D		X		X			
Ghaderi *et al.* [7]	D		X					X
Hinojosa *et al.* [11]	D		X					
Canel *et al.* [2]	D		X					
Melachrinoudis *et al.* [15]	D		X		X			X
Syam [26]	D	X	X					
Ambrosino *et al.* [1]	D	X	X					
Martel [14]	D	X	X	X	X		X	
Vila *et al.* [32]	D	X	X	X	X			
Pirard *et al.* [19]	D		X		X			
Hinojosa *et al.* [12]	D		X		X			
Thanh *et al.* [27,28]	D	X	X		X	X		
Suon [24]	D	X	X	X	X			
Pan *et al.* [18]	D		X					
Shankar *et al.* [23]	D	X	X					
Tsao *et al.* [29]	S	X	X					
Hameur-Lavoie *et al.* [10]	S	X	X					
Ramezani *et al.* [20]	S		X					

- **opening/closing** (OC): possibility of opened or closed the facilities (suppliers, warehouses, production plants, ...) during the planning horizon,
- **technology** ($Tech$): storage and/or production of some products that require particular equipments,
- **modular capacities** (MC): the facility capacity may change between two periods,
- **sub-contracting** (SC): a part of production can be done by another company,
- **marketing policies** (MP): price that a market will bear for each product.
- **budget constraints** (BC): the budget to enable the network growth is limited.

- The performance criteria (γ field)
  The main objectives are:
  - minimizing total costs (C),
  - minimizing Carbon Emissions (CE),
  - maximizing economic Profit (P),
  - maximizing the Quality of Service (QoS),
  - maximizing the facility Fill Rate (FR),
  - minimizing the Rate of Faulty Raw material (RFR) (this criterion appears in models that include reverse logistics).

Table 3. Performance criteria and costs.

	C	P	QoS	FR	RFR	Tr	F	OC	Inv	$Inter$	AR
Dias *et al.* [3]	X						X	X			
Saldanha-da-Gama *et al.* [22]	X						X	X			
Melo *et al.* [16]	X					X	X	X	X		
Ghaderi *et al.* [7]	X					X	X	X			
Hinojosa *et al.* [11]	X					X	X				
Canel *et al.* [2]	X					X	X	X			
Melachrinoudis *et al.* [15]	X					X	X	X	X		
Syam [26]	X					X	X		X		
Ambrosino *et al.* [1]	X					X	X	X	X		
Martel [14]		X				X	X	X	X	X	X
Vila *et al.* [32]		X				X	X	X	X		X
Pirard *et al.* [19]		X				X	X	X	X	X	X
Hinojosa *et al.* [12]	X					X	X	X			
Thanh *et al.* [27], [28]	X		X			X	X	X	X		
Suon [24]	X					X	X	X		X	
Pan *et al.* [18]	X					X	X		X		
Shankar *et al.* [23]	X			X		X	X	X	X		
Tsao *et al.* [29]	X					X	X		X		
Hameur-Lavoie *et al.* [10]	X					X		X	X		
Ramezani *et al.* [20]		X	X		X	X	X	X			X

The considered costs to assess the economical objective are given between brackets. For instance, $C(OC, Inv)$ describes a model when the objective is to minimize the total costs computed using opening and closure (OC) and inventory (Inv) costs.

- **transport and distribution** costs (Tr) between the facilities of the logistic network,
- **processing facility** costs (F), the use of facility costs,
- **opening/closure** costs (OC) for a facility,
- **inventory** costs (Inv) for raw materials, semi-finished and finished products,
- **import and export** costs ($Inter$) related to the movement of goods in an international network,
- **amount received for the sales** (AR): the costs of the products depend on the customers at each periods.

Table 1 summarizes the characteristics of the logistic network in the studied models from the literature. Table 2 lists the management rules and Table 3 lists the performance criteria taken into account in the different models.

To the best of our knowledge, no multi-period models with an environmental criterion exists. We will include this criterion in our notation because it is increasingly becoming important. Indeed, we can find single period models such as the one considered by Xifen *et al.* [34]. Using the presented notation, this problem is denoted $MEN, 1E, 1P, F.FC, SA|DD|C(Tr, F), CE, QoS$. The authors present a single-echelon single-product model on one period. It is an extension of the $UPLP$ with three performance criteria. The customer location is known and each customer is supplied by only one facility. The problem consists in opening some facilities selected from a set of available locations. The three objectives are :

- minimizing costs noted (C),
- maximizing the quality of service (QoS),
- minimizing carbon emissions (CE).

In the following section, models with deterministic demands are presented.

3. Models with deterministic demands

3.1. Single-echelon models

A deterministic single-product model called *Simple Dynamic Location Problem* proposed by Saldanha da Gama and Captivo in 1998 [22] is first presented. By using the exposed notation this model is noted $MEN, 1E, 1P, SA|DD, OC|C(F, OC)$. It is an extension of *UPLP* within the frame of a multi-period horizon. The network is composed of warehouses. The goal is to locate facilities by minimizing the sum of opening costs and processing costs. The authors made the following hypotheses:

- opening and closure are instantaneous,
- the establishment (respectively removal) of a facility must happen at the beginning (respectively end) of a time period,
- the facilities have no capacity,
- a facility can change its status only once during the planning horizon,
- from now until the beginning of the planning horizon it is possible to remove facilities,
- no facility will be closed in the last period of the planning horizon.

