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NEW NOTATION AND CLASSIFICATION SCHEME FOR VEHICLE ROUTING PROBLEMS

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Abstract. Vehicle Routing Problems have been some of the most studied problems in combinatorial optimisation because they have many applications in transportation and supply chain. They are usually known as Vehicle Routing Problems or VRPs. The related literature is quite large and diverse both in terms of variants of the problems and in terms of solving approaches. To identify the different variants of routing problems, authors generally use initialisms, in which various prefixes and suffixes indicate the presence of different assumptions or constraints. But this identification based on initialisms is inefficient. For example, two variants of a problem may be identified by the same abbreviation, whereas different abbreviations may be assigned to the same problem. This paper proposes a new notation and a new formalism to identify and to classify instances of routing problems. This contribution aims at filling in the gaps of the current identification system. The goal is to allow everyone to position his work accurately in the literature, and to easily identify approaches and results comparable to his research. The proposed notation is inspired by the scheduling formalism. It has four fields ($\pi/\alpha/\beta/\gamma$), respectively describing the type and horizon of the problem, the system structure, resources and demands, constraints and objectives to be optimized. 26 papers from the literature chosen

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for their disparity are classified using this notation to illustrate its usefulness and a software tool is proposed to make its use easier.

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1. Introduction

The objective of this paper is to propose a new notation and classification schema for vehicle routing problems. We note that our aim is not to make a state of the art of the literature because there are already several recent surveys interested in vehicle routing problems [5, 28, 36, 43, 68, 73, 83, 88, 91].

The basic model of all variants of vehicle routing problems is the Traveling Salesman Problem or TSP [25], which consists in defining the tour of a salesman visiting all cities of a predefined set of cities and returning to the departure city (usually called depot). The aim is to fix the order in which cities will be visited such as each city will be visited once (the tour is a Hamiltonian cycle), while minimizing the travel distance. The TSP is defined in an undirected, valued and complete graph. Many other vehicle routing problems have been defined by extending this basic problem, adding different assumptions, constraints and/or objectives. The literature dedicated to these problems is vast. In addition, many problems in various application areas can be advantageously modelled and solved by using modelling and resolutions of the traveling salesman problem (TSP) or one of its variants. On the other hand, during the last decade, the growing interest in the fields of transportation and logistics has increased the number of published research works on different variants of routing problems further. However, authors use initialisms, in which various prefixes and suffixes indicate the presence of different assumptions or constraints, to distinguish various variants of vehicle routing problems. The multiplication of complex variants in the literature has increased the number and the length of used initialisms. This identification system based on initialisms then shows its limits. As there is no common standard, and therefore no control over the relevance and the consistency of the used initialisms, redundancy and/or inconsistencies appear. New suffixes or prefixes are often created as needed by each author. No record of existing initialisms is maintained. Note that prefixes or suffixes can surround the initialism TSP or the initialism VRP. Two different variants of the problem may be designated by the same initialism or otherwise separate initialisms can be assigned to two strictly identical problems depending on the authors. In addition, using an initialism becomes insufficient to represent complex variants.

In this paper, we propose a new notation and classification schema based on a $\pi/\alpha/\beta/\gamma$ pattern, such as those traditionally used for production scheduling.

The π field characterizes the problem, α describes the structure of the system, the resources and the demands, β specifies the constraints and γ describes the criteria to be optimized (objectives). The rest of the paper is organised as follows. Section 2 presents a review of classification schemes for routing problems. Section 3 describes the new classification and details its parameters, the semantic of its symbols and the construction rules. An application of the proposed notation on 26 selected papers from the literature is proposed in Section 4. Section 5 compares the proposed classification with those found in the literature. Section 6 gives a conclusion and discusses some prospects.

2. Overview of existing classification schemes for vehicle routing problems

2.1. Classification scheme by initialisms

The most used classification for vehicle routing problems is the identification by initialisms. Prefixes or/and suffixes can surround either the initialism TSP or the initialism VRP. In fact, the initialism VRP itself is already a source of difficulty and confusion. The VRP was formulated in 1959 by [24] but like other initialisms, he has various definitions in the literature. VRP is a generalization of TSP that can be described as defining routes of several vehicles at a minimal cost. Each route starts from the depot, visits a set of geographically dispersed clients and returns to the depot. The objective is to find a set of routes visiting these sites to minimize the total cost. According to the authors, this generalization of the TSP can be distinguished either by: 1) only the fact that many vehicles are available; or 2) by vehicle capacity constraint and autonomy constraint (see for example [44, 79, 92] or [15]; or 3) only by the vehicle capacity constraint ([37] or [40]), or 4) only by vehicle autonomy [50]; or 5) by various types of constraints that may limit the tour (capacity, total time, total distance, a maximum number of customers ...) (see for example [62, 74]). [86] refers to four types of VRP which can be deduced from the literature: basic VRP corresponds to the first case in the previous list, capacitated Distance-Constrained CDVRP corresponds to the second case, capacitated VRP (CVRP) corresponds to the third case, and Constrained Distance VRP (DVRP) corresponds to the fourth case.

Various prefixes and suffixes can be combined to indicate the presence of different assumptions or constraints [73]. For example, the name DSDVRPTW [70] means "Discrete Split Delivery Vehicle Routing Problem with Time Windows", in which each customer can be visited several times, but under constraints of time windows. The name VRPMVTTW [51] (or Vehicle Routing Problem with Multiple Vehicle Types and Time Windows) is another problem similar to the above except that the constraint "Split Delivery" is replaced by a constraint "heterogeneous fleet of vehicles". These examples demonstrate that the length of the initialisms became increasingly long. These initialisms are not sufficient to accurately represent the conjunction of assumptions, constraints and objectives of variants addressed in practice.

Reviewing the literature reveals that there exist many conflicting definitions for some abbreviations. In particular, initialisms can be used to represent two different problems. [85] studied a case of vehicle routing problem where the customer demands and visiting times are stochastic (VRP with stochastic demands and stochastic travel times). [34] treated a vehicle routing problem where the tours may not include all clients but where each customer must be visited within a given time window. In the latter case, the visited customers are chosen to optimize multiple objectives: the total profit, total distance, total time and total vehicle load. The differences between the characteristics of the two problems addressed by [34, 85] are very large, but the same notation SVRP is used in both cases.

Similarly, the SD prefix is used by some authors to describe the stochastic nature of such requests [10], while other authors such as [7] use it to define variants of vehicle routing problems in which customer demand can be satisfied by several tours (Vehicle Routing Problem with Split Delivery). [86] uses the initialism DVRP to mean that the total distance is limited by an upper bound (Distance-Constrained VRP), while [47] uses the same initialism DVRP to design the dynamic case of VRP (Dynamic vehicle routing problem or DVRP). Then one could think that the initialism DARP designs either a Distance-constrained Arc Routing Problem or a dynamic arc routing problem. In fact it represents the Dial-a-Ride Problem [17, 19, 21], a particular case of Vehicle Routing Problem with Pickup and Delivery for which [39] recently proposed a survey. [66] has identified several existing notations in the literature for the VRP with Backhauls and Mixed Linehauls: PDP (Pickup and Delivery Problem) in [56] and MVRPB (Mixed Vehicle Routing Problem with Backhauls) in [82, 84]. Similarly, in the literature, there are several initialisms to denote the fact that vehicles can do several tours during the same period as VRPMT (Vehicle Routing Problem with Multiple Trips), in [61], or MTVRP (Multi-Trip Vehicle Routing Problem) in [2] and VRPMVT or VRP with Multiple Vehicle Trips in [51].

Because of the limits presented above, we propose in this paper a new formalism and notation for vehicle routing problems, which may facilitate the identification of variants of problems, the communication between researchers and the comparison of proposed approaches. The main difficulty in the proposition of this notation is to select the most relevant parameters to make the classification be sufficiently meaningful and discriminating, while being as concise as possible.

2.2. Other existing classification schemas for routing problems

Because of the large research dedicated to routing problems, the classifications most often proposed in the literature are either detailed but restricted to subclasses of problems, or comprehensive but using a quite low level of detail. A review of some previous classification efforts and taxonomies can be found in [28, 91].

