RAIRO-Oper. Res. 48 (2014) 615–639
DOI: 10.1051/ro/2014021

RAIRO Operations Research
www.rairo-ro.org

AN SMDP MODEL FOR A MULTICLASS MULTI-SERVER QUEUEING CONTROL PROBLEM CONSIDERING CONVERSION TIMES

Zhicong Zhang[1], Na Li[2], Shuai Li[1], Xiaohui Yan[1] and Jianwen Guo[3]

Abstract. We address a queueing control problem considering service times and conversion times following normal distributions. We formulate the multi-server queueing control problem by constructing a semi-Markov decision process (SMDP) model. The mechanism of state transitions is developed through mathematical derivation of the transition probabilities and transition times. We also study the property of the queueing control system and show that optimizing the objective function of the addressed queueing control problem is equivalent to maximizing the time-average reward.

Keywords. Queueing control, semi-Markov decision process, reward.

Mathematics Subject Classification. 60K25, 68M20.

1. Introduction

Multi-server queueing problems assuming that customers arrive with Poission process are widely applied to manufacturing and service industries. These problems have received remarkable attention from researchers. In literature, research

Received September 29, 2013. Accepted March 27, 2014.

[1] Department of Industrial Engineering, School of Mechanical Engineering, Dongguan University of Technology, Songshan Lake District, Dongguan 523808, Guangdong Province, China. stephen1998@gmail.com; yanxh@dgut.edu.cn; lishuai@dgut.edu.cn

[2] Department of Industrial Engineering, Shanghai Jiao Tong University, Shanghai 200240, P.R. China. na-li03@sjtu.edu.cn

[3] School of Mechanical Engineering, Dongguan University of Technology, Songshan Lake District, Dongguan 523808, Guangdong Province, P.R. China. guojw@dgut.edu.cn

Article published by EDP Sciences

on multi-server queueing problems is mainly categorized into two types. Some research focuses on analyzing the performance measures of the queueing systems in stationary state [7, 9, 18] contending for resources under a given policy, *e.g.* to analyze the performance measures such as the utilization of the servers, the workload balance of the servers, the queue length, the waiting time of the customers or the flow time of the customers under FCFS (First Come First Served) dispatching rule. Some research pursues a good policy by comparing several candidate control policies [5]. For example, Borst *et al.* [3] study a system of parallel servers handling users of various classes and compare the server assignment only policy (SA), the coordinated scheduling only policy (CS) and the combined server assignment and scheduling policy (SACS). Andriansyah *et al.* [1] study open zero-buffer multi-server queueing networks in the semi-process and process industries. They evaluate the performance of the queueing networks in terms of throughput using the generalized expansion method (GEM) and compare the results with a simulation method.

Some queueing networks control problems are solved by heuristics such as genetic algorithm [4] or formulated into fluid networks models [13, 16, 17]. However, many researchers study queueing networks with Markovian property. These queueing networks control problems are formulated into Markov chain models [2] or Markov decision process (MDP) models [10, 12, 19] and solved by analytical methods such as matrix analysis [14] or handled by simulation methods. Harchol-Balter *et al.* [8] present the first near-exact analysis of an M/PH/k queue with $m > 2$ preemptive-resume priority classes. They introduce Recursive Dimensionality Reduction (RDR) whose key idea is that the m-dimensionally infinite Markov chain, representing the m class state space, is recursively reduced to a 1-dimensionally infinite Markov chain, that is easily and quickly solved. Xu and Zhang [20] consider a Markovian multi-server queue with a single vacation (e, d)-policy. They formulate the service or manufacturing systems with servers' maintenance performed during their idle time as quasi-birth-and-death (QBD) processes and develop the various stationary performance measures for these systems. They also prove several conditional stochastic decomposition properties. Koçagă and Ward [11] formulate a M/M/N+M queue with abandonment as a Markov decision process (MDP), show that the optimal policy is of threshold form, and provide a simple and efficient iterative algorithm. They solve the approximating diffusion control problem (DCP) and obtain a convenient analytic expression for the infinite horizon expected average cost as a function of the threshold level. Choudhury and Pallabi [6] and Pallabi and Choudhury [15] analyze a single server Markovian queuing system and a multiserver Markovian queuing system with limited waiting space under the assumption that customers may balk as well renege.

In the following, we address a queueing control problem with unrelated parallel servers considering sequence-dependent conversion times so as to minimize the weighted mean flow time. We describe the queueing control problem in Section 2, formulate the problem as a detailed semi-Markov decision process (SMDP) model, analyze the state transition mechanism and develop the transition probabilities in Section 3, prove the property of the queueing control system in Section 4, and present a numerical example in the Section 5. The main contributions of this paper are obtaining analytic expression of the mechanism of state transitions, *i.e.* the explicit formulation of state transition probabilities and the probability distribution of transition times, revealing the property of the multi-server queueing control problem with normal distributed conversion times and service times, and showing the equivalence of the objective of the queueing control problem and the objective of the corresponding SMDP model.

2. Problem statement

In some manufacturing and service industries, *e.g.* machinery industry and semiconductor industry, many production or service control problems consider dynamic jobs arrival and conversion times. Take the example of semiconductor industry, a semiconductor test station usually contains several testers which test many types of semiconductor products. Different types of products arrive at the test station following Poission processes. Any product must be tested on one tester in the station. To test a semiconductor product, a tester, a handler and an enabler are simultaneously in need. A product can be tested with one or more [tester, handler, enabler] combinations, and one [tester, handler, enabler] combination is qualified to test one or more products. Therefore, when a tester finishes testing a product, *e.g.* product P1, and intends to test another product, *e.g.* product P2, a qualified handler and a qualified enabler for product P2 need to be setup on the tester, which takes a specific amount of conversion time. In some circumstance, both the conversion times and the testing times follow normal distributions.