The notations used in the model are:

TP	set of time period,
I	set of customers,
J	set of locations where facilities can be established,
c_{ij}^t	cost of satisfying the demand of customer i by a facility processing at location j in time period t,
o_j^t	cost to open a facility at location j in time period t,

The subset J_f (resp. J_o) is the set of locations where facilities can be removed (resp. established). $J = J_o \cup J_f$. The decision variables are :

$x_{ij}^t = 1$	if the demand of customer i is satisfied by a facility located at j in time period t, 0 otherwise
$u_j^t = 1$	if there is a used facility at location j in time period t, 0 otherwise

The authors present a linear formulation. The objective function is:

$$\min \sum_{t\in TP}\sum_{i\in I}\sum_{j\in J} c_{ij}^t.x_{ij}^t + \sum_{t\in TP}\sum_{j\in J} o_j^t.u_j^t \tag{3.1}$$

and the set of constraints is:

$$\sum_{j\in J} x_{ij}^t = 1 \quad \forall i\in I, \forall t\in TP \tag{3.2}$$

$$x_{ij}^t - u_j^t \leq 0 \quad \forall i\in I, \forall j\in J, \forall t\in TP \tag{3.3}$$

$$u_j^{t+1} - u_j^t \leq 0 \; \forall j\in J_f, \forall t\in\{1,\ldots,T-1\} \tag{3.4}$$

$$u_j^{t+1} - u_j^t \geq 0 \; \forall j\in J_o, \forall t\in\{1,\ldots,T-1\} \tag{3.5}$$

$$x_{ij}^t \in\{0,1\} \quad \forall i\in I, \forall j\in J, \forall t\in TP \tag{3.6}$$

$$u_j^t \in\{0,1\} \quad \forall j\in J, \forall t\in TP. \tag{3.7}$$

The considered performance criteria is the sum of the costs (3.1). Constraints (3.2) ensure that the demand is satisfied by one facility. Constraints (3.3) guarantee that no customer will be assigned to a closed facility. Constraints (3.4) and (3.5) guarantee that an opened facility cannot be closed and a closed facility cannot be opened. Additionally, the variables are binary according to the constraints (3.6) and (3.2). Then the authors complete the model by taking into account opening and closure costs, $MEN, 1E, 1P|DD, OC|C(F, Tr, OC)$. To solve the problem, the authors propose a heuristic based on two steps. The first step is an heuristic called DROP described by Domschke and Drexl in 1983 [4]. The second step is a local search. The dataset is randomly generated. $|I|$ varies between 10 and 500, $|J|$ takes the value 5, 10 or 20. Saldanha da Gama and Captivo compare their results with the ones obtained by Van Roy and Erlenkotter in 1982 [31]. The gap between the two methods is between 0.13 and 6.77 and the two-step-heuristic of [22] is faster.

In 2006, Melo *et al.* [16] proposed a model in which the facility capacities are considered as a decision variable, $MEN, 1E, nP, F.FC|DD, OC, MC|C(F, OC, Tr, Inv)$. Thus, facilities have an evolving finite capacity. A part of the capacity movement is allowed between two opened facilities. Moreover, they include distribution and transport, inventory and processing facility costs. In the second part of the paper, they generalize the model with two echelons. This type of model is described in the following section.

In 2006, Dias *et al.* [3] described a deterministic model. The characteristics of the models are single-echelon, single-product with opening and closing facilities, $MEN, 1E, 1P, F.FC|DD, OC, MC|C(F, OC)$. In these models, more than one facility can supply a customer, a closed facility can be reopened, and the authors include modular capacity.

Recently Ghaderi *et al.* [7] propose a multi-period single-echelon model to design a logistic network, $MEN, 1E, 1P|DD, OC, BC|C(Tr, F, OC)$. They include budget constraints. The aim is to create care centers in developing countries in order to facilitate the access to health care. In each period, the company have a maximum budget to open facilities and to build distribution roads. The remaining budget may be reallocated to the next period. The considered costs are opening facilities, processing facilities and transport costs. The model is non-linear due to budget constraints. The authors proposed two methods to determine the logistic network. The first method consists in fixing some variables and implementing the model in a solver. The second method is a hybrid simulated annealing heuristic.

In this section, we described multi-period single-echelon models to design logistic networks. But the networks have an increasing complexity, the warehouse location depends on not only the customers but also the production plants, suppliers, ... The logistic network must be considered as a whole. Design models evolve by integrating the location of several levels from the network. Multi-echelon models should be able to respond to this new trend.

3.2. Multi-echelon models

The first model was proposed by Hinojosa *et al.* in 2000 [11], $MEN, 2E, nP, F.FC|DD, OC|C(Tr, F)$. It is a multi-product model with two echelons. It is possible to open or close facilities. The network is composed of production plants, warehouses and customers and modeled such as a multi-echelon network. The assumptions made by the authors are:

- the facilities have a finite capacity which depends on the period,
- a minimum number of warehouses must be opened at the beginning of the first period and at the end of the last period,
- a closed (respectively opened) facility may be opened (respectively closed) at the beginning (resp. end) of each period,
- a facility can change its status only once in the planning horizon.