Authors in [65, 66] considered the General Pickup and Delivery Problem or GPDP. They proposed a classification limited to Vehicle Routing Problem with Backhauls (VRPB) and Vehicle Routing Problem with Pickup and Delivery (VRPPD). Note that the latter is also sometimes denoted in the literature by the shorter initialism PDP (Pick up and Delivery Problem) [6]. These two variants of problems are characterized by the presence of two types of sites, linehauls or backhauls. They are distinguished by the role of the depot. [8] proposed to classify the static pickup and delivery problem according to three fields Structure/Visits/Vehicles. The "structure" field indicates the number of origins and destinations of the commodities. The second field describes the service at customers vertices (PD if pickup and delivery operation must be performed in one visit to the customer, P-D if the pickup and delivery operation may be done together or separately, and P/D if either pickup or delivery operation is performed at each vertex).

Ghiani *et al.* [31] classified all variants of VRP problems but only depending on the nature of the problem. If some data of the problem is random, the problem is called stochastic. Otherwise, it is deterministic. In addition, if all data are known from the beginning of the resolution, it is called static. Finally, it is dynamic if some data depends on time. In this case, no data are available at the beginning of the resolution; new data are available randomly during the resolution. Psaraftis [75] also proposed a taxonomy useful for characterising input informations parameters. In the same way, Pillac *et al.* [68] classified routing problems from the perspective of information quality and evolution.

Min *et al.* [53] provided a hierarchical taxonomy and classification scheme to better identify location-allocation-routing problems and they also proposed a related taxonomy for solving methods. Other reviews and classifications dedicated to a class of routing problems need to be mentioned. Andersson *et al.* [5] provided a classification and a literature review for combined inventory management and routing. Hoff *et al.* [36] reviewed and classified a fleet composition and routing problems. Schmid *et al.* [83] reviewed some extensions of routing problems in the context of supply chain, including lotsizing, scheduling, packing, inventory and intermodality. Authors in [42, 57] provided a classification scheme for periodic routing problems.

A recent classification can be found in [91]. These authors introduced an attribute-classification system. The proposed classification makes relationships between problem attributes and recent heuristics. The paper provided a survey of heuristics and meta-heuristics for Multi Attribute Vehicle Routing Problems (MAVRP). MAVRP includes many variants with additional characteristics and constraints to complement the Capacitated VRP variant. Three main classes attributes are used in the proposed classification. The first class, "Assignment of customers and routes to resources", regroups some attributes as multiple depots, heterogeneous fleets, split delivery. The second class, "Sequence Choice", regroups attributes that impact the nature and structure of the routes like multi-trips or intermediate facility, and the last class, "Evaluation of fixed sequences" regroups attributes that impact a large variety of evaluations and constraints that must be verified once the route and orders within the route are chosen like time windows, open routes. The attribute-classification allows the representation of the most studied variants in the literature and provides a concept to analyse MAVRP. The performed analysis already covers a large part of Vehicle Routing literature. But, this contribution was rather a method-oriented classification than an application-oriented one. Thus, the main goal of this approach was to establish relationships between problem attributes and the performances of recent heuristics. Therefore, the scope of the analysis was limited to single objective problems in which demands are on nodes, with complete and exact data. In addition, MAVRPs with combined attributes are not adressed.

Only two classifications proposed for all vehicle routing with high levels of detail were found in [27, 28]. [27] propose a language for describing classes of problems. They use four fields describing respectively the addresses (depot and clients sites), the vehicles, other characteristics of the problem (those not linked to addresses or vehicles) and the objectives. A hierarchical structure is used, consisting of fields, sub-fields and the parameters, which are non-terminal symbols. Terminal symbols are the values of the parameters for a given problem instance. Logical operators (or exclusive, and) are used to link the terminal symbols. A default value, which is the most known and most frequently used value in the literature, is used to make this notation be more concise. This notation is very interesting, especially for its high level of detail. These authors illustrate its usefulness through fourteen examples to show that it can represent complex variations of VRP. The notation of [27] allows to specify: 1) if demands are on the nodes, on the edges or are origin-destination pairs; 2) if it is necessary to select a subset of customers, possibly specifying whether to serve at least one client in each subset of a given partition; 3) if vehicles have a dedicated or interchangeable compartments; 4) if demands and/or vehicles are subject to constraints of time windows specifying when the service can be made. One major inconvenient of the notation of [27] is its limitation to static and dynamic cases. Only the description of demands includes a parameter indicating a possible stochastic nature. On the other hand, some terminal symbols are numeric or expressed by operators ($\pm$, /) rather than by words or abbreviations of words, which makes them difficult to memorize. Although the level of detail of this classification is high, it does not represent some important variants of VRP, such as periodic problems. These authors indicate only whether the problem is periodic or not within the constraints of the "address" field but information on service planning are essential to understand the complexity of the treated case.

Eksioglu *et al.* [28] proposed an interesting taxonomy of the VRP literature. This taxonomy is built in an arborescent way with at most three levels. The first level is composed of five different aspects: 1) type of study; 2) scenario characteristics; 3) problem physical characteristics; 4) information characteristics, and 5) data characteristics. The first category "type of study" is divided into four subcategories (theory, applied methods, implementation documentation, survey review or meta-research). The subcategory "applied methods" is subdivided into five attributes: exact methods, heuristics, simulation, and real time solution methods. Within the second category, "scenario characteristics" a description of the problem is given by 9 attributes and 26 final parameters. As an example of these attributes, we can note "the number of stops on route" subdivided into parameters (known, partially known, partially probabilistic), "The Load splitting constraint" subdivided into parameters (splitting allowed, splitting not allowed). The third category "problem physical characteristics" is divided into 11 attributes and 34 final parameters. This category includes factors that directly affect the solution. Some used attributes in this third category are "number of vehicles", divided into parameters (exactly n vehicles, up to n vehicles, unlimited number of vehicles), and "travel time", divided into (deterministic, function dependent, stochastic, unknown). The fourth category is dedicated to information characteristics and it is divided into 4 attributes and 10 final parameters. The last category "Data characteristics" is divided into 2 attributes and 3 final parameters. This taxonomy is used to classify 30 papers from the VRP literature. The classification of [28] allows identifying which attributes are more, less or never investigated in the literature. It represents a very large framework which enables to represent different aspects of VRP problems including the characteristics of problem, information about instances, characteristics of information, and solution methods. Despite this, this taxonomy fails to underscore the intrinsic characteristic of the variants of VRP problems. In fact, it provides a comparison of research studies but does not highlight a comparison according to variants of VRP. In addition, the objectives are not represented; only attributes as "travel time" and "transportation cost" are given. Also, the characteristic of periodic problem cannot be represented. No information about planning service is given. Some variants cannot be identified by this taxonomy like "multi-trip VRP", which represents the possibility for a vehicle to do several tours, and "selective VRP" designing the case when it is not possible to visit all customers. In fact, as mentioned by these authors, it is possible to evolve the proposed taxonomy and to add some attributes, but it seems difficult to identify VRP variants.

None of the cited classifications has been used to cover the large literature dedicated to vehicle routing. They all fail in identifying differences and similarities between the VRP problems studied in the literature. This is why the new notation proposed in this paper is based on a level of detail quite similar to the one used in [27, 28] while using more standard mnemonics and schemes to facilitate its adoption by research communities. Note that our classification uses a smaller number of parameters than [28] but covers more VRP variants. We have chosen the most significant parameters and we have defined rules and operators to represent the combination of various complex problems. We give more precision on the way to use and to combine the parameters according to systematic rules using operators. Besides, a software tool (`http://comordo.fr/classeur/accueil.php`) was developed to facilitate the use of the notation. It permits the user (industrial, researcher) to easily describe the variant of vehicle routing problem. Then, the software tool automatically generates the associated notation, and then compares it with the notations of the papers previously classified in order to position it precisely.

3. Presentation of the new classification schema for routing problems

The proposed notation is summarized in the tables presented from Figure 8 to Figure 11. It is based on a standard and simple scheme. Systematic rules and semantic symbols are used to facilitate its use. The structure of the proposed notation is an adaptation of a well-known notation used in scheduling. This classical formalism was proposed by [81], and described in [11, 33]. The notation of [81] includes different fields $\alpha/\beta/\gamma$ that describe respectively the production environment, tasks and resources, and finally the criterion to optimize. It has already been adapted to classify various types of problems like Hoist Scheduling Problem or HSP in [58] and Assembly Line balancing problem or ALB in [12, 13]. In the context of vehicle routing problems, we propose to use four fields $\pi/\alpha/\beta/\gamma$, respectively, to describe the context, the resources (fixed or mobile), the demands and the constraints other than those specific for resources or requests and finally the objectives to optimize. The following sections detail the proposed notation.