The above problem can be modeled as a multi-server queueing control problem described as follows. There are n classes of customers to be served at a service station containing m unrelated parallel servers. Different classes of customers arrive at the service station following independent Poission processes and the jth class of customers arrive following Poission process with rate $\lambda_j (1 \leqslant j \leqslant n)$. All the arrived customers wait in a queue until they are selected to be served by a server. Each customer needs to be served on one server only and each server is qualified to provide service for the specific classes of customers. The service time of customer $j(1 \leqslant j \leqslant n)$ on server $i(1 \leqslant i \leqslant m)$, denoted $p_{i,j}$, is stochastic and follows normal distribution $\mathrm{N}(\mu^p_{i,j}, (\sigma^p_{i,j})^2)$. The servers are unrelated, that is, $p_{i,j}$ is independent of $p_{k,j}$ for any customer j and all servers $i \neq k$, and $p_{i,j}$ is independent of $p_{i,l}$ for any server i and all customers $j \neq l$. A conversion takes place when a server finishes serving a customer and selects another customer to serve. The conversion times are customer-sequence-dependant and independent of servers. For any server, the conversion time between customer classes j and l $(1 \leqslant j, l \leqslant n)$, denoted $s_{j,l}$, is stochastic and follows normal distribution $\mathrm{N}(\mu^s_{j,l}, (\sigma^s_{j,l})^2)$. Suppose $s_{j1,l1}$ and $s_{j2,l2}$ are independent if $j1 \neq l1$ or $j2 \neq l2$. Trivially, $s_{j,j}$ is zero for any customer j. Moreover, the conversion times and the service times are independent, that is, $p_{i,j}$ is independent of $s_{j,l}$ and $s_{l,j}$ for any server $i(1 \leqslant i \leqslant m)$ and any customers j

and $l(1 \leqslant j, l \leqslant n)$. If a server completes serving a customer, then the customer leaves the service station immediately. The customer classes are classified by their importance weights. Let $J_{j,k}$ denote the kth arriving customer of class j, then the objective function of this queueing control problem is to minimize the expected weighted mean flow time of all customers defined as:

$$\text{Min } E[\bar{f}] = E\left[\frac{1}{\sum_{j=1}^{n} N_j} \sum_{j=1}^{n} \sum_{k=1}^{N_j} w_j \left(c_{j,k} - d_{j,k}\right)\right], \qquad (2.1)$$

where $d_{j,k}$ is the arrival time of customer $J_{j,k}$, $c_{j,k}$ is the completion time of serving customer $J_{j,k}$, w_j is the weight of customer class j, N_j is the number of customers of class j having been served, and $c_{j,k} - d_{j,k}$ is the flow time of customer $J_{j,k}$.

3. The SMDP model

In this section the queueing control problem is formulated into an SMDP model. An SMDP model includes the following key elements: states, policies, transition probabilities, and the reward function. A policy determines the action selected at each state. The transition probabilities indicate the behavior of state transitions. The reward function evaluates the instant impact of an action on the system at a transition from one state to another and the value function specifies the value of a state in the long run which is usually the expected total reward or the expected average reward from this state.

3.1. State representation

A state describes the characteristics of the system including the information of the servers and the customers. It is capable of tracking the variation of the system status. The system state at a decision-making epoch can be represented by a vector of state variables defined as:

$$s = [q_j(1 \leqslant j \leqslant n); B_i(1 \leqslant i \leqslant m); T_i(1 \leqslant i \leqslant m); t_i(1 \leqslant i \leqslant m)], \qquad (3.1)$$

where q_j denotes the number of customers of class $j(1 \leqslant j \leqslant n)$ waiting in the queue, $B_i(1 \leqslant i \leqslant m)$ denotes the customer class that the last customer having been completely served on server $i(1 \leqslant i \leqslant m)$ belongs to, $T_i(1 \leqslant i \leqslant m)$ denotes the customer class that the customer being served on server $i(1 \leqslant i \leqslant m)$ belongs to (T_i equals zero if server i is idle), and $t_i(1 \leqslant i \leqslant m)$ denotes the time starting from the beginning of the latest conversion on server $i(1 \leqslant i \leqslant m)$ (t_i equals zero if $T_i = 0$, *i.e.* server i is idle). There are $3m + n$ components in the state vector.

3.2. Actions

We define the action taken at decision-making state s as a_Ω, where $\Omega = \{(i, j) | \text{server i is selected to serve a customer of class j}, 1 \leqslant i \leqslant m, 1 \leqslant j \leqslant n\}$. If

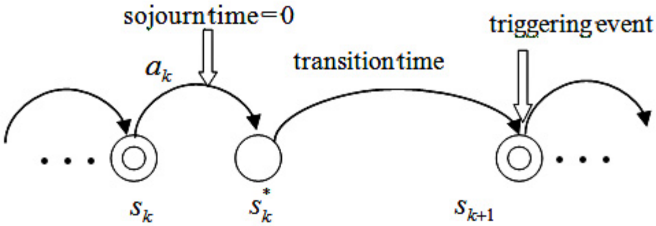

FIGURE 1. Scheme of state transitions.

$T_i \neq 0$ for any server $i(1 \leqslant i \leqslant m)$, then Ω is empty ($\Omega = \varphi$), *i.e.* a server cannot select a customer if it is serving another customer. For any server $i(1 \leqslant i \leqslant m)$, if $T_i = 0$, then $|\{(i,j)|(i,j) \in \Omega,\ 1 \leqslant j \leqslant n\}| \leqslant 1$, *i.e.* a server cannot select two or more customers synchronously, where $|\Delta|$ is the cardinality (size) of the set Δ. Trivially, for any customer $j(1 \leqslant j \leqslant n)$, we have $|\{(i,j)|(i,j) \in \Omega, 1 \leqslant i \leqslant m\}| \leqslant q_j$, *i.e.* the number of customers selected is not more than the number of customers waiting in the queue.

Apparently a natural action is to select no customer. For example, when a server is serving a customer in a state, it is not allowed to select another customer to serve synchronously. Or if no customer is in the queue in a state, all the idle servers have to keep idle in this circumstance. That is, if $\{i|T_i = 0, 1 \leqslant i \leqslant m\} = \varphi$ or $\{j|q_j > 0, 1 \leqslant j \leqslant n\} = \varphi$, then a_Ω selects no customer, *i.e.* $\Omega = \varphi$. In this case, we also use a_φ to denote a_Ω

3.3. The mechanism of state transitions

This section develops the mechanism of state transitions, focusing on the transition probabilities and transition times. The transition probabilities and transition times determine the trace of state transitions. There are two types of system states: the decision-making states and the interim states. Let s_k and s_k^* respectively denote the kth decision-making state and the kth interim state. As shown in Figure 1, when the system arrives at decision-making state s_k, an action, denoted a_k, is selected immediately at state s_k and the system immediately transfers from s_k to an interim state, denoted s_k^*, with probability 1. The system stays at state s_k^* for some time, called the transition time or the sojourn time, and it transfers into the next decision-making state, denoted s_{k+1}, and receives reward r_k when a triggering event for state transition takes place. The sojourn time of a decision-making state is zero.

Specifically, suppose s_k is represented as

$$s_k = [q_{j,k}(1 \leqslant j \leqslant n); B_{i,k}(1 \leqslant i \leqslant m); T_{i,k}(1 \leqslant i \leqslant m); t_{i,k}(1 \leqslant i \leqslant m)]\,. \quad (3.2)$$

Let τ_k denote the time at the kth decision-making state. Let $\Omega_k = \{(i,j)|\text{action}\ a_k\ \text{selects server i to serve a customer of class j},\ 1 \leqslant i \leqslant m,\ 1 \leqslant j \leqslant n\}$.