The notations used in the model are:

TP	set of time periods
I	set of customers
J	set of possible warehouses
K	set of possible production plants
L	set of product types
d_{il}^t	demand of product of type $l \in L$ by customer $i \in I$ at a time period $t \in TP$
w_j^t	capacity of warehouse $j \in J$ at a time period $t \in TP$
s_k^t	capacity of production plant $k \in K$ at a time period $t \in TP$
f_j^t	processing cost of a warehouse opened at location $j \in J$ at a time period $t \in TP$
g_k^t	processing cost of a production plant opened at location $k \in K$ at a time period $t \in TP$

c^t_{ijl} costs of transport for a type of product $l \in L$ from warehouse $j \in J$ to customer $i \in I$ at a time period $t \in TP$

b^t_{jkl} costs of transport for a product $l \in L$ from production plant $k \in K$ to warehouse $j \in J$ at a time period $t \in TP$

ND^1 and NC^1 (respectively ND^T and NC^T) are the minimum number of warehouses and production plants opened at the beginning of the first time period (respectively the last time period). The subsets J_f (set of facilities which can be closed) and J_o (set of facilities which can be opened) form a partition of the set J. Similarly, K_o and K_f form a partition of the set K.

The decision variables are:

x^t_{ijl} fraction of the type of product $l \in L$ delivered to customer $i \in I$ from the warehouse $j \in J$ at a time period $t \in TP$

y^t_{jkl} fraction of the type of product $l \in L$ sent to a production plant $k \in K$ from warehouse $j \in J$ at a time period t

u^t_j $= 1$ if a warehouse $j \in J$ is opened at the beginning of time period t, 0 otherwise

v^t_k $= 1$ if a production plant $k \in K$ is opened at the beginning of time period t, 0 otherwise

$$C_1 = \sum_{t\in TP}\sum_{i\in I}\sum_{j\in J}\sum_{l\in L} c^t_{ijl}.x^t_{ijl}.d^t_{il} \tag{3.8}$$

$$C_2 = \sum_{t\in TP}\sum_{j\in J}\sum_{k\in K}\sum_{l\in L} b^t_{jkl}.y^t_{jkl}.s^t_j \tag{3.9}$$

$$C_3 = \sum_{t\in TP}\sum_{j\in J} f^t_j.u^t_j + \sum_{t\in TP}\sum_{k\in K} g^t_k.v^t_k \tag{3.10}$$

Objective function:

$$\text{minimize } C_1 + C_2 + C_3 \tag{3.11}$$

under constraints:

$$\sum_{j\in J} x^t_{ijl} \geq 1 \qquad \forall i \in I, \forall l \in L, \forall t \in TP \tag{3.12}$$

$$\sum_{i\in I}\sum_{l\in L} d^t_{il}.x^t_{ijl} \leq w^t_j.u^t_j \quad \forall j \in J, \forall t \in TP \tag{3.13}$$

$$\sum_{k\in K} w^t_j.y^t_{jkl} \geq \sum_{i\in I} d^t_{il}.x^t_{ijl} \ \forall j \in J, \forall l \in L, \forall t \in TP \tag{3.14}$$

$$\sum_{j\in J}\sum_{l\in L} w^t_j.y^t_{ijl} \leq s^t_k.v^t_k \quad \forall k \in K, \forall t \in TP \tag{3.15}$$

$$\sum_{j\in J} u^1_j \geq ND^1 \tag{3.16}$$

$$\sum_{j\in J} u_j^T \geq ND^T \tag{3.17}$$

$$\sum_{k\in K} v_k^1 \geq NC^1 \tag{3.18}$$

$$\sum_{k\in K} v_k^T \geq NC^T \tag{3.19}$$

$$u_j^1 = 1, u_j^t - u_j^{t+1} \geq 0 \ \forall j \in J_f, \forall t \in \{1,\ldots,T-1\} \tag{3.20}$$

$$u_j^t - u_j^{t+1} \leq 0 \qquad \forall j \in J_o, \forall t \in \{1,\ldots,T-1\} \tag{3.21}$$

$$v_k^1 = 1, v_k^t - v_k^{t+1} \geq 0 \ \forall k \in K_f, \forall t \in \{1,\ldots,T-1\} \tag{3.22}$$

$$v_k^t - v_k^{t+1} \leq 0 \qquad \forall k \in K_o, \forall t \in \{1,\ldots,T-1\} \tag{3.23}$$

$$x_{ijl}^t \geq 0, y_{jkl}^t \geq 0 \qquad \forall i \in I, \forall j \in J, \forall k \in K, \ \forall l \in L, \forall t \in TP \tag{3.24}$$

$$u_j^t, v_k^t \in \{0,1\} \qquad \forall j \in J, \forall k \cup K, \forall t \in TP. \tag{3.25}$$

The goal is to locate facilities (production plants and warehouses) and to define the fraction of a product delivered to each customer from a warehouse by minimizing the sum of transport costs between warehouses and customers (3.8), transport costs between production plants and warehouses (3.9) and processing facility costs (3.10). Equation (3.11) gives the objective function.