3.1. Syntax and construction rules

The proposed notation is built using the simple and systematic rules described below. It has four fields $\pi/\alpha/\beta/\gamma$. The field (α) is divided into subfields ($\alpha1$, $\alpha2$ and $\alpha3$). Each field (π, α, β, γ) or subfield ($\alpha1$, $\alpha2$ and $\alpha3$) contains several parameters. The separation between fields is done by "/". The separation between subfields or between parameters of the same fields is done by ",". Each parameter corresponds to a pair (key, value) where the key is the name of this parameter. Separators "/" are present even when some of the previous fields or subfields are absent. For example $\pi/\alpha1$, $\alpha2$, $\alpha3$ / / γ expresses the absence of β field. The next Section 3.2 describes precisely the set of pairs used. Each key (this means parameter name) is chosen to facilitate memorisation using if possible initialisms already used in the literature or abbreviations of words whose meaning is close to the parameter. All parameters used in the notation are summarized in the Appendix presented in Section 6. To instantiate the value of a given parameter, there are three possible levels:

- Level 1: the parameter does not appear in the notation, which means it takes its default value, corresponding to the case most frequently encountered in the literature. The goal is to avoid any overload and to get the most concise notation, especially for basic problems of VRP.
- Level 2: only the key appears with unspecified value, which means that the corresponding hypothesis, constraint or objective appears in the problem as static and deterministic.
- Level 3: the key is followed by one or more values that are grouped in this case by a concatenation operator ":" to describe more complex cases. The values

Parameter	Default value	General values
Planning Horizon (H)	1 period	**MP** (Multi Period)

FIGURE 1. Notation's parameters: field π.

used are of three types. They can be: 1) "values of the key": after the concatenation operator ":", a general set of possible values can be used by everyone to bring additional information. These values are chosen from a set described in braces, in which the values are separated by "-". For some parameters values are not mutually exclusive, several values can be used together. For example, general values for the parameter Tw (describing constraints of time windows) can be chosen in the set Ve (designing vehicles) − Wo (designing workers) − Dp (designing depots) − C (designing customers) − Net (designing network), it is possible to use both Ve and C to specify that there are constraints of time windows for vehicles but also for customers. 2) "generic value OT", meaning "Other", to represent the cases encountered in the literature that can not be represented using the previous general predefined values. Furthermore, the addition of the letters D or S specifies the dynamic and stochastic nature. The abbreviations defined in the detailed description may be added to specify on which the stochastic and/or dynamic character is defined. Note that the rule of application of (:D) and (:S) can be applied to all parameters, even if in the following tables, we do not detail all possible values with the application of (:D) and/or (:S), in order to prevent these tables from being too loaded. 3) Finally, the "$\leftrightarrow$" represents the existence of a relationship between two types of elements. For example, when there are constraints of compatibility between vehicles and customers (which is denoted Ve $\leftrightarrow$ C).

3.2. Semantics of symbols

The following paragraphs detail the keys and values of the parameters constituting the different fields and sub-fields. They are illustrated by examples. The appendix presented in Section 6 shows the meaning of existing initialisms in these tables.

3.2.1. π Field

The field π describes the problem (see Fig. 1). It has the initials VR to indicate that it refers to the classification of the Vehicle Routing problems. It is different from the initialism VRP, already associated with several conflicting definitions in the literature, as previously mentioned in 2.1. Besides, it permits to precise that this notation is limited to vehicle routing. It is not able for instance to represent all types of "routing problems", such as those encountered in the communication

Parameter	Default Value	General values
Type of Depot	Backhaul	**Dp:Li** (Linehaul) **Dp:(B, Li)** (Two types: Backhaul, Linehaul) **Dp: (B+Li)** (Combined Backhaul and Linehaul) **Dp:W** (Warehouse)
Number of Depots (Dp)	1 depot	**MDp** (Multi-depot)
Network (Net)	Undirected network	**ONet** (Directed network) **MixNet** (Mixed network)
Network Level (E)	1 echelon	**2E** (2-Echelon) **MultiE** (Number of echelons >2)

FIGURE 2. Notation's parameters: subfield $\alpha 1$.

field. But it permits to distinguish this category of problems from the great variety of optimization problems which can be met in transportation field, such as [59] for instance, who study the planning of public transportation systems, particularly the assessment of connection times, rather than building routes. In addition, the π field contains one parameter, which is the planning horizon H, which by default is one period, but can also be Multi period (MP). This corresponds to the Multi-periodic Vehicle Routing Problem (Periodic Vehicle Routing Problem or PVRP) in which each customer must be visited k times during the planning horizon ($1 \leq k \leq M$), where M is the given number of periods, and the daily demands are fixed [42]. Several references on the PVRP can be found for example in [16, 30].

3.2.2. α Field

The α field defines the structure of the system; it is divided into three subfields
$\alpha 1$: fixed resources (depot (Dp) and network (Net));
$\alpha 2$: mobile resources (vehicles (Ve) and workers (Wo));
$\alpha 3$: demands (Dd) (Customer (C), quantity (Q) and location (Lo)).

i. Subfield $\alpha 1$ (fixed resources)

The $\alpha 1$ subfield has three parameters (Fig. 2):

- *The type of depot*: if all depots are backhaul, the parameter takes the default value. If all depots are linehaul, the value is Dp: Li. If there is two different types of depots (some backhaul, other linehaul) the value is Dp(B, Li). If some depots can be both backhaul and linehaul, the value is Dp:(Li + B). If the depot is not dedicated to deliver or to receive any demands, it takes the value Dp: W (warehouse).

Parameter	Default value	General values
Number of vehicles (NVe)	Unlimited	**NVe :** 1 (Exactly one vehicle) **NVe** (Fixed number of vehicles > 1)
Vehicle compartment (Cp)	1 compartment	**MCp** (Multi-compartment)
Type of fleet	Homogeneous	**Het** (Heterogeneous)
Number of Workers (NWo)	Unlimited	**NWo** (Limited, static and deterministic)

FIGURE 3. Notation's parameters: subfield $\alpha 2$.

- *The number of depots for each type of depot*: is equal to one (default value) or (Multi-depots MDp). This allows representing the Multi-Depot Vehicle Routing Problem (MDVRP). See for example ([22, 35, 55, 90]).
- *The type of network (Net)*: The network can be undirected (default value), oriented (ONet), or mixed (MixNet) containing both edges and arcs.
- *The number of network levels*: most studies consider one-echelon networks but some variants in the context of supply chain and logistic management consider multi-echelon networks. In this case, the delivery from one depot to the customers is managed by intermediate depots [67].

ii. Subfield $\alpha 2$ (mobile resources)

The $\alpha 2$ subfield (Fig. 3) determines all information concerning mobile resources [vehicle (Ve) and workers (Wo)]. It has three parameters.

- *The number of vehicles*: can be unlimited (Default Value), or limited by a constant greater than 1 in the static and deterministic case (NVe), or limited by value equal to 1 (NVe: 1), or limited by a variable over time in the dynamic case (NVe: D), or limited by a variable defined by a stochastic law in the stochastic case (NVe: S).
  In many papers from the literature, the number of vehicles is limited by a constant value (NVe) as in [1–3, 9, 23, 26, 30, 38, 45, 49, 54, 63, 71, 87, 93]. Another example for the value (NVe: 1) can be found in [38].
- *Vehicle compartment*: can be equal to 1 (Default Value) which corresponds to the case without compartment. The value MCp means that the vehicle has a number of compartments >1 (Cp: MCp). Each compartment can be dedicated or not to a type of product. This allows modeling several applications like fuel delivery [52].
- *The fleet type*: it can be homogeneous (identical vehicles), which is a default value, or heterogeneous, where the vehicles have different characteristics, for example in terms of capacity or time (Het). This parameters is considered for example in [2, 26, 69].
- *The number of workers*: may be unlimited (DefaultValue) or limited by a static and deterministic constant (NWo), or limited by a dynamic value (NWo: D), or limited by a stochastic value (NWo: S) [89].