After taking action a_k, the system immediately transfers form s_k to an interim state s_k^* denoted as

$$s_k^* = \left[q_{j,k}^*(1 \leqslant j \leqslant n); B_{i,k}^*(1 \leqslant i \leqslant m); T_{i,k}^*(1 \leqslant i \leqslant m); t_{i,k}^*(1 \leqslant i \leqslant m)\right] \quad (3.3)$$

where $B_{i,k}^* = B_{i,k}(1 \leqslant i \leqslant m)$, $t_{i,k}^* = t_{i,k}(1 \leqslant i \leqslant m)$, $q_{j,k}^*$ is defined as:

$$q_{j,k}^* = \begin{cases} q_{j,k} - 1 & \text{if } \exists i \in \{1, 2, \ldots, m\}, \text{ s.t.} (i, j) \in \Omega \\ q_{j,k} & \text{otherwise} \end{cases} \quad (1 \leqslant j \leqslant n), \quad (3.4)$$

and $T_{i,k}^*$ is defined as:

$$T_{i,k}^* = \begin{cases} j & \text{if } \exists j \in \{1, 2, \ldots, n\}, \text{ s.t.} (i, j) \in \Omega \\ T_{i,k} & \text{otherwise} \end{cases} \quad (1 \leqslant i \leqslant m). \quad (3.5)$$

There are two kinds of events triggering a transition from an interim state to the next decision-making state: arrival of a new customer and completion of serving a customer on any server. The next decision-making state, denoted s_{k+1}, is represented as

$$s_{k+1} = [q_{j,k+1}(1 \leqslant j \leqslant n); B_{i,k+1}(1 \leqslant i \leqslant m); \\ T_{i,k+1}(1 \leqslant i \leqslant m); t_{i,k+1}(1 \leqslant i \leqslant m)] \quad (3.6)$$

If the triggering event for the transition from s_k^* to s_{k+1} is completion of serving a customer on some server, we have $\{i | T_{i,k}^* = 0, 1 \leqslant i \leqslant m\} \neq \varphi$ In the following we derive the analytical expression of the transition probabilities and the transition times respectively when the two kinds of events take place.

3.3.1. The triggering event of state transitions: arrival of a customer

If the triggering event of the state transition from s_k^* to s_{k+1} is arrival of a new customer (say, belonging to customer class J), we use s_{k+1}^J to denote the next decision-making state s_{k+1}. Let X_J denote the transition time from s_k^* to s_{k+1}^J, then

$$q_{j,k+1} = \begin{cases} q_{j,k}^* + 1 & \text{if } j = J \\ q_{j,k}^* & \text{otherwise} \end{cases} \quad (1 \leqslant j \leqslant n), \quad (3.7)$$

$$t_{i,k+1} = \begin{cases} t_{i,k}^* + X_J & \text{if } T_{i,k}^* > 0 \\ t_{i,k}^* & \text{if } T_{i,k}^* = 0 \end{cases} \quad (1 \leqslant i \leqslant m), \quad (3.8)$$

and for all $1 \leqslant i \leqslant m$, we have $B_{i,k+1} = B_{i,k}^*$ and $T_{i,k+1} = T_{i,k}^*$.

Let $F_J(u)$ denote the probability of that the triggering event of the state transition from s_k^* to s_{k+1}^J under action a_k is arrival of a customer of class J and the transition time from s_k^* to s_{k+1}^J is not longer than u, then

$$F_J(u) = P\{X_J \leqslant u, \text{the trigger event of the state transition from } s_k^* \text{ to } s_{k+1}^J$$

$$\text{is arrival of a customer of class } J\} = P\{0 \leqslant X_J \leqslant u, X_J < X_j(\forall j \neq J), s_{B_i^*,T_i^*}$$

$$+ p_{i,T_i^*} - t_i^* > X_J(\forall T_i^* \neq 0) | s_{B_i^*,T_i^*} + p_{i,T_i^*} - t_i^* > 0\,(\forall T_i^* \neq 0)\}$$

$$= \frac{P\{0 \leqslant X_J \leqslant u, X_J < X_j(\forall j \neq J), s_{B_i^*,T_i^*} + p_{i,T_i^*} - t_i^* > X_J(\forall T_i^* \neq 0), s_{B_i^*,T_i^*}}{P\{s_{B_i^*,T_i^*} + p_{i,T_i^*} - t_i^* > 0\,(\forall T_i^* \neq 0)\}},$$

$$+ \frac{p_{i,T_i^*} - t_i^* > 0\,(\forall T_i^* \neq 0)\}}{P\{s_{B_i^*,T_i^*} + p_{i,T_i^*} - t_i^* > 0\,(\forall T_i^* \neq 0)\}} \tag{3.9}$$

where $X_j (1 \leqslant j \leqslant n)$ denotes the length of the time interval from τ_k to the time when the next customer of class j arrives. Since the customers of class J arrive following Poission process with rate λ_J, X_J is a continuous random variable having exponential distribution with parameter λ_J. Hence,

$$F_J(u) =$$
$$\frac{\int_0^u P\{X_J < X_j(\forall j \neq J), s_{B_i^*,T_i^*} + p_{i,T_i^*} - t_i^* > X_J(\forall T_i^* \neq 0) | X_J = x\}\lambda_J \mathrm{e}^{-\lambda_J x} \mathrm{d}x}{P\{s_{B_i^*,T_i^*} + p_{i,T_i^*} - t_i^* > 0\,(\forall T_i^* \neq 0)\}}$$
$$= \frac{\int_0^u P\{X_j > x\,(\forall j \neq J), s_{B_i^*,T_i^*} + p_{i,T_i^*} - t_i^* > x(\forall T_i^* \neq 0)\}\lambda_J \mathrm{e}^{-\lambda_J x} \mathrm{d}x}{P\{s_{B_i^*,T_i^*} + p_{i,T_i^*} - t_i^* > 0\,(\forall T_i^* \neq 0)\}}. \tag{3.10}$$

Since customers of class j $(1 \leqslant j \leqslant n)$ arrive following Poission process with rate λ_j, we have

$$\begin{aligned} & P\{X_j > x\,(\forall j \neq J), s_{B_i^*,T_i^*} + p_{i,T_i^*} - t_i^* > x\,(\forall T_i^* \neq 0)\} \\ =& P\{X_j > x\,(\forall j \neq J)\} P\{s_{B_i^*,T_i^*} + p_{i,T_i^*} - t_i^* > x\,(\forall T_i^* \neq 0)\} \\ =& P\{s_{B_i^*,T_i^*} + p_{i,T_i^*} - t_i^* > x\,(\forall T_i^* \neq 0)\} \prod_{1 \leqslant j \leqslant n, j \neq J} P\{X_j > x\} \\ =& P\{s_{B_i^*,T_i^*} + p_{i,T_i^*} - t_i^* > x\,(\forall T_i^* \neq 0)\} \prod_{1 \leqslant j \leqslant n, j \neq J} \mathrm{e}^{-\lambda_j x}. \end{aligned} \tag{3.11}$$