Constraints (3.12) ensure that a customer receives his demand for each type of products at each period. The capacity of warehouses and production plants are satisfied in accordance with constraints (3.13) and (3.15). Constraints (3.14) make sure that production plants give enough products to warehouses. Constraints (3.16) to (3.19) ensure that the minimum number of opened facilities at the first and the last period is respected. Constraints (3.21) to (3.23) verify that a facility can change its status maximum once in the planning horizon. The variables x_{ijl}^t and y_{jkl}^t are positive and the variables u_j^t and v_k^t are binary according to constraints (3.24) and (3.25). The authors use a Lagrangian relaxation to compute a lower bound and a two-step-heuristic to obtain a feasible solution. The method has been applied on randomly generated datasets. The number of customers is between 10 and 75, the number of warehouses is between 5 and 40, the number of production plants is between 5 and 40 and the number of products types is 2 or 3. The gap between the lower bound and the solution found with the heuristic varies between 0.24% and 5%. For the small and medium datasets the gap with the optimal solution is between 0.17% and 2.7%. In this case the optimal solution was found by a solver. However, for the large datasets, the solver does not find the optimal solution.

In 2001, Canel *et al.* [2] proposed a single-product model, $MEN, 2E, 1P, F.FC, Tr.FC|DD, OC|C(Tr, F, OC)$. They added transport constraints and capacity to the roads. In addition they considered that opening and closure facility costs depend on the period. The method to solve the problem is composed of three phases. The first phase is to identify the facilities to be opened or closed. The second phase is to find a feasible solution. The third phase uses dynamic programming to obtain the optimal solution (evaluation of the different networks obtained in the second phase). The algorithm was tested on the datasets described by Sweeny *et al.* in 1976 [25]. Sweeney *et al.* solve the problem with a Bender's decomposition and dynamic programming. The size of the datasets is: 3 production plants, 5 warehouses, 15 customers and 5 periods. The three-phase-heuristic by Canel *et al.* finds the same solutions with shorter computational times.

In 2002, Syam [26] also proposed a multi-product model with two echelons, $MEN, 2E, nP, F.FC, Tr.FC|DD, OC, Inv|C(Tr, F, Inv)$. The additional characteristics concern the inventory, the purchase and the consolidation of transport. The inventory costs are added to the objective function. Products that one delivered with the same frequency, along the same road, are delivered together. This leads to economies of scale on the costs of transport. The goal of the model is to determine the network structure, the flow between the facilities and the delivery frequency for products while minimizing costs. The suggested method for this problem is composed of two phases: a simulated annealing to determine the facilities to open (production plants and warehouses) and a Lagrangian relaxation to find out the optimal consolidation policies. The datasets are randomly generated from 10 to 100 production plants and 2 to 20 warehouses. The company manufactures 5 types of products. For the small datasets, the gap between the optimal solution and the solution obtained with Lagrangian relaxation is between 0.42% and 1.66% and the computational time is between 0.5 and 1 second. For the proposed method, the gap is between 0.35% and 4.17% for the same computanional time. For the large datasets, computational times are between 2 and 152 seconds for the Lagrangian relaxation and between 5 and 329 seconds for the heuristic. Lagrangian relaxation obtains better results in 90% of cases.

In 2005, Pirard *et al.* [19] propose a hybrid heuristic method to reconfigure a logistic network, $GLN, 3E, nP, F.FC, Inter, BOM|DD, OC, MC|P(Tr, F, OC, Inv, Inter, AR)$. This is a multi-echelon and multi-product model and it can be used whatever the number of echelons is. The goal is to maximize the profit after taxation. The model includes inventory, distribution, production costs, import and export taxes, and amount received for the sales. It is possible to transfer a fraction of capacity from a facility to another. It takes into account the bill of materials. To solve the problem, the authors use a branch and bound but this method generates higher computational times for industrial datasets. To solve this problem, they take an iterative approach. The problem is separated into subproblems. Each subproblem is solved with a local search and simplex algorithm. They obtain a quality feasible solution in a reasonable time. This method is applied in the case of a company composed of 16 facilities. The company makes 20 types of products based on 45 components. It has also 20 customers. The components are bought from 10 suppliers. The planning horizon is divided into five periods. The gap between the optimal solution and the heuristic solution is between 0.04% and 0.19%.

Computational times are between 12h58 and 243h18 to find out the optimal solution with an exact method. The heuristic method offers results between 2h56 and 3h02.

In 2005, Martel [14] proposes a logistic network design model for an international company, $GLN, 2E, 1P, F.FC, BOM, Inter|DD, Inv, OC, Tech, MC, MP|P(Tr, F, OC, Inv, Inter, AR)$. Instead of a multi-echelon network, the author considers a general logistic network. The author considers three types of nodes located in several countries such as external suppliers, internal potential facilities and customers. Three types of inventory are modeled: safety stocks, seasonal stocks and order cycle stocks. The author considers the costs of transfer between facilities (transport) and import and export costs. Then, he introduces a choice of marketing policies. A marketing policy defines the costs of the product depending on the customer at each period. This describes the amount received for all the products' sales. He also includes technologies necessary to manufacture the products. It is possible to install a technology or reconfigure an open facility. The goal is to determine the logistic network while maximizing the profit. The model is non-linear. The author proposes to linearize non-linear constraints and to solve the resulting problem with a commercial linear solver.