Parameter	Default value	General values
Localisation (Lo)	Nodes	**A** (Arcs) **Mix** (Mixed: Arcs+Nodes)
Type of Customers (C)	Linehaul	**C:B** (Backhaul) **C:(Li,B)** (Two types: Backhaul, Linehaul) **C:(Li+B)** (Combined Backhaul and Linehaul)
Number of types of products (Pr)	1 type of product	**MPr** (Multi-product)
Type of demand (Dd)	Static	**Dd : S: C** (The "number of clients" is stochastic) **Dd : S: Q** (The "quantity" is stochastic) **Dd : S: Lo** (The "localisation of clients" is stochastic) **Dd : D: C** (The "number of clients" is dynamic) **Dd : D: Q** (The "quantity" is dynamic) **Dd : D: Lo** (The "localisation of clients" is dynamic)
Service cost (Ser)	Ser:T (Static and deterministic service cost time)	**Ser: Ø** (No service cost is considered) **Ser:T:S** (Stochastic service time cost) **Ser:T:D** (Dynamic service time cost) **Ser: OT** (Other type of service cost, static and deterministic, **: S** other type of stochastic service cost, **:D** other type of dynamic service cost)
Transportation cost (Tr)	Tr:Cost (Static and deterministic transportation time or distance)	**Tr: Ø** (No transportation cost is considered) **Tr: Cost** (Stochastic (**:S**) or dynamic (**:D**) transportation time or distance) **Tr:Dis** (Static and deterministic transportation distance, **:D** dynamic (**: S** stochastic) transportation distance) **Tr: T** (Static and deterministic transportation time, **:D** dynamic (**:S** stochastic) transportation time)

FIGURE 4. Notation's parameters: subfield $\alpha 3$.

iii. Subfield $\alpha 3$ (requests)

Figure 4 below summarizes all elements related to $\alpha 3$ field described above.

This subfield represents demands (Dd), Customers (C), quantity (Q) and locations (Lo) in six parameters:

- *Location (Lo)* of the demands may be on nodes such as in VRP (DefaultValue), or on the edges, like the ARP [78] (A), or on arcs and nodes as in General Routing Problem (Mix).

- *The type of customers (C)* specifies whether the customers are linehauls (DefaultValue) or not. If all clients are backhauls, the value is (C: B). If some clients are linehauls, and other backhauls, the value is C: (Li, B) as in [38, 64] or customers can be both linehauls and backhauls C: (Li + B) as in arc routing applications.
- *The number of products types (Pr)*: it can take a default value equal to 1, or greater than 1 (Mpr).
- *The nature of the demand (Dd)* can be static (DefaultValue) or stochastic (Dd: S), or dynamic (Dd: D). Note that in this case, it is possible to specify which parameter carries the dynamic or stochastic aspects: the number of customers (Dd: D: C or Dd: S: C), the quantities (Dd: D: Q or Dd: S: Q) or locations (Dd: D: Lo or Dd: S: Lo)), or a combination of these.
- *The cost of service (Ser)* can be equal to the time during which the vehicle is stopped at the customers sites (Operation time)(DefaultValue), or to 0, or other costs, such as loading/unloading, fixed or proportional to the load (Ser: OT). Many examples provided in this paper correspond to the default value [1, 3, 9, 14, 23, 26, 30, 35, 41, 45, 46, 48, 49, 63, 64, 71, 87].
  Note that if the cost of service is different from zero (Ser: T or Ser: OT), it is (by default) static and deterministic. Otherwise, it can be defined dynamically (Ser: T: D or Ser: OT: D), or defined by a stochastic law (Ser: T: S or Ser: OT: S).
- *The cost of transport (Tr)* can be equal to a cost in terms of distance or in terms of travel time (many authors use this solution, without specifying it precisely) (DefaultValue), or can be equal to zero, and can potentially include fixed costs (ex. the purchase of vehicles) and a variable value proportional to the number of products. The notation provides two types of cost conventionally used: the distance (Tr: Dis) and travel time (Tr: T), or other cases (Tr: OT). Note that if the cost of transport is non zero, it is by default static and deterministic (Tr: Dis, Tr: T or Tr: OT). Otherwise, it can be dynamic (Tr: Dis: D, Tr: T: D or Tr: OT: D) or stochastic (Tr: Dis: S, Tr: T: S or Tr: OT: S). Several articles use (Tr: Dis) as optimization criterion [1–3, 9, 14, 23, 45, 46, 48, 49, 60, 63, 64, 87, 93] and (Tr: T) as [1, 3, 9, 14, 23, 26, 30, 35, 38, 45, 46, 48, 49, 63, 64, 71, 87].

3.2.3. β Field

The third field, β (Figs. 5 and 6) describes the constraints of the problem. In general, a constraint is a restriction or condition on a variable of the problem. In the literature of routing problems, many restrictions exist depending on the studied problem.

This sub-section attempts to identify the constraints encountered in the literature:

- *Capacity (Cap)* (vehicle, depot, or clients) is either unlimited (default value) or limited by a static and deterministic value. For example Cap: Ve, Cap: Dp, Cap: C, Cap: Ve: C, indicate respectively vehicle capacity, depot capacity, client capacity (storage site capacity) and vehicle and clients capacities.

Parameter	Default value	General values
Capacity (Cap)	Unlimited	**Cap: Ve** (Capacity of the vehicle) **Cap:Dp** (Capacity of the depot) **Cap:C**(Capacity of customer sites) **Cap:Ve:C** (Capacity of vehicle and capacity of customer sites)
Autonomy (Aut)	Unlimited	**Aut:Ve** (Autonomy of the vehicle) **Aut:Wo** (Maximum working time of workers)
Time windows (Tw)	∅	**Tw: Ve** (TW for vehicle availability) **Tw: Wo** (TW for working hours of workers) **Tw: Dp** (TW for the opening hours of the depot) **Tw:** C (TW for clients) **Tw: Net** (TW for some parts of the network)
Number of not served customers (NoSer)	∅	**NoSer** (>=0)
Traffic conditions (Tra)	∅	Tra (Static and deterministic)
Split Delivery (SD)	∅	**SD** (Splitting allowed per demand and per product) **SD: NoPr** (Compartment: splitting allowed per demand only)
Multi-trips (MT)	∅	**MT**
Vehicle return to its original depot	Obligatory	**NoR** (Not obligatory)
Original depot for the vehicle	The same depot	**MDpP** (Vehicle can start its trip from different depots)

Figure 5. Notation's parameters: subfield β, part 1.

- *Autonomy (Aut)* represents either vehicle range (maximum time between the start from depot and the return to it) or the maximum working time of workers, and can be unlimited (default value), or limited by a static and deterministic value, (Aut: Ve, Aut: Wo). For (Aut: Ve), see for example [1, 2, 23, 64, 71].
- *Time windows (Tw)* specify one (or more) intervals of time to satisfy, as client service (Tw: C), the beginning service on the depot (Tw: Dp), the respect

Parameter	Default value	General values
Planning service	Ø	F (Service frequency) **Spa:Min—Max** (Minimum (∞ if unlimited) and maximum (∞ if unlimited) spacing time between two successive services for the same customer) **Comb** (Choice of service according to a set of combinations of periods)
Initial depot of a vehicle durin the planning horizon	A vehicle always starts from the same depot	**DpVar** (Vehicle doesn't start its trip from the same depot)
Link between demand	Unpaired	**Paired**
Compatibility	Ø	**α i ↔ α j** i,j ∈{1,2,3} α1∈{Dp,Net}, α2∈{Ve,Wo}, α3∈{C, Pr}
Origin of products	Depot	C (Customer) **(Dp,** C) (Depot and customer)
Precedence (Prec)	Ø	**Prec**

FIGURE 6. Notation's parameters: subfield β, part 2.

of the regulations of working time (Tw: Wo), the availability of vehicles (Tw: Ve), or time constraints on some parts of the network (Tw: Net). This allows to represent the most studied variants of VRP when time constraints are related to the customer, but also some variants of the problem including time windows on the opening hours of the depot or the working hours of staff [63].

- *The number of no served customers* represents the selective nature of the problem. In this variant, all customers are not necessarily served if the number and the capacity of vehicles do not allow it. In this case, only a subset of clients can be served [34]. By default, this number is zero, otherwise it is potentially non-zero (NoSer) as in [49].
- *The traffic conditions (Tra)*: by default, the problem does not take into account traffic conditions. Otherwise, we must specify the nature of these conditions. They can be static (Tra), stochastic (Tra: S) or dynamic (Tra: D).
- *The splitting of demands (split delivery) (SD)*: by default, customer demand must be satisfied on one visit, but in some variants of the problem each demand (itself composed of several products) can be divided into several vehicles (SD). In the case of SD, each customer can be visited more than once if necessary;

therefore, the demand of each customer can be greater than the vehicle capacity [7]. In other variants such as the VRP with compartment [29], the demand is composed by several products, and the split is not allowed within the same product (SD: NoPr).