Because the conversion times and the service times on all servers are independent random variables, $s_{B_i^*,T_i^*} + p_{i,T_i^*}$ and $s_{B_k^*,T_k^*} + p_{k,T_k^*} (T_i^* \neq 0, T_k^* \neq 0, 1 \leqslant i \neq k \leqslant m)$ are independent random variables. Hence,

$$P\left\{s_{B_i^*,T_i^*} + p_{i,T_i^*} - t_i^* > x\,(\forall T_i^* \neq 0)\right\} = \prod_{1 \leqslant i \leqslant m, T_i^* \neq 0} P\left\{s_{B_i^*,T_i^*} + p_{i,T_i^*} > t_i^* + x\right\}, \tag{3.12}$$

$$P\left\{s_{B_i^*,T_i^*}+p_{i,T_i^*}-t_i^*>x\,(\forall T_i^*\neq 0)\right\}=\prod_{1\leqslant i\leqslant m,T_i^*\neq 0}P\left\{s_{B_i^*,T_i^*}+p_{i,T_i^*}>t_i^*\right\}. \tag{3.13}$$

Since $s_{B_i^*,T_i^*}$ follows normal distribution $\mathrm{N}(\mu^s_{B_i^*,T_i^*},(\sigma^s_{B_i^*,T_i^*})^2)$ and p_{i,T_i^*} follows normal distribution $\mathrm{N}(\mu^p_{i,T_i^*},(\sigma^p_{i,T_i^*})^2)$,$s_{B_i^*,T_i^*}+p_{i,T_i^*}$ follows normal distribution $\mathrm{N}(\mu^s_{B_i^*,T_i^*}+\mu^p_{i,T_i^*},(\sigma^s_{B_i^*,T_i^*})^2+(\sigma^p_{i,T_i^*})^2)$ and thus

$$P\left\{s_{B_i^*,T_i^*}+p_{i,T_i^*}>t_i^*+x\right\}=\int_{t_i^*+x}^{+\infty}\frac{1}{\sqrt{2\pi\left[(\sigma^s_{B_i^*,T_i^*})^2+\left(\sigma^p_{i,T_i^*}\right)^2\right]}}\times\exp\left\{-\frac{(y-\mu^s_{B_i^*,T_i^*}-\mu^p_{i,T_i^*})^2}{2\left[\left(\sigma^s_{B_i^*,T_i^*}\right)^2+\left(\sigma^p_{i,T_i^*}\right)^2\right]}\right\}\mathrm{d}y, \tag{3.14}$$

$$P\left\{s_{B_i^*,T_i^*}+p_{i,T_i^*}>t_i^*\right\}=\int_{t_i^*}^{+\infty}\frac{1}{\sqrt{2\pi\left[\left(\sigma^s_{B_i^*,T_i^*}\right)^2+\left(\sigma^p_{i,T_i^*}\right)^2\right]}}\times\exp\left\{-\frac{(y-\mu^s_{B_i^*,T_i^*}-\mu^p_{i,T_i^*})^2}{2\left[\left(\sigma^s_{B_i^*,T_i^*}\right)^2+\left(\sigma^p_{i,T_i^*}\right)^2\right]}\right\}\mathrm{d}y. \tag{3.15}$$

By equations (3.11)–(3.15), equation (3.10) yields

$$F_J(u)=\frac{\lambda_J\int_0^u\exp\left(-\sum_{j=1}^n\lambda_jx\right)\prod_{1\leqslant i\leqslant m,T_i^*\neq 0}\int_{t_i^*+x}^{+\infty}\frac{1}{\sqrt{2\pi\left[\left(\sigma^s_{B_i^*,T_i^*}\right)^2+\left(\sigma^p_{i,T_i^*}\right)^2\right]}}}{\prod_{1\leqslant i\leqslant m,T_i^*\neq 0}\int_{t_i^*}^{+\infty}\frac{1}{\sqrt{2\pi\left[\left(\sigma^s_{B_i^*,T_i^*}\right)^2+\left(\sigma^p_{i,T_i^*}\right)^2\right]}}\exp\left\{-\frac{\left(y-\mu^s_{B_i^*,T_i^*}-\mu^p_{i,T_i^*}\right)^2}{2\left[\left(\sigma^s_{B_i^*,T_i^*}\right)^2+\left(\sigma^p_{i,T_i^*}\right)^2\right]}\right\}\mathrm{d}y}.$$

$$\times\ \exp\left\{-\frac{\left(y-\mu^s_{B_i^*,T_i^*}-\mu^p_{i,T_i^*}\right)^2}{2\left[\left(\sigma^s_{B_i^*,T_i^*}\right)^2+\left(\sigma^p_{i,T_i^*}\right)^2\right]}\right\}\mathrm{d}y\mathrm{d}x \tag{3.16}$$

Let $P_F(s_k^*,a_k,s_{k+1}^J)$ denote the transition probability from s_k^* to s_{k+1}^J by taking action a_k, then

$$P_F(s_k^*,a_k,s_{k+1}^J)=F_J(+\infty). \tag{3.17}$$

3.3.2. The trigger event of state transitions: completion of serving a customer

If the triggering event of the next state transition is completion of serving a customer on a server (say, server I), we use s_{k+1}^I to denote the next decision-making state s_{k+1}. Let Y_I denote the transition time from s_k^* to s_{k+1}^I, then

$$B_{i,k+1} = \begin{cases} T_{i,k}^* & \text{if } i = \text{I} \\ B_{i,k}^* & \text{if } i \neq \text{I} \end{cases} \quad (1 \leqslant i \leqslant m), \tag{3.18}$$

$$T_{i,k+1} = \begin{cases} 0 & \text{if } i = \text{I} \\ T_{i,k}^* & \text{if } i \neq \text{I} \end{cases} \quad (1 \leqslant i \leqslant m), \tag{3.19}$$

$$t_{i,k+1} = \begin{cases} t_{i,k}^* + Y_I & \text{if } T_{i,k}^* > 0 \quad \text{and} \quad i \neq \text{I} \\ 0 & \text{otherwise} \end{cases} \quad (1 \leqslant i \leqslant m), \tag{3.20}$$

and for any customer class $j(1 \leqslant j \leqslant n)$, we have $q_{j,k+1} = q_{j,k}^*$.