In 2008, Thanh *et al.* [27, 28] addresses with a three echelons model; $GLN, 3E, nP, F.FC, BOM|DD, Inv, OC, MC, SC \ |C(Tr, F, OC, Inv), QoS$. The network is composed of suppliers, production plants, warehouses and customers and transfers between production plants are possible so the production may be shared in several steps made by different production plants. This model allows one to locate and open or close facilities, optimize material flows while minimizing the network total costs. The management of the network is controlled by:

- a purchasing strategy: several suppliers can supply the same plant for the same product. In the case of the purchase of several raw materials, suppliers may offer a discount,
- a production strategy: batch production,
- an inventory strategy: no stock in the production plant, seasonal and safety stocks in the warehouses,
- a distribution strategy: customers are supplied by warehouse except for the big customers supplied by production plants.

In this work, the modeling cost is more detailed. Thanh adds supplier selection costs, discount policies according to ordered quantities, sub-contracting costs and processing facilities costs. The assumptions made by the author are:

- facilities have a finite capacity,
- each facility produces one or more type of products,
- the facility capacity is flexible,
- a facility has a minimum and a maximum percentage of use,
- a facility can only change its status once in the planning horizon,
- the capacity of an opened facility progressively increases over each period,
- a production plant may sub-contract a part of its production,

- two types of warehouses are available: public (leased by the company) or private (property of the company),
- a warehouse has two types of capacity: storage and processing of transit stock.

To solve the problem, the author proposes three methods, a linear relaxation, D.C. programming (Difference of Convex functions) and a Lagrangian relaxation. These three methods are tested with randomly generated datasets of three sizes (small, medium and large). The planning horizon is composed of five periods. The datasets are composed of 15 to 27 suppliers, 10 to 22 production plants, 5 to 13 warehouses, 100 to 270 customers and 10 to 18 types of products. The results are presented according to two criteria which are the computational time and the value of the objective function. For the small datasets, the linear relaxation is faster. For the other datasets, D.C. programming is faster. Concerning the objective function value, Lagrangian relaxation gives the best results and linear relaxation is the least effective method for all datasets.

Recently, Pan *et al.* [18] give a multi-level model, multi-echelon network. The model can be used with any number of levels. At each level, an operation is performed according to a bill of materials, $MEN, nE, 1P, F.FC, BOM, LT,\ Tr.FC| DD, OC|C(Tr, F, Inv)$. The production time added to the travel time to reach the next level corresponds to a period (lead-time). The authors assume that the model is single-product, the transport is single-modal with a capacity. The considered costs in this model are related to the transport, the production, the storage and the purchase of raw materials. At each level, the choice of which facilities should be opened to manufacture the desired quantity of products in order to minimize the sum of the costs is determined. To do that, the authors proposed a Lagrangian relaxation and tested this method on randomly generated datasets.

In 2013, Shankar *et al.* [23] proposed a bi-objective model of logistic network design with three echelons (suppliers, production areas, warehouses and customers), $MEN, 3E, 1P, F.FC, BOM|DD, Inv, OC|C(Tr, F,\ OC, Inv), FR$. Potential sites and their capacity are set by the managers. The objectives are to define the quantity of raw materials to purchase from each supplier, the facility location to be opened and the roads to be used in order to minimize the fixed and variable costs such as the procurement of raw materials, transport, production and storage and to maximize fill rate of the opened facilities. The problem is formulated as a mixed integer mathematical model. The proposed resolution method is a multi-objective particle swarm optimization (MOHPSO) determining a set of dominant solutions.

Many other works can be quoted related to multi-period and multi-echelon logistic network design models. The specificity of the proposed model in [1] lies in the allocation of stored quantities in different platforms, the choice of distribution routes and the possibility to classify customers according to the ordered volume. Then, Vila *et al.* [32] proposed a model that integrates the choice of suppliers, manages the inventory in a seasonal activity within an international network. Hinojosa *et al.* in 2008 [12] consider the maintenance costs in the processing costs of the facilities. Finally, [24] introduces investments in his works taking into account

the depreciation on the considered horizon.
According to the notation, these models can be classified as follow:

- $GLN, 2E, 1P, F.FC, Tr.FC, Tr.M|DD, Inv, OC|C(Tr, F, OC, Inv)$ [1]
- $GLN, nE, nP, F.FC, Inter, BOM, Tr.FC|DD, Inv, OC, Tech, MC|P(Tr, F, OC, Inter, Inv, AR)$ [32],
- $MEN, 2E, nP, F.FC, BOM|DD, OC, MC|C(Tr, F, OC)$ [12],
- $MEN, 2E, nP, F.FC, Inter, BOM, Tr.FC, TR.M|DD, Inv, Tech, MC, OC, |C(Tr, F, Inter, OC)$ [24].