- *The possibility of having several tours (multiple trips) per vehicle (MT)* states that vehicles can do several tours as in [2].
- *The return of each vehicle to its original depot* is required by default but in some cases, as in OVRP (Open Vehicle Routing Problem), vehicles are not required to return to the depot, or when it is necessary to return, they revisit customers in reverse order (NoR).
- *The origin of vehicle*: by default, during the same period, all vehicles start from the same depot, but for some variants vehicles can start from different depots (MDpP) as in [35].
- *The Planning service*: in some variants of VRP as the "Periodic VRP" or "Split Delivery VRP", the demand can be satisfied by several vehicles (several trips). A planning service constraint is then defined by frequency (F) (a number of visits to meet over a period of one or more times), see for instance [2, 30, 54, 93], or by a minimum and/or maximum spacing time between two successive services (Spa: Min $\sim$ Max) as in [30], or by a list of combinations of periods (Comb) like in [2, 54].
- *The allocation of depot*: by default, a vehicle is assigned to a depot, and does not change depot. In other cases, the assignment of a vehicle to depots can change from one period to another (DpVar).
- *Paired demands*: the points to visit during tours are generally independent (unpaired), but sometimes the client requests are linked (Paired). Thus, in the PDP or DARP, origin and destination are specified by the request of each customer as in [38, 64].
- *Compatibility constraints*: by default, there are no compatibility constraints. Otherwise, there exist several types of constraints; they joint some fields $\alpha 1$, $\alpha 2$ and $\alpha 3$ to express relationships. This allows representing different cases, for example, a vehicle access constraint to streets, or to some parts of the network or to depots. Another example is the compatibility constraints between workers (skill required) and demands, or compatibility constraint between vehicles and products. For example in [2,26] there is a constraint linking the customer service to some vehicles (Ve $\leftrightarrow$ C).
- *The source of the products*: products can be provided from one or several depots (Default Value), from clients as in PDP and DARP (C), or from depots and clients (Dp,C).
- *The precedence constraints (Prec)*: by default, there is no precedence constraints between clients, but in some variants vehicles must visit some clients before others, such as the VRP with Clustered Backhauls (VRPCB), the Pickup and Delivery Problem or PDP as in [38,64] and Dial-A-Ride Problem or DARP.

Parameter	Default value	General values
Objectives (minimisation)	Find a feasible solution	**Tr:Dis** (Transportation distance) **NVe** (Number of vehicles) **NoSer** (Number of non served clients) **Ser:T** (Service time) **Tr:T** (Transportation time) **Tr: Cost** (Transportation cost, time or distance) **WT** (Waiting time at customer site) **Over:** β [Violation of constraint: (TW, Cap, Aut...)] **ODp** (Opening cost of depots) **OT** (Other)

Figure 7. Notation's parameters: subfield γ.

3.2.4. γ Field (objectives)

This field γ (Fig. 7) describes the criteria to optimize.
Indeed, the literature reports various objective functions such as minimizing:

- The number of vehicles (NVe).
- The total distance or travel time (not specified) (Tr:cost).
- The total distance (Tr: Dis).
- The travel time (Tr: T).
- The time service (Ser: T).
- The number of customers not served (NoSer) as in [49].
- The waiting time for customers (Waiting Time) (WT).
- The opening cost of depots (ODp).
- The violation of constraint (Over). In this case, it is possible to specify which constraint belonging to the field β. For example, the use of (Over: Tw) means that the objective is to minimize customers delays for VRPTW as [9, 38, 49]. The parameter (Over: Aut: Wo) means that the objective is to minimise the workers overtime. The parameter (Over: Cap) indicates the minimisation of capacity violation constraint as in [1, 38].

Note that some of these criteria are already listed in the previous fields. This is necessary to distinguish the parameters that must be taken into account (but not necessarily as an objective to optimize) of those who actually are the objectives of the problem. For example, the cost of transportation (Tr: Dis) could be considered to check the constraint of autonomy in a problem for which the objective is to minimize the number of customers not served (NoSer). Then, (Tr: Dis) appear in the $\alpha 3$ subfield, while (NoSer) appear in β and γ. Inversely, (Tr: Dis) could be the objective to minimize (then appearing in $\alpha 3$ and γ) in a problem where (NoSer)

	Parameter	Default value	General values
Π	Planning Horizon (H)	1 period	**MP (Multi Period)**
Fixed resources α1	Type of Depot	Backhaul	**{ Li - (B+Li) - (B, Li) –W }** **Dp:Li** (Linehaul) **Dp:(B, Li)** (Two types: Backhaul, Linehaul) **Dp: (B+Li)** (Combined Backhaul and Linehaul) **Dp:W** (Warehouse)
	Number of Depots (Dp)	1 depot	**MDp** (Multi-depot)
	Network (Net)	Undirected network	**{ ONet – MixNet }** **ONet** (Directed network) **MixNet** (Mixed network)
	Network Level (E)	1 echelon	**{2E - MultiE}** **2E** (2-Echelon) **MultiE** (Number of echelons >2)
Mobile resources α2	Number of vehicles (NVe)	Unlimited	**{ NVe, NVe : 1 } (limited, static and deterministic)** **NVe : 1** (Exactly one vehicle) **NVe** (Fixed number of vehicles >1)
	Vehicle compartment (Cp)	1 compartment	**MCp** (Multi-compartment)
	Type of fleet	Homogeneous	**Het** (Heterogeneous)
	Number of Workers (NWo)	Unlimited	**NWo** (limited, static and deterministic)
Demands α 3	Localisation (Lo)	Nodes	**{ A - Mix }** **A** (Arcs) **Mix** (Mixed: Arcs+Nodes)
	Type of Customers (C)	Linehaul	**C: {B - (Li,B) - (Li + B) }** **C:B** (Backhaul) **C:(Li,B)** (Two types: Backhaul, Linehaul) **C:(Li+B)** (Combined Backhaul and Linehaul)
	Number of Types of products (Pr)	1 type of product	**MPr** (Multi-product)

Figure 8. Summary of the notation: part 1.

appears only in β , to represent the selective nature of the problem, without being itself an objective.

4. Applications of the proposed classification schema

Figure 12 shows the application of the proposed notation on 26 articles from the literature on the VRP, chosen for their disparity. These papers were selected to

	Parameter	Default value	General values
Demands α 3	Type of Demand (Dd)	Static	**Dd : { S - D:{C – Q - Lo}}** **Dd : S: C** (The "number of clients" is stochastic) **Dd : S: Q** (The "quantity" is stochastic) **Dd : S: Lo** (The "localisation of clients" is stochastic) **Dd : D: C** (The "number of clients" is dynamic) **Dd : D: Q** (The "quantity" is dynamic) **Dd : D: Lo** (The "localisation of clients" is dynamic)
	Service Cost (Ser)	Ser:T (Static and deterministic cost time service)	**Ser: {Ø , T-OT : {D,S}}** **Ser: Ø** (No service cost is considered) **Ser:T:S** (Stochastic service time cost) **Ser:T:D** (Dynamic service time cost) **Ser: OT** (Other type of service cost, static and deterministic, **: S** for other type of stochastic (**:D** dynamic) service cost)
	Transportation Cost (Tr)	Tr:Cost (Static and deterministic transportation time or distance)	**Tr:{ Ø , Cost – Dis - T: {D,S}}** **Tr: Ø** (No transportation cost is considered) **Tr: Cost** (Stochastic (:S) or dynamic (:D) transportation time or distance) **Tr:Dis** (Static and deterministic transportation distance, **:D** for dynamic (**:S** stochastic) transportation distance) **Tr: T** (Static and deterministic transportation time, **:D** for dynamic (**: S** stochastic) transportation time)
Constraints β	Capacity (Cap)	Unlimited	**Cap: {Ve- Dp – C - Ve:C} (Limited, static et deterministic)** **Cap: Ve** (Capacity of the vehicle) **Cap:Dp** (Capacity of the depot) **Cap:C** (Capacity of customer sites) **Cap:Ve:C** (capacity of vehicle and capacity of customer sites)
	Autonomy (Aut)	Unlimited	**Aut: {Ve - Wo} (Limited static and deterministic)** **Aut: Ve** (Autonomy for vehicles) **Aut: Wo** (Maximum working time of workers)
	Time windows (Tw)	Ø	**Tw :{Ve- Wo- Dp- C- Net}** **Tw: Ve** (TW for vehicle availability) **Tw: Wo** (Tw for working hours of workers) **Tw: Dp** (TW for the opening hours of the depot) **Tw: C** (TW for the opening hours of clients) **Tw: Net** (TW for some parts of the network)
	Number of no served customers (NoSer)	Ø	**NoSer** (⩾0)

FIGURE 9. Summary of the notation: part 2.