Let $G_I(u)$ denote the probability of that the triggering event of the state transition from s_k^* to s_{k+1}^I under action a_k is completion of serving a customer on server I and the transition time from s_k^* to s_{k+1}^I is not longer than u, then

$$G_I(u) = P\{Y_I \leqslant u, \text{the trigger event of the next state transition is completion of serving a customer on machine I}\}. \tag{3.21}$$

Since $Y_I = s_{B_I^*,T_I^*} + p_{I,T_I^*} - t_I^*$, equation (3.21) yields

$$\begin{aligned} G_I(u) &= P\{s_{B_I^*,T_I^*} + p_{I,T_I^*} - t_I^* \leqslant u, X_j > s_{B_I^*,T_I^*} + p_{I,T_I^*} - t_I^*(1 \leqslant j \leqslant n), s_{B_i^*,T_i^*} \\ &\quad + p_{i,T_i^*} - t_i^* > s_{B_I^*,T_I^*} + p_{I,T_I^*} - t_I^*\,(\forall T_i^* \neq 0, i \neq I) | s_{B_i^*,T_i^*} \\ &\quad + p_{i,T_i^*} - t_i^* > 0\,(\forall T_i^* \neq 0)\} \\ &= \frac{P\{0 < s_{B_I^*,T_I^*} + p_{I,T_I^*} - t_I^* \leqslant u, X_j > s_{B_I^*,T_I^*} + p_{I,T_I^*} - t_I^*(1 \leqslant j \leqslant n), s_{B_i^*,T_i^*}}{P\{s_{B_i^*,T_i^*} + p_{i,T_i^*} - t_i^* > 0\,(\forall T_i^* \neq 0)\}} \\ &\quad + \frac{p_{i,T_i^*} - t_i^* > s_{B_I^*,T_I^*} + p_{I,T_I^*} - t_I^*(\forall T_i^* \neq 0, i \neq I)\}}{P\{s_{B_i^*,T_i^*} + p_{i,T_i^*} - t_i^* > 0\,(\forall T_i^* \neq 0)\}}. \end{aligned} \tag{3.22}$$

Let $H(x)$ denote the probability distribution function of $s_{B_I^*,T_I^*}+p_{I,T_I^*}$, *i.e.* $H(x) = P\{s_{B_I^*,T_I^*}+p_{I,T_I^*} \leqslant x\}$, then equation (3.22) yields

$$G_I(u) = \frac{\int_{t_I^*}^{t_I^*+u} P\{X_j > s_{B_I^*,T_I^*}+p_{I,T_I^*}-t_I^*(1\leqslant j\leqslant n), s_{B_i^*,T_i^*}+p_{i,T_i^*}-t_i^* > s_{B_I^*,T_I^*}}{P\{s_{B_i^*,T_i^*}+p_{i,T_i^*}-t_i^*>0\,(\forall T_i^*\neq 0)\}}$$
$$+\frac{p_{I,T_I^*}-t_I^*(\forall T_i^*\neq 0, i\neq I)|s_{B_I^*,T_I^*}+p_{I,T_I^*}=x\}\mathrm{d}H(x)}{P\{s_{B_i^*,T_i^*}+p_{i,T_i^*}-t_i^*>0\,(\forall T_i^*\neq 0)\}}$$
$$=\frac{\int_{t_I^*}^{t_I^*+u} P\{X_j > x-t_I^*(1\leqslant j\leqslant n), s_{B_i^*,T_i^*}+p_{i,T_i^*} > x}{P\{s_{B_i^*,T_i^*}+p_{i,T_i^*}>t_i^*(\forall T_i^*\neq 0)\}}$$
$$+\frac{t_i^*-t_I^*(\forall T_i^*\neq 0, i\neq I)\}\mathrm{d}H(x)}{P\{s_{B_i^*,T_i^*}+p_{i,T_i^*}>t_i^*(\forall T_i^*\neq 0)\}}. \tag{3.23}$$

Since the customer of class $j(1\leqslant j\leqslant n)$ arrive following independent Poission process with rate λ_j, it follows that

$$\begin{aligned}
&P\left\{X_j > x-t_I^*(1\leqslant j\leqslant n), s_{B_i^*,T_i^*}+p_{i,T_i^*} > x+t_i^*-t_I^*(\forall T_i^*\neq 0, i\neq I)\right\}\\
&= P\left\{X_j > x-t_I^*(1\leqslant j\leqslant n)\right\}P\{s_{B_i^*,T_i^*}+p_{i,T_i^*} > x+t_i^*-t_I^*(\forall T_i^*\neq 0, i\neq I)\}\\
&= P\{s_{B_i^*,T_i^*}+p_{i,T_i^*} > x+t_i^*-t_I^*(\forall T_i^*\neq 0, i\neq I)\}\prod_{1\leqslant j\leqslant n} P\{X_j > x-t_I^*\}\\
&= P\{s_{B_i^*,T_i^*}+p_{i,T_i^*} > x+t_i^*-t_I^*(\forall T_i^*\neq 0, i\neq I)\}\prod_{1\leqslant j\leqslant n} \mathrm{e}^{-\lambda_j(x-t_I^*)}\\
&= P\{s_{B_i^*,T_i^*}+p_{i,T_i^*} > x+t_i^*-t_I^*(\forall T_i^*\neq 0, i\neq I)\}\exp\left[(t_I^*-x)\sum_{j=1}^{n}\lambda_j\right].
\end{aligned} \tag{3.24}$$

Similar to the derivation of equations (3.12) and (3.13), we have

$$P\{s_{B_i^*,T_i^*}+p_{i,T_i^*} > x+t_i^*-t_I^*(\forall T_i^*\neq 0, i\neq I)\}$$
$$= \prod_{1\leqslant i\leqslant m, T_i^*\neq 0, i\neq I} P\{s_{B_i^*,T_i^*}+p_{i,T_i^*} > x+t_i^*-t_I^*\}, \tag{3.25}$$

$$P\{s_{B_i^*,T_i^*}+p_{i,T_i^*} > t_i^*(\forall T_i^*\neq 0)\} = \prod_{1\leqslant i\leqslant m, T_i^*\neq 0} P\left\{s_{B_i^*,T_i^*}+p_{i,T_i^*} > t_i^*\right\}. \tag{3.26}$$

By equations (3.24)–(3.26), equation (3.23) yields

$$G_I(u) =$$
$$\frac{\int_{t_I^*}^{t_I^*+u} \exp\left[(t_I^*-x)\sum_{j=1}^{n}\lambda_j\right]\prod_{1\leqslant i\leqslant m, T_i^*\neq 0, i\neq I} P\{s_{B_i^*,T_i^*}+p_{i,T_i^*} > x+t_i^*-t_I^*\}\mathrm{d}H(x)}{\prod_{1\leqslant i\leqslant m, T_i^*\neq 0} P\left\{s_{B_i^*,T_i^*}+p_{i,T_i^*} > t_i^*\right\}}. \tag{3.27}$$