4. Models with stochastic demands

In 2006, Hameur-Lavoie *et al.* [10] get interest in a multi-level, multi-product location problem considering the inventory and stochastic demands, $MEN, 2E, nP, F.FC, LT, Tr.FC, Tr.M|SD, Inv, OC|C(Tr, OC, Inv)$. The network consists of production sites (known and located) and platforms (to be located). Products are transported from the production plants to the distribution centers using a transport mode and a route. Customers are delivered by distribution centers in the same way. An available transport mode list is defined for each road. The customers demand is modeled by a normal distribution function. The objective is to locate the sites where opened platforms to satisfy the demands while minimizing costs. To solve this nonlinear model, the authors proposed a linear approximation of the variance of the demand. Then, they use a separation method to solve the problem. The datasets are randomly generated according to four criteria:

- the size of the problem (24, 32 or 48 customers ; 8, 10 or 16 plants, warehouses and types of product),
- the network capacity (number of plants that may produce a product type, number of platforms that may handle a product type, number of available types of platform on each potential sites),
- the safety stocks according to change in demand and lead-times,
- the planning horizon of four periods (quarterly) or several periods (monthly).

There is one tranport mode between production plants and warehouses. However, three transport modes are available between warehouses and customers. The authors note that the computational times may vary across the size of the datasets. Moreover, the capacity criterion related to the number of plants and the number of platforms is the one that has the largest impact on the solving problem difficulty.

In 2012, Tsao *et al.* [29] proposed a three-echelon model of location platforms with stochastic demands, MEN, $3E$, $1P$, LT, $Tr.FC$, $Tr.M$, $SA|SD$, Inv, $OC|C(Tr, F, Inv)$. The network consists in suppliers who are delivering national platforms (known and fixed). They distribute products to regional platforms (to locate) which then distribute them to the stores. A regional platform has a circular impact area (centered on the platform) and distributes products to every store under its influence. The national and regional platforms have no capacity constraint. The stores demand is modeled by a Poisson distribution function.

Lead-times between national and regional platforms are ignored. However, the lead-times between regional platforms and their stores are to be considered. Carriers discount is evaluated based upon transported quantities between national and regional platforms. The costs between regional platforms and stores are divided into fixed and variable costs. To solve this problem, the authors used a two-phase approximated heuristic proposed by [30]. The first step consists in dividing the territory according to the density of stores around each national platform. Each region is assigned to a national platform. In the second step, regional platforms are located and stocks are distributed.

In 2013, Ramezani *et al.* [20] present a stochastic multi-objective, multi-product and multi-period model, $MEN, 2E, nP, F.FC, Tr.FC|SD, OC|P(Tr, F, OC, AR)$, QoS, RFR. The network is modeled such as a multi-echelon network. In this work, the authors consider the location of sites within a framework of reverse logistic. According to Pohlen and Farris, "reverse logistics represents the process by which organization recovers by-products and residuals for reuse, resale, remanufacturing, recycling or disposal". This model differs from the previous ones because, in addition to the demand, several various model parameters are stochastic. The assumptions are:

- the number of opened facilities on each level is limited,
- the facilities have a finite capacity,
- the stochastic data are: the product prices, the costs of the facilities, the provision and evaluation of raw materials, the costs of returns,
- opening and transport costs are known and fixed.

The approach proposed by the authors is a stochastic programming method. The model includes two types of variables: binary variables for network designs and continuous ones for quantities that will be transported on the network. The proposed model allows to locate facilities in each level and to define their ability of optimizing three criteria: maximization of the total profit, the customer service rate and the minimization of the rate of faulty raw materials. The stochastic parameters are modeled by a uniform distribution function. To solve the problem, the authors use the ε-*constraint* method.

In this section, three location models with stochastic demands have been shown. One of them includes the dimension of reverse logistics. In the first two cases, the furthest end customers (production sites or national platform) are known and fixed. The number and the location of the facilities of a single level must therefore be determined. The demand is modeled by a normal or Poisson distribution function. In the previous quoted work, many parameters are stochastic and modeled by a uniform distribution function.

5. Concluding remarks

In Table 4, the methods quoted in this paper are described. Six authors use the integer linear programming (ILP), [1,10,15,16,24,28]. The authors find an optimal

TABLE 4. Resolution methods.

	ILP	Heuristics	Lagrangian relaxation	Local search	Simulated annealing	MOHPSO	ϵ-constraints
Saldanha-da-Gama *et al.* [22]		X					
Melo *et al.* [16]	X						
Ghaderi *et al.* [7]		X			X		
Hinojosa *et al.* [11]		X	X				
Canel *et al.* [2]		X					
Melachrinoudis *et al.* [15]	X						
Syam [26]			X		X		
Ambrosino *et al.* [1]	X						
Pirard *et al.* [19]		X					
Hinojosa *et al.* [12]		X					
Thanh *et al.* [27,28]	X	X	X				
Suon [24]	X			X			
Pan *et al.* [18]		X	X				
Shankar *et al.* [23]						X	
Tsao *et al.* [29]		X					
Hameur-Lavoie *et al.* [10]	X						
Ramezani *et al.* [20]							X