	Parameter	Default value	General values
Constraints β	Traffic conditions (Tra)	Ø	**Tra (static and deterministic)**
	Split Delivery (SD)	Ø	**{SD-SD :NoPr}** **SD** (Splitting allowed per demand and per product) **SD:NoPr** (Compartment : splitting allowed per demand only)
	Multi-trips (MT)	Ø	**MT**
	Vehicle return to its original depot	Obligatory	**NoR** (Not obligatory)
	Original depot for vehicles	The same depot	**MDpP** (The vehicle can start its trip from different depots)
	Planning service	Ø	**{ F - Spa:Min~Max - Comb }** **F** (Service frequency) **Spa:Min~Max** (Minimum (∞ if unlimited) and maximum (∞ if unlimited) spacing time between two successive services for the same customer) **Comb** (Choice of service according to a set of combinations of periods)
	Initial depot of a vehicle during the planning horizon	A vehicle always starts its trip from the same depot	**DpVar** (The vehicle doesn't start its trips from the same depot)
	Link between demand	Unpaired	**Paired**
	Compatibility	Ø	α i ↔ α j (i,j ∈{1,2,3}) α1∈{Dp,Net}, α2∈{Ve,Wo}, α3∈{C, Pr}
	Origin of products	Depot	**{ C- (Dp ,C) }** C (Customer) (Dp, C) (Depot and customer)
	Precedence (Prec)	Ø	**Prec**

FIGURE 10. Summary of the notation: part 3.

cover a variety of the most important variants encountered in the literature. The first column gives the reference of the article, the second one gives the initialism used by the authors to describe the variant of VRP, and the third column presents the notation obtained using the proposed classification schema. The table in Figure 12 has been sorted for grouping variants designated by identical initialisms.

Authors in [1, 23] studied both the VRP. The notation in Figure 12 shows that these two variants of problems, although otherwise identical, do not optimize the

Objectives γ	Objective (minimisation)	Find a feasible solution	**{ Tr:Dis - NVe - NoSer - Ser:T - Tr:T - WT – Over: β – Dp - OT }** **Tr:Dis** (Transportation distance) **NVe** (Number of vehicles) **NoSer** (Number of non served clients) **Ser:T** (Service time) **Tr:T** (Transportation time) **Tr: Cost** (Transportation cost, time or distance) **WT** (Waiting time at customer site) **Over: β** [Violation of constraint: (TW, Cap, Aut...)] **ODp** (Opening cost of depots) **OT** (Other)

FIGURE 11. Summary of the notation: part 4.

same objective. [23] considered that the capacity constraint is a hard constraint and included in the minimisation function transportation time or distance and service cost (Tr: T and Ser: T in field γ). Alba and Dorronsoro [1] relaxed the capacity and autonomy constraint and included in the objective the minimization of the capacity and autonomy overload (Over: Cap and Over: Aut: Ve in the field γ). A comparison of the results of these two articles is difficult. It is necessary to ensure that the final solutions respect the capacity constraints and autonomy constraints before comparing these solutions in terms of objectives and computing time. Thus, the notations highlight that using only the initialism VRP does not disambiguate this problem identification.

Prins [71] studied the Distance Constrained VRP (DVRP), named by some authors VRP (Sect. 2.1). The DVRP varies from TSP only by the existence of autonomy constraints (Aut: Ve in the field β).

Ho *et al.* [35] presented the Multi Depots VRP (MDVRP). In this variant, there are several depots (MDpP in the field β) in which the vehicles are located. Each vehicle must start and end its trip at the same depot. In addition, each customer must be visited exactly once by one of the vehicles.

Najera [60] studied the Capacitated Vehicle Routing Problem (CVRP). In this variant, vehicles have a limited capacity (Cap: Ve in the field β). Each customer must be assigned to one trip and each trip must be done by one vehicle. This vehicle starts and ends its tour in a depot. As VRP, CVRP is one of the variants for which there are contradictory definitions in the literature. For some authors [20,92], CVRP is characterized by the existence of capacity constraints and the absence of autonomy constraint. For others [18], CVRP is characterised by the occurrence of both capacity constraints and autonomy constraint.

Several articles related to (VRP with Time Windows) VRPTW are presented in Figure 12 but note that the resulting notations are all different except for [3, 87]. This shows the importance of the notation and its ability to highlight the characteristics of the problems and to identify without ambiguity the similarity and the difference between variants of VRP. In the case of VRPTW, the constraints

Ref.	Acronym	Notation
79	VRP	VR/NVe/ Cap:Ve, Aut:Ve/ Tr:Cost, Ser:T
80	VRP	VR/ NVe/ Cap:Ve, Aut:Ve/ Tr:Dis, Ser :T, Over:Cap, Over:Aut:Ve.
23	DVRP	VR/ Nve, Tr :T/ Cap:Ve, Aut:Ve/ Tr:T,Ser :T
81	MDVRP	VR/ MDp, Tr:T/ Cap:Ve, MDpP/ Tr:T
88	CVRP	VR/ Ser:Ø ,Tr:Dis,/ Cap:Ve/ Tr:Dis
67	CVRP	VR/ NVe, Ser:Ø, Tr:Dis/ Cap:Ve/ Tr:Dis
	VRPTW	VR/ NVe/ Cap:Ve, Tw:C, Tw:Dp/ Tr:Dis
69	VRPTW	VR/ NVe/Cap:Ve, Tw:C,Tw:Dp/ Tr:Dis
82	VRPTW	VR//Cap:Ve,Tw :C,Tw :Dp/Tr :Dis
	VRPTW	VR//Cap:Ve,Tw :C,Tw :Dp/NVe,Tr :Dis
68	VRPTW	VR/NVe/Cap:Ve,Tw:C,Tw:Dp/NVe,Tr:Dis
70	VRPTW	VR/ NVe/ Cap:Ve, Tw:C,Tw:Dp/ Nve, Tr:Cost, WT, OT
71	VRPTW	VR/ NVe/ Cap:Ve, Tw:C,Tw:Dp/NVe, Tr:Dis, Over:TW, OT
83	VRPTW	VR//Cap:Ve, Tw:C,TW:Dp/ NVe, Tr:Dis , WT
72	m-VRPTW	VR/ NVe/ Cap:Ve, TW:C, TW:Dp, NoSer/ Nve, Tr:Dis, NoSer, Over:TW, OT
84	VRPMVTTW	VR/Het/ Cap:Ve, Tw:C,TW:Dp / Tr:Dis, OT
73	FSMVRPTW	VR/NVe, Het,Tr :T/ Cap:Ve,TW:C / Tr:T, Ser:T, NVe,WT
78	PDPTW	VR/ Dp:W, C:(Li,B)/ Cap:Ve, Tw:C, Tw:Dp, Aut:Ve, Paired, Prec/ Tr:Dis
74	1-PDPTW	VR/ Dp:W,ONet, NVe:1,C:(Li,B), Ser: Ø , Tr:T /Cap:Ve, Tw:C, NoR, Paired, Prec/ Tr:T, WT, Over:TW, Over:Cap.
93	VRPDSPTW	VR/Dp:(Li+B),ONet,C:(Li+B),Ser:OT,Tr:T/Cap:Ve,TW:C,NoSer/Tr:T, Ser:OT
60	PVRP	VR(MP)/NVe, Ser: Ø ,Tr:Dis/ Cap:Ve,F/Tr:Dis
52	PVRP-SC	VR(MP)/ NVe, Tr:T/ Cap:Ve, F, Spa:Min~Max / Ser:T, Tr:T, OT
29	MDPVRP	VR(MP)/MDp,NVe, Ser: Ø ,Tr:OT/ Cap:Ve, F, Comb/ OT
37	SDMTPVRP or PVRPMVTAR	VR(MP)/ NVe, Het, Ser : Ø ,Tr:Dis/ Cap:Ve, Aut:Ve, MT, F, Comb, Ve↔C/ Tr:Dis
87	CARP	RP /Dp:Li, A,C:B, Tr :T / Cap:Ve/ Ser:T, Tr:T
94	OVRP	RP/NVe, Ser: Ø ,Tr:Dis/Cap:Ve,Aut:Ve,NoR/Tr:Dis,NVe
75	LRP-MPPD	RP/Dp:(B+Li),MDp,MPr, Ser: Ø ,Tr:Dis/Cap:Ve,Cap:Dp,Aut:Ve/Tr:Dis,NVe,ODp
95	MC-VRPSD	RP/MCp,MPr,Dd:S, Ser:Ø ,Tr:Dis/Cap:Ve, Aut:Ve,Pr↔Cp/Tr:Dis:S

FIGURE 12. Applications of the notation.