Similar to equation (3.14), we have

$$P\left\{s_{B_i^*,T_i^*}+p_{i,T_i^*}>x+t_i^*-t_I^*\right\}=\int_{x+t_i^*-t_I^*}^{+\infty}\frac{1}{\sqrt{2\pi\left[\left(\sigma^s_{B_i^*,T_i^*}\right)^2+\left(\sigma^p_{i,T_i^*}\right)^2\right]}}\times\exp\left\{-\frac{\left(y-\mu^s_{B_i^*,T_i^*}-\mu^p_{i,T_i^*}\right)^2}{2\left[\left(\sigma^s_{B_i^*,T_i^*}\right)^2+\left(\sigma^p_{i,T_i^*}\right)^2\right]}\right\}\mathrm{d}y, \tag{3.28}$$

$$P\left\{s_{B_i^*,T_i^*}+p_{i,T_i^*}>t_i^*\right\}=\int_{t_i}^{+\infty}\frac{1}{\sqrt{2\pi\left[\left(\sigma^s_{B_i^*,T_i^*}\right)^2+\left(\sigma^p_{i,T_i^*}\right)^2\right]}}\times\exp\left\{-\frac{(y-\mu^s_{B_i^*,T_i^*}-\mu^p_{i,T_i^*})^2}{2\left[\left(\sigma^s_{B_i^*,T_i^*}\right)^2+\left(\sigma^p_{i,T_i^*}\right)^2\right]}\right\}\mathrm{d}y. \tag{3.29}$$

By the definition of $H(x)$, we have

$$\mathrm{d}H(x)=\frac{1}{\sqrt{2\pi\left[\left(\sigma^s_{B_I^*,T_I^*}\right)^2+\left(\sigma^p_{I,T_I^*}\right)^2\right]}}\exp\left\{-\frac{\left(x-\mu^s_{B_I^*,T_I^*}-\mu^p_{I,T_I^*}\right)^2}{2\left[\left(\sigma^s_{B_I^*,T_I^*}\right)^2+\left(\sigma^p_{I,T_I^*}\right)^2\right]}\right\}\mathrm{d}x. \tag{3.30}$$

Hence, it follows from equations (3.27)–(3.29) that

$$G_I(u)=\frac{\int_{t_I^*}^{t_I^*+u}\exp\left[\left(t_I^*-x\right)\sum_{j=1}^{n}\lambda_j\right]\prod_{1\leqslant i\leqslant m,T_i^*\neq 0,i\neq I}\int_{x+t_i^*-t_I^*}^{+\infty}\frac{1}{\sqrt{2\pi\left[\left(\sigma^s_{B_i^*,T_i^*}\right)^2+\left(\sigma^p_{i,T_i^*}\right)^2\right]}}}{\prod_{1\leqslant i\leqslant m,T_i^*\neq 0}\int_{t_i^*}^{+\infty}\frac{1}{\sqrt{2\pi\left[\left(\sigma^s_{B_i^*,T_i^*}\right)^2+\left(\sigma^p_{i,T_i^*}\right)^2\right]}}\exp\{-\frac{\left(y-\mu^s_{B_i^*,T_i^*}-\mu^p_{i,T_i^*}\right)^2}{2\left[\left(\sigma^s_{B_i^*,T_i^*}\right)^2+\left(\sigma^p_{i,T_i^*}\right)^2\right]}\}\mathrm{d}y},$$

$$\times\exp\left\{-\frac{\left(y-\mu^s_{B_i^*,T_i^*}-\mu^p_{i,T_i^*}\right)^2}{2\left[\left(\sigma^s_{B_i^*,T_i^*}\right)^2+\left(\sigma^p_{i,T_i^*}\right)^2\right]}\right\}\mathrm{d}y\mathrm{d}H(x) \tag{3.31}$$

where $\mathrm{d}H(x)$ is shown as equation (3.30).

Let $P_G(s_k^*, a_k, s_{k+1}^I)$ denote the transition probability from s_k^* to s_{k+1}^I under action a_k, then

$$P_G(s_k^*, a_k, s_{k+1}^I) = G_I(+\infty). \tag{3.32}$$

3.3.3. The probability distribution of transition times

Assume that random variable $V(V = \tau_{k+1} - \tau_k)$ denote the transition time from state s_k to state s_{k+1}, and $F(x)$ is the probability distribution function of V, *i.e.* $F(x) = P\{V \leqslant x\}$. Thus,

$$\begin{aligned} F(x) = \sum_{j=1}^{n} P\{&0 \leqslant X_j \leqslant x, \text{the trigger event of the next state transition is arrival} \\ &\text{of a customer belonging to class } j\} + \sum_{1\leqslant i\leqslant m, T_i^*\neq 0} P\{0 \leqslant Y_i \leqslant x, \text{the trigger event} \\ &\text{of the next state transition is completion of serving a customer on server } i\} \\ = \sum_{J=1}^{n} F_J(x) &+ \sum_{1\leqslant I\leqslant m, T_I^*\neq 0} G_I(x). \end{aligned} \tag{3.33}$$

3.4. The reward function

A reasonable reward function indicates the instant impact of an action on the queueing system, that is, to link an action with immediate reward. Moreover, the accumulated reward or the average reward indicates the performance of a control policy. We define the reward function as follows and prove the property of the reward function in the fourth section.

Definition 3.1. Let r_k(k=1, 2, ...) denote the reward received at the transition from state s_k to s_{k+1} at time τ_{k+1}. r_k is defined as

$$r_k = -(\tau_{k+1} - \tau_k)\sum\nolimits_{j=1}^{n} w_j q_{j,k}^*, \tag{3.34}$$

where τ_k is the time at the kth decision-making state, w_j is the weight of customer class j, and $q_{j,k}^*$ is the number of waiting customers of class j at the kth interim state which is a component of vector s_k^* as equation (3.3).

From the definition of the reward function, r_k is computed due to s_k^* and the transition time from s_k^* to s_{k+1}. Because s_k^* is determined by s_k and a_k, r_k is also determined by s_k and a_k. Let S denote the state space. It is easy to shown that for any $s \in S$ we have

$$\begin{aligned} P\{s_{k+1} = s, \tau_{k+1} - \tau_k \leqslant t, r_k \leqslant r | s_0, \tau_0, a_0; s_1, \tau_1, a_1; \ldots; s_k, \tau_k, a_k\} \\ = P\{s_{k+1} = s, \tau_{k+1} - \tau_k \leqslant t, r_k \leqslant r | s_k, \tau_k, a_k\}, \end{aligned} \tag{3.35}$$

where $\tau_{k+1} - \tau_k$ is the transition time from s_k to s_{k+1}. That is, the decision process associated with (s, τ, a) is a semi-Markov decision process.