solution with this method for a few datasets. For instance, Ambrosino *et al.* [1] find an optimal solution with ILP in a small dataset (30 customers, 2 available central depots and 5 available regional depots). The computational time is less than 16 minutes. If the number of customers or available facilities increases, the ILP does not find an optimal solution and computational time increase significantly. For 60 customers (2 available central depots and 5 available regional depots), the computational time is more than 10 hours and the gap is around 3.2%. For the higher dataset (135 customers, 5 available central depots and 23 available regional depots), the gap is around 42.96% and the computational time is more than 257 h. Melo *et al.* [16] solve optimally in less than 5 hours for a number of randomly generated datasets (50 to 150 customers, 5 to 20 available facilities). In the case of multi-objective models, the objective function is a weighted sum of each criteria. The heuristic methods are most commonly used [2,7,11,12,18,19,22,28,29]. Four papers propose methods based on Lagrangian relaxation [11,18,26,28]. Four references use metaheuristics such as simulated annealing, [7,26], local search [24] or multiobjective hybrid PSO [23]. To solve a multiobjective problem, Shankar *et al.* implement MOHPSO with a Pareto front determining dominating solutions. Ramzani *et al.* [20] use ϵ-constraints method for multiobjective problems.

Datasets used to test the methods have different sizes and are randomly generated by the authors. The size of datasets presented in the literature are gathered in Table 5. For each paper, the number of periods, product types and echelons are given. For each echelon, the number of available sites and customer are detailed. We see that the sizes may differ drastically. The number of customers varies between 1 and 500. There are between 1 and 20 product types. For papers including

Table 5. Size of datasets.

	Periods	Product types	Echelons number	Available sites for each echelon 1	2	3	Customers
Saldanha-da-Gama *et al.* [22]	15	1	1	5 to 50	5 to 50		10 to 500
Melo *et al.* [16]	3 to 6	3 to 6	1		10 to 20		50 to 150
	3 to 5	5 to 10	2	5	8 to 20		50 to 150
Ghaderi *et al.* [7]	5 to 20	1	1		46 to 162		20 to 40
Hinojosa *et al.* [12]	4	2 to 3	2	5 to 40	5 to 40		10 to 75
Canel *et al.* [2]	5	1	2	3	15		15
Syam [26]	5	5	2	10 to 100	2 to 20		12 to 120
Ambrosino *et al.* [1]		1	2	2 to 5	5 to 23		30 to 135
Pirard *et al.* [19]	5	20 + 45*	3	10	6	10	20
Hinojosa *et al.* [11]	2 to 8	2 to 12	2	5 to 40	5 to 40		10 to 125
Thanh *et al.* [27,28]		10 to 18	3	15 to 27	10 to 22	5 to 13	100 to 270
Suon [24]	4	5 + 451*	2	7	7		79
Pan *et al.* [18]	3 to 10	1	3 to 10		2 to 20		1 to 5
Shankar *et al.* [23]		1	3	3	5	6	7
Hameur-Lavoie *et al.* [10]	4 to 12	8 to 16	2	8 to 16	8 to 16		24 to 48
Ramezani *et al.* [20]		2 + 5*	2	6	5		10

* $n + m$: n types of product and m components.

both products and components, we note $x + y$ (where x is the number of product types and y is the number of components) in column "Product types".

A notation and a classification for logistic network design that details the structure, management rules and performance criteria has been presented. This notation has been used in twenty papers and it details the multi-period models found in the literature. Most articles deal with deterministic, multi-level models and only few of them include the international aspect of logistics, lead-times or subcontracting. Heuristics are most commonly used to solve facility location problems but Griffis *et al.* [9] consider that the growing complexity of the models does not allow a more satisfactory resolution using exact or heuristic methods, concluding that metaheuristics provide more suitable methods to optimize the supply chain as a whole. Concerning environmental policies, single-period models with gas emission criteria are reported ([5, 33, 34]) but no multi-period models. However in an environmental context we find a multi-period model satisfying the objective to maximize the raw materials quality in a network that includes reverse logistics. This criterion may generate future works on multi-period models.

References

[1] D. Ambrosino and M.G. Scutellà, Distribution network design: New problems and related models. *Eur. J. Oper. Res.* **165** (2005) 610–624.

[2] C. Canel, B.M. Khumawala, J. Law and A. Loh, An algorithm for the capacitated, multicommodity multi-period facility location problem. *Comput. Oper. Res.* **28** (2001) 411–427.

[3] J. Dias, M.E. Captivo and J. Clìmaco, Capacitated dynamic location problems with opening, closure and reopening of facilities. *IMA J. Manage. Math.* **17** (2006) 317–348.

[4] W. Domschke and A. Drexl, ADD-heuristics' starting procedures for capacitated plant location models. *Eur. J. Oper. Res.* **21** (1985) 47–53.

[5] S. Elhedhli and R. Merrick, Green supply chain network design to reduce carbon emissions. *Transp. Res. D* **17** (2012) 370–379.

[6] D. Erlenkotter, A dual-based procedure for uncapacitated facility location. *Oper. Res.* **26** (1978) 992–1009.

[7] A. Ghaderi and M.S. Jabalameli, Modeling the budget-constrained dynamic uncapacitated facility location network design problem and solving it via two efficient heuristics: A case study of health care. *Math. Comput. Model.* **57** (2013) 382–400.