of time windows (Tw in the field β)) signify time intervals during which a service (for example loading or unloading of goods) must (or can) be done. These time windows constraints can represent the arrival time and/or departure time at the customers sites (Tw: C), the arrival time and/or departure time at the depot (opening hours of depot, Tw: Dp), and can represent the working hours of staff (Tw: Wo) [89]. The literature distinguishes between "hard constraints" (hard time window constraints) and "soft constraints" (soft time window constraints). In the first case, a vehicle arriving before the start of the time window must wait until the service is possible. In this case, the waiting time should be taken into account in the objective to minimize (WT in the field γ) [26]. If the vehicle arrives after the end of the time window, the service cannot be made and the corresponding customer will never be satisfied. In the second case, vehicles can serve the customer outside the time window, but penalty must be considered. For example, [63] studied a VRPTW as a multi-objective problem in which the number of vehicles and the total cost in terms of distance must be minimised (NVe and Tr: Dis in the field γ). Authors in [14,45] investigated the vehicle routing problem with hard time windows (WT in the field γ) and [9] addressed soft time windows constraints (Over: Tw in field γ). Lau *et al.* [49] considered the m-VRPTW, a variant with time windows (Tw: C and Tw:Dp in the field β) and a limited number of vehicles (NVe in field α). A feasible solution is a solution that may contain no served customers (NoSer in the field β) and/or delay penalty if the time windows constraints are relaxed (Over: Tw in the field γ). Liu and Shen [46] presented a vehicle routing problem with heterogeneous fleet (Het in the field α) and time windows (Tw: C and Tw: Dp in the field β) (Vehicle Routing Problem with Multiple Vehicle Types and Time Windows or VRPMVTTW). The objective is to minimize the sum of the possession costs of vehicles (OT in the notation) and the total distance travelled (Tr:Dis in field γ). Dell'Amico *et al.* [26] addressed the VRP with heterogeneous vehicles (Het in the field α) and time windows for customers (Tw: C in the field β) (Fleet Size and Mix Vehicle Routing Problem with Time Windows or FSMVRPTW).

The PDPTW and the 1-PDPTW belong to the family of the Vehicle Routing Problems with Pickup and Delivery (VRPPD) [65] when the client requests are dependent (paired pickup and delivery points). The Pickup and Delivery Problem (PDP) focuses on the collection and distribution of goods [4]. Each customer can either need or supply goods. The objective in the PDP is to compute a set of routes such as: 1) each route starts and ends at the depot; 2) all pickups and delivery requests must be satisfied; 3) each request must be satisfied by one vehicle; and 4) the load of the vehicle never exceeds its capacity. In the PDPTW studied in [64], time windows constraints are related to the depot and to the customers (Tw:C and Tw:Dp in the field β). The 1-PDPTW studied in [38], is a mono vehicle PDPTW (NVe:1 in the subfield $\alpha 1$): only one vehicle with finite capacity is available (Cap:Ve in the field β). Gutiérrez-Jarpa *et al.* [32] studied a more general extension of the PDP, named Vehicle Routing Problem with Deliveries, Selective Pickups and Time Windows (VRPDSPTW). In this variant, every customer has a delivery and pickup demand (C:(Li+B) in the field α) and performing pickups at customer yields revenue (this service cost is designed by Ser: OT in the γ field of the notation). All deliveries must be performed, whereas pickups are selective (NoSer in the field β) and partial pickups are not authorized.

The Periodic Vehicle Routing Problem (PVRP) [93] considers a planning horizon of M periods (MP in the field π). Each customer must be visited k times during the planning horizon ($1 \leq$ k $\leq$ M), and the daily demands are fixed. In [30] the authors present a periodic vehicle routing problem with services choices (PVRP-SC). This problem is mainly characterized by the fact that the service is determined by the frequency of visits to clients (Spa:Min $\sim$ Max in the field β).

Mingozzi [54] studied the Multi depots periodic Vehicle Routing problem (MD-PVRP, VR(MP) in the field π and MDp in the field α). The objective is to minimize the total cost, but the paper does not clearly explains what this cost includes, which explains the use of "OT" in the notation.

Alonso *et al.* [2] considered a periodic vehicle routing problem that includes the possibility for a vehicle to make several trips per day (MT in the field β) as long as the maximum working time of the vehicle is not exceeded (Aut:Ve) while satisfying some constraints of vehicle access to some customers. Indeed, these authors considered the constraints of compatibility between vehicles (mobile resource $\alpha 2$ = Ve) and customers (an element of requests $\alpha 3$ = C) (Ve $\leftrightarrow$ C in the field β). These authors called this problem SDMTPVRP (the Site-Dependent Multi-Trip Periodic Vehicle Routing Problem or Periodic Vehicle Routing Problem with Multiple Vehicle Trips and Accessibility Restrictions).

Note that the same parameter indicating a heterogeneous fleet of vehicles (Het in the subfield $\alpha 2$) appears in the same way in [2,26,46] while it is represented by the letters: 1) "MVT", which means Multiple Vehicle Types in the initialism VRP-MVTTW for (Vehicle Routing Problem with Multiple Vehicle Types and Time Windows) used by [26, 46]; 2) "MV", which means Mix Vehicle in FSMVRPTW initialism for (Fleet Size and Mix Vehicle Routing Problem with Time Windows); and 3) It does not appear in the initialism PVRPMVTAR (for Periodic Vehicle Routing Problem with Multiple Vehicle Trips and Accessibility Restrictions) used by [2], in which "MVT" means Multiple Vehicle Trips.

Lacomme *et al.* [41] treated the Capacitated Arc Routing Problem (CARP). Contrary to the VRP, in which goods must be delivered to client nodes in a network, the CARP consists in visiting a subset of edges (A in the field α). The quantities are associated with the edges and they are either delivered from a single depot to edges as in the case of winter gritting, or picked from the edges to the depot as in the waste collection. The aim is to compute a set of tours of minimum total cost, such as: 1) each tour is performed by one vehicle that starts and ends its tours in the depot; 2) the amount of requests handled by a tour does not exceed the vehicle capacity (Cap:Ve); 3) each edge is treated by a single vehicle without splitting.

Salari *et al.* [80] addressed the Open Vehicle Routing Problem (OVRP), a variant of VRP in which the vehicles are not required to return to the depot (NoR in the field β). M open routes (path) must be computed such as; 1) each path is associated with one vehicle, starting at the depot and ending at one of the customers; 2) each route duration must not exceed the autonomy of the vehicle (Aut:Ve); and 3) the total demand does not exceed the capacity of the vehicle (Cap:Ve). The objective used in this article is the minimization of the total distance (Tr:Dis) and of the fixed costs associated with the vehicles used to serve the customers (NVe in the field γ).

An extended variant of Location-Routing Problem namely LRP with Multi-Product and Pickup and Delivery (LRP- MPPD) is presented in [76, 77]. This variant of the LRP deals with simultaneously selecting one or more facilities from a set of potential hubs (locations), assigning customers to the selected hubs and defining routes of the vehicles for serving multi-product customers demand (MPr in the field α) in such a way that each selected hub must be visited once for delivering, though it can be visited many times for picking up. In the γ field of the notation, ODp indicates the minimisation of opening cost of hubs.

Mendoza *et al.* [52] studied the Multi-Compartment Vehicle Routing Problem with Stochastic Demand (MC-VRPSD), which consists of designing routes to serve the demands for multiple products of a set of customers. Products that are not of the same type must be transported in independent vehicle compartments. The authors considered that the demand of each customer for each product is stochastic. This demand is modelled by a random variable with a known probability distribution. The multi-compartment constraint is presented by Cp:MCp in the subfield $\alpha 2$ of the notation.

5. Comparison of the proposed classification with the classification of [27, 28]

As explained in Section 2.2, Desrochers *et al.* [27] use the following fields: "addresses" for depots and customers sites, "vehicles", "other characteristics", and "objectives". There is no separation between the constraints and the characteristics of the problem. These may appear in the same field. For example in the field "addresses", we can find subfields or parameters that concern the characteristics of the problem such as the number of depots and other subfields or parameters concerning the constraints of the problem such as time window constraints. In addition, some constraints can appear in several fields, such as (tw) that appears in "addresses" field to indicate a time window constraint on demand and in "vehicles" field to indicate a time constraint on the vehicle. In our notation, we proposed another classification schema to better highlight the characteristics of the problem and its constraints. For example "tw constraint" appears only once. We can indicate on which resources and/or on which demands it can be applied (depot, vehicle, customer, ...). The notation of [27] uses some symbols such as $\pm$ to represent a value of parameters which make them difficult to memorize.