4. The property of the queuing control system

In Section 3 the queuing problem is formulated as an SMDP problem. In the following we investigate the property of the queuing control system and also show the relationship of the reward function and the objective of the discussed queueing control problem.

Lemma 4.1. *Let N_T^j denote the number of customers of class j arriving by time T, N_T denote the number of customers of all classes arriving by time T. Then*

(1)

$$\lim_{T\to+\infty} P\{N_T^j = k\} = 0 \quad \forall 1 \leqslant j \leqslant n, \mathrm{k} \in \mathrm{Z}^+ \cup \{0\} \tag{4.1}$$

(2)

$$\lim_{T\to+\infty} P\{N_T = k\} = 0 \quad \forall \mathrm{k} \in \mathrm{Z}^+ \cup \{0\} \tag{4.2}$$

Proof.

(1) Since the customers of class j arrive following Poission process with rate λ_j, it follows that

$$P\{N_T^j = k\} = \frac{1}{k!}\mathrm{e}^{-\lambda_j T}(\lambda_j T)^k. \tag{4.3}$$

Hence, for any $1 \leqslant j \leqslant n, k \in Z^+ \cup \{0\}$,

$$\lim_{T\to+\infty} P\{N_T^j = k\} = \lim_{T\to+\infty} \frac{1}{k!}\mathrm{e}^{-\lambda_j T}(\lambda_j T)^k = \frac{1}{k!}\lim_{T\to+\infty} \mathrm{e}^{-\lambda_j T}(\lambda_j T)^k = 0 \tag{4.4}$$

(2) Since the customers of class j arrive following Poission process with rate λ_j, N_T^j follows Poission distribution with parameter $\lambda_j T (1 \leqslant j \leqslant n)$ and thus N_T follows Poission distribution with parameter $\sum_{j=1}^n \lambda_j T$. Similarly to the proof of part (2.1) we obtain equation (4.2).

Lemma 4.2. *Suppose $C > 0$, $D > 0$, q is a positive integer ($q \geqslant 1$), K is an non-negative integer ($0 \leqslant K < q$) and j is an integer ($1 \leqslant j \leqslant n$). Then*

(1)

$$\lim_{T\to+\infty} E\left[\frac{C}{N_T^j + D}\right] = 0 \qquad \forall 1 \leqslant j \leqslant n \tag{4.5}$$

(2)

$$\lim_{T\to+\infty} E\left[\frac{C}{N_T + D}\right] = 0 \tag{4.6}$$

(3)

$$\lim_{T\to+\infty} \sum_{k=q}^{+\infty} \frac{C}{k-K} P\{N_T^j = k\} = 0 \tag{4.7}$$

(4)

$$\lim_{T\to+\infty}\sum_{k=q}^{+\infty}\frac{C}{k-K}P\{N_T=k\}=0 \tag{4.8}$$

Proof.
(1) By the definition of mathematical expectation, we have

$$\begin{aligned}E\left[\frac{C}{N_T^j+D}\right]&=\sum_{k=0}^{+\infty}\frac{C}{k+D}P\{N_T^j=k\}\\&=\sum_{k=0}^{+\infty}\frac{C}{k+D}\frac{(\lambda_jT)^k}{k!\mathrm{e}^{\lambda_jT}}.\end{aligned} \tag{4.9}$$

Since $C>0$ and $D>0$, it follows that

$$\frac{C}{k+D}P\{N_T^j=k\}\geqslant 0\quad(\forall k\in Z^+\cup\{0\}). \tag{4.10}$$

Hence,

$$E\left[\frac{C}{N_T^j+D}\right]\geqslant 0\quad(\forall T>0). \tag{4.11}$$

For arbitrary ε>0, there exists a positive integer W satisfying that $W>\max\{C/\varepsilon-D,0\}$. Hence,

$$\frac{C}{W+D}<\varepsilon. \tag{4.12}$$

By equation (4.9) we obtain

$$\begin{aligned}\left[\frac{C}{N_T^j+D}\right]&=E\sum_{k=0}^{W}\frac{C}{k+D}\frac{(\lambda_jT)^k}{k!\mathrm{e}^{\lambda_jT}}+\sum_{k=W+1}^{+\infty}\frac{C}{k+D}\frac{(\lambda_jT)^k}{k!\mathrm{e}^{\lambda_jT}}\\&<\frac{C}{D}\sum_{k=0}^{W}\frac{(\lambda_jT)^k}{k!\mathrm{e}^{\lambda_jT}}+\frac{C}{K+D}\sum_{k=W+1}^{+\infty}\frac{(\lambda_jT)^k}{k!\mathrm{e}^{\lambda_jT}}.\end{aligned} \tag{4.13}$$

From inequality (4.12), we have

$$\begin{aligned}\frac{C}{K+D}\sum_{k=K+1}^{+\infty}\frac{(\lambda_jT)^k}{k!\mathrm{e}^{\lambda_jT}}&<\varepsilon\sum_{k=K+1}^{+\infty}\frac{(\lambda_jT)^k}{k!\mathrm{e}^{\lambda_jT}}\\&<\frac{\varepsilon}{\mathrm{e}^{\lambda_jT}}\sum_{k=0}^{+\infty}\frac{(\lambda_jT)^k}{k!}\\&=\varepsilon.\end{aligned} \tag{4.14}$$

Since $\lim_{T\to+\infty} \frac{C}{D}\sum_{k=0}^{W} \frac{(\lambda_j T)^k}{k!\mathrm{e}^{\lambda_j T}} = \frac{C}{D}\sum_{k=0}^{W} \lim_{T\to+\infty} \frac{(\lambda_j T)^k}{k!\mathrm{e}^{\lambda_j T}}$ =0, it follows from inequalities (4.13) and (4.14) that as $T \to +\infty$, $E[C/(N_T^j + D)] < \varepsilon$ for arbitrary $\varepsilon >$ 0. According to the definition of limit and the arbitrariness of ε, we arrive at equation (4.5).

(2) Since N_T follows Poission distribution with parameter $\sum_{j=1}^{n} \lambda_j T$, similar to the derivation of equation (4.5) we obtain equation (4.6).