[8] Graham, L. Ronald, Lawler, L. Eugene, Lenstra, J. Karel and Kan, AHG. Optimization and approximation in deterministic sequencing and scheduling: a survey. *Ann. Discrete Math.* **5** (1979) 287–326.

[9] S.E. Griffis, J.E. Bell and D. Closs, Metaheuristics in Logistics and Supply Chain Management. *J. Business Logistics* **33** (2012) 90–106.

[10] G. Hamer-Lavoie and J.F. Cordeau, Un modèle pour la conception d'un réseau de distribution avec localisation, affectation et stocks. *Inf. Syst. Oper. Res.* **44** (2006) 1–18.

[11] Y. Hinojosa, J. Puerto and F.R. Fernandez, A multiperiod two-echelon multicommodity capacitated plant location problem. *Eur. J. Oper. Res.* **123** (2000) 271–291.

[12] Y. Hinojosa, J. Kalcsics, S. Nickel, J. Puerto and S. Velten, Dynamic multi-echelon facility-location. *Comput. Oper. Res.* **35** (2008) 373–391.

[13] A. Kuehn and M. Hamburger, A heuristic program for locating warehouses. *Manage. Sci.* **9** (1963) 643–666.

[14] A. Martel, The design of production-distribution networks: A mathematical programming approach. *Supply Chain Optimization* **98** (2005) 265–305.

[15] E. Melachrinoudis and H. Min, The dynamic relocation and phase-out of a hybrid, two echelon plant/warehousing facility. A multi objective approach. *Eur. J. Oper. Res.* **123** (2000) 1–15.

[16] M.T. Melo, S. Nickel and F. Saldanha-Da-Gama, Dynamic multi-commodity capacitated facility location: a mathematical modeling framework for strategic supply chain planning. *Comput. Oper. Res.* **33** (2006) 181–208.

[17] M.T. Melo, S. Nickel and F. Saldanha-Da-Gama, Facility location and supply chain management: A review. *Eur. J. Oper. Res.* **196** (2009) 401–412.

[18] F. Pan and R. Nagi, Multi-echelon supply chain network design in agile manufacturing. *Omega* **41** (2013) 969–983.

[19] F. Pirard, D. Duvivier and O. Roux, Une heuristique hybride pour la reconfiguration des réseaux logistiques dans le cadre d'entreprises multi-sites. *Actes du 6e congrés international de génie industriel, Besançon France*, 2005.

[20] M. Ramezani, M. Bashiri and R. Tavakkoli-Moghaddam, A new multi-objective stochastic model for a forward/reverse logistic network design with responsiveness and quality level. *Appl. Math. Model.* **37** (2013) 328–344.

[21] G. Sá, Branch-and-bound and approximate solutions to the capacitated plant-location problem. *Oper. Res.* **17** (1969) 1005–1016.

[22] F. Saldanha-Da-Gama and M.E. Captivo, A heuristic approach for the discrete dynamic location problem. *Location Sci.* **6** (1998) 211–223.

[23] B.L. Shankar, S. Basavarajappa, J.C.H. Chen and R.S. Kadadevaramath, Location and allocation decisions for multi-echelon supply chain network - A multi-objective evolutionary approach. *Expert Syst. Appl.* **40** (2013) 551–562.

[24] M. Suon, *Optimisation de la logistique internationale à horizon stratégique. Application à un constructeur automobile.* Thèse de doctorat de l'université de Clermont-Ferrand, France (2011).

[25] D. Sweeney and R. Tatham, An improved long-run model for multiple warehouse location. *Manage. Sci.* **22** (1978) 748–758.

[26] S.S. Syam, A model and methodologies for the location problem with logistical components. *Comput. Oper. Res.* **29** (2002) 1173–1193.
[27] P.N. Thanh, N. Bostel and O. Péton, A dynamic model for facility location in the design of complex supply chains. *Int. J. Prod. Econ.* **113** (2008) 678–693.
[28] P.N. Thanh, N. Bostel and O. Péton, A DC programming heuristic applied to the logistics network design problem. *Int. J. Prod. Econ.* **136** (2012) 94–105.
[29] Y.C. Tsao and J.C. Lu, A supply chain network design considering transportation cost discounts. *Transp. Res. E* **48** (2012) 401–414.
[30] Y.C. Tsao, D. Mangotra, J.C. Lu and M. Dong, A continuous approximation approach for the integrated facility-inventory allocation problem. *Eur. J. Oper. Res.* **222** (2012) 216–228.
[31] T. Van Roy and D. Erlenkotter, A dual-based procedure for dynamic facility location. *Manage. Sci.* **28** (1982) 1091–1105.
[32] D. Vila, A. Martel and R. Beauregard, Designing logistics networks in divergent process industries: A methodology and its application to the lumber industry. *Int. J. Prod. Econ.* **102** (2006) 358–378.
[33] F. Wang, Lai X and N. Shi, A multi-objective optimization for green supply chain network design. *Decis. Support Syst.* **51** (2011) 262–269.
[34] T. Xifeng, Z. Ji and X. Peng, A multi-objective optimization model for sustainable logistics facility location. *Transp. Res. D* **22** (2013) 45–48.