Note that our notation covers all aspects (characteristics and constraints) considered in the [27] notation, without using the same level of detail. Figure 13 shows

Parameter or terminal symbol of [45]	**Corresponding parameter of our notation**
Considering several types of time windows: 1) single tw, 2) multiple tw, 3) fixed departure time, 4) different time windows for vehicles.	Time windows (Tw)
Considering if: 1) a given subset of addresses must be visited, 2) at least one address in each subset of a given partition must be visited.	Number of no served customers (NoSer)
Travel time matrix satisfying the triangle inequality or not.	Transportation cost (Tr)
Splitting of demand is allowed priori or posteriori	Split Delivery (SD)
Type of strategy: 1) no backhauling, full loads required, 2) backhauling in case of node routing, 3) full loads, in case of task routing	Type of depot Type of customers Precedence (Prec)

FIGURE 13. "Parameters or terminal symbols" of [27] not included with the same details in the proposed notation.

in column 1 "parameters or terminal symbols" of [27] not included in our notation with the same level of details. In column 2 of Figure 13, the corresponding parameter in the proposed notation which is more general is represented.

Compared to the notation of [27], we have added parameters and/or values for modelling important variants of vehicle routing as multi-commodity vehicle routing, dynamic vehicle routing, multi-periodic vehicle routing, stochastic vehicle routing, and multi-echelon vehicle routing. These new parameters and values are listed below:

- Type of Depot (Dp:(B, Li),Dp: (B+Li), Dp:W).
- Network Level (E).
- Number of Workers (NWo).
- Number of Type of products (Pr).
- Type of Demand (Dd: D: C, Dd: D: Q, Dd: D: Lo).
- Type of Demand (stochastic with more details).
- Service Cost (more details).
- Transportation Cost (more details).
- Capacity (Cap:Dp,Cap:C).
- Autonomy (Aut:Wo).
- Time windows (Tw:Wo, Tw:Dp,Tw:Net).
- Traffic conditions (Tra).
- Split Delivery (SD:NoPr).
- Original depot for vehicles,
- Planning service.
- Initial depot of a vehicle during the planning horizon.
- Compatibility (more details).
- Origin of products.
- Objective (NVe,NoSer, WT,Over,ODp).

Eksioglu *et al.* [28] have proposed a detailed taxonomy (see Sect. 2.2). This taxonomy represents a very large framework to classify and to compare research on VRP using 5 fields and 80 final parameters. It uses a table as representation schema. Each line of table represents an "article title" and each column of the table represent a "final parameter". Cells are marked if a final parameter is considered in the given article. This classification contains two fields to characterize the VRP variant. It concerns "scenario characteristics" with 25 final parameters, and "Problem physical characteristics" with 34 final parameters. By using less parameters, our classification includes all aspects covered by the two fields cited above except "geographical location of customers: urban, or rural, or mixed". Our classification does not include the three other fields of [28] classification ("type of study", "Information characteristics", and "data characteristics"). These fields provide more details to compare research study instead of VRP characteristic variants. Despite the level of details of the classification of [28], it does not take into account significant parameters that we have included in our notation and which are cited below:

- Type of Depot (Dp: (B, Li), Dp: (B+Li), Dp:W).
- Network Level (E).
- Compartment (Cp).
- Number of Workers (NWo).
- Type of Customers (C: (Li,B), C: (Li+B).
- Number of Type of product (Pr).
- Capacity (Cap: Dp, Cap: C).
- Time windows (Tw:Dp, Tw:Net).
- Number of no served customers (NoSer).
- Traffic conditions (Tra).
- Split Delivery (SD: NoPr).
- Multi-trips (MT).
- Vehicle returns to its original depot,
- Planning service.
- Initial depot of a vehicle during the planning horizon.
- Compatibility (more details).
- Origin of products.
- Objective (NVe, NoSer, Over, ODp).

Figure 14 shows that only 6 vehicle routing variants could be classified by the classification of [27, 28].

6. Conclusion

This paper responds to the need of a precise identification system for vehicle routing problems. In fact, the use of initialisms to describe variants of vehicle routing problems has many inconvenients. Redundancies and contradictions have been detected in numerous papers from the literature of the VRP. All proposed classifications for VRP fail to identify all intrinsic characteristics of VRP variants.

Reference	Acronym	[45]	[6]
79	VRP	x	x
80	VRP		
23	DVRP	x	x
81	MDVRP	x	x
88	CVRP	x	x
67	CVRP	x	x
	VRPTW		
69	VRPTW		
82	VRPTW		
	VRPTW		
68	VRPTW		
70	VRPTW		
71	VRPTW		
83	VRPTW		
72	m-VRPTW		
84	VRPMVTTW		
73	FSMVRPTW		
78	PDPTW		
74	1-PDPTW		
93	VRPDSPTW		
60	PVRP		
52	PVRP-SC		
29	MDPVRP		
37	SDMTPVRP or PVRPMVTAR		
87	CARP	x	x
94	OVRP		
75	LRP-MPPD		
95	MC-VRPSD		

FIGURE 14. Comparison table in which Vehicle routing variants of Figure 12 that could be classified by [27, 28] are marked.

They all fail in identifying differences and similarities between the VRP problems studied in the literature.A new notation and classification schema has been proposed in this article to identify vehicle routing problems without ambiguity. It describes the problems addressed through their assumptions, constraints and objectives rather than abbreviations. It is based on a $\pi/\ \alpha/\ \beta/\ \gamma$ pattern. The π field characterizes the type of problem, α describes the structure of the system resources and the demands, β and γ describe the constraints and the criteria to optimize (objectives). This common formalism can be used by anyone, especially young researchers who wish to position their problem among other variants. The proposed notation is intended to facilitate communication among researchers, to permit a better comparison between solving approaches, and to allow identifying the research directions that have not yet been explored in the literature. The notation proposed in this paper is based on a level of detail quite similar to the one used in [27] and [28] while using more standard mnemonics and schemas. We have chosen the most significant parameters and we defined systematic rules and operators to represent the combination of parameters involved in various complex problems. We give more precision by the way to use and to combine the parameters according to these systematic rules using operators. This facilitates its evolution to cover other aspects of routing problems as inventory routing problems, not yet tested by the current notation. For now, this notation has been applied on 26 papers chosen for their disparity. It will be interesting to use it to classify some thematic literature for vehicle routing problem and to test its capacity to identify the relationships between characteristics of problems identified in the notation and the solving approaches, like [72] for scheduling problems. Finally, using systematic rules and combinations permitted to develop a software tool (`http://comordo.fr/classeur/accueil.php`) to facilitate the use of the notation and to increase its chances to be helpful for the whole community interested in vehicle routing.

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Appendix A. Symbols used in the notation sorted by alphabetic order

2E: 2-echelon network
Cp: compartment
F: frequency of service
H: planning horizon
Het: heterogeneous fleet of vehicles
Li: linehaul
Lo: location
MCp: multi-compartments
MDp: multi-depots
MDpP: vehicle starts from different depots during the planning horizon
Mix: mixed localisation of demands (arcs + nodes).
MixNet: mixed network
MP: multi-periods
MPr: multi-products
MT: multi-trips
MultiE: number of echelons of the network >2
Net: network
NoR: return to original depot is not obligatory

NoSer: number of not served customers
NWo: number of workers
NVe: 1: number of vehicles fixed to 1
NVe: number of fixed vehicles >1
ODp: opening cost of depot
ONet: oriented network
OT: other
Over: β**:** violation of a constraint
Over: violation
Over: Aut: violation of autonomy constraint
Over: Aut: Wo: violation of workers time constraint
Over: Cap: violation of capacity constraint
Over: Tw: violation of time windows constraint
P: period
Paired: paired demands
Pr: product
Prec: precedence
Q: quantity
S: stochastic
SD: split delivery allowed per demand and per product
SD: NoPr: splitting allowed by demand only
Ser: service cost
Ser: OT: other type of service cost
Ser: T: service time
Spa: Min $\sim$ **Max:** minimum (∞ if unlimited) and maximum (∞ if unlimited) spacing time between two successive services for the same Customer
Tr: transport costs
Tr: cost: total distance or transport time (not specified)
Tr: Dis: total distance
Tr: T: transport time
Tra: traffic conditions
Tw: time windows
Tw: C: time windows for customers
Tw: Dp: time windows for the opening hours of the depot
Tw: Net: time windows for some parts of the network
Tw: Ve: time windows for vehicle availability
Tw: Wo: time windows for the working hours of workers
Unpaired: independent demands
Ve: vehicles.
W: warehouse
Wo: workers
WT: waiting time in customers sites

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