(3) Since $q > K$, there exist a number $M > 1$, *e.g.* $M = (k+1)/(k-K)+1$, such that for arbitrary $k \geqslant q$ we have $1/(k-K) < M/(k+1)$. Then by equation (4.3) we have

$$\begin{aligned}\sum_{k=q}^{+\infty} \frac{C}{k-K} P\{N_T^j = k\} &\leqslant \sum_{k=q}^{+\infty} \frac{MC}{k+1} P\{N_T^j = k\} \\ &= \sum_{k=q}^{+\infty} \frac{MC(\lambda_j T)^k}{(k+1)!\mathrm{e}^{\lambda_j T}}. \end{aligned} \tag{4.15}$$

Obviously,

$$\sum_{k=q}^{+\infty} \frac{C}{k-K} P\{N_T^j = k\} \geqslant 0. \tag{4.16}$$

Hence,

$$0 \leqslant \sum_{k=q}^{+\infty} \frac{C}{k-K} P\{N_T^j = k\} \leqslant MC \sum_{k=q}^{+\infty} \frac{(\lambda_j T)^k}{(k+1)!\mathrm{e}^{\lambda_j T}}. \tag{4.17}$$

Since

$$\begin{aligned}\sum_{k=q}^{+\infty} \frac{(\lambda_j T)^k}{(k+1)!\mathrm{e}^{\lambda_j T}} &= \sum_{k=0}^{+\infty} \frac{(\lambda_j T)^k}{(k+1)!\mathrm{e}^{\lambda_j T}} - \sum_{k=0}^{q-1} \frac{(\lambda_j T)^k}{(k+1)!\mathrm{e}^{\lambda_j T}} \\ &= E\left[\frac{1}{N_T^j + 1}\right] - \sum_{k=0}^{q-1} \frac{(\lambda_j T)^k}{(k+1)!\mathrm{e}^{\lambda_j T}} \end{aligned} \tag{4.18}$$

and from equation (4.5) we have

$$\lim_{T\to+\infty} E\left[\frac{1}{N_T^j + 1}\right] = 0, \tag{4.19}$$

by taking the limit of the two sides of (4.18) we obtain

$$\lim_{T\to+\infty} \sum_{k=q}^{+\infty} \frac{(\lambda_j T)^k}{(k+1)!\mathrm{e}^{\lambda_j T}} = \lim_{T\to+\infty} E\left[\frac{1}{N_T^j + 1}\right] - \lim_{T\to+\infty} \sum_{k=0}^{q-1} \frac{(\lambda_j T)^k}{(k+1)!\mathrm{e}^{\lambda_j T}} = 0. \tag{4.20}$$

Then inequality (4.17) and equation (4.20) yield equation (4.7).

(4) Since N_T follows Poission distribution with parameter $\sum_{j=1}^{n} \lambda_j T$, similar to the derivation of equation (4.7) we obtain equation (4.8).

Lemma 4.3. *Let f_g denote the flow time of the gth arriving customer, w_g denote the weight of the gth arriving customer. Suppose K is an arbitrary integer and $V = \max_{1\leqslant j\leqslant n}\{w_j\}$. If there exists a positive number U such that $E[f_g] \leqslant U$ for arbitrary $g(g \in Z^+)$, then*

$$E\left[\frac{1}{N_T+K}\sum_{g=1}^{N_T+K} w_g f_g\right] = E\left[\frac{1}{N_T}\sum_{g=1}^{N_T} w_g f_g\right] \quad \text{as} \quad T \to +\infty$$

.

Proof. According to the definition of mathematical expectation, we have

$$E\left[\frac{1}{N_T+K}\sum_{g=1}^{N_T+K} w_g f_g\right] = \frac{1}{K}\sum_{g=1}^{K} w_g E[f_g] P\{N_T = 0\} + \sum_{k=1}^{+\infty}\frac{1}{k+K}\sum_{g=1}^{k+K} w_g E[f_g] P\{N_T = k\}. \tag{4.21}$$

In the following we show inequalities (4.22) and (4.23) respectively.

$$E\left[\frac{1}{N_T+K}\sum_{g=1}^{N_T+K} w_g f_g\right] \leqslant E\left[\frac{1}{N_T}\sum_{g=1}^{N_T} w_g f_g\right] \text{ as } T \to +\infty \tag{4.22}$$

$$E\left[\frac{1}{N_T+K}\sum_{g=1}^{N_T+K} w_g f_g\right] \geqslant E\left[\frac{1}{N_T}\sum_{g=1}^{N_T} w_g f_g\right] \text{ as } T \to +\infty \tag{4.23}$$

Trivially,

$$\sum_{k=1}^{+\infty}\frac{1}{k+K}\sum_{g=1}^{k+K} w_g E[f_g] P\{N_T = k\} \leqslant \sum_{k=1}^{+\infty}\frac{1}{k}\sum_{g=1}^{k+K} w_g E[f_g] P\{N_T = k\}$$

$$= \sum_{k=1}^{+\infty}\frac{1}{k}\sum_{g=1}^{k} w_g E[f_g] P\{N_T = k\} + \sum_{k=1}^{+\infty}\frac{1}{k}\sum_{g=k+1}^{k+K} w_g E[f_g] P\{N_T = k\}. \tag{4.24}$$

Since $E[f_g] \leqslant U$ and $w_g \leqslant V$ hold for arbitrary g ($g \in \mathrm{Z}^+$), then

$$\sum_{k=1}^{+\infty}\frac{1}{k}\sum_{g=k+1}^{k+K} w_g E[f_g] P\{N_T = k\} \leqslant \sum_{k=1}^{+\infty}\frac{KUV}{k} P\{N_T = k\}. \tag{4.25}$$

According to the definition of mathematical expectation, we have

$$E\left[\frac{1}{N_T}\sum_{g=1}^{N_T} w_g f_g\right] = \sum_{k=1}^{+\infty}\frac{1}{k}\sum_{g=1}^{k} w_g E[f_g] P\{N_T = k\}. \tag{4.26}$$

It follows from equations (4.21), (4.24)–(4.26) that

$$E\left[\frac{1}{N_T+K}\sum_{g=1}^{N_T+K} w_g f_g\right] \leqslant \frac{1}{K}\sum_{g=1}^{K} w_g E[f_g] P\left\{N_T=0\right\} + E\left[\frac{1}{N_T}\sum_{g=1}^{N_T} w_g f_g\right] + \sum_{k=1}^{+\infty}\frac{KUV}{k}P\{N_T=k\}. \quad (4.27)$$

It follows from Lemma 4.1 that

$$\lim_{T\to+\infty} P\{N_T=0\}=0. \quad (4.28)$$

Hence,

$$\lim_{T\to+\infty}\sum_{g=1}^{K} w_g E[f_g] P\{N_T=0\}=0. \quad (4.29)$$

By equation (4.8), we have

$$\lim_{T\to+\infty}\sum_{k=1}^{+\infty}\frac{KUV}{k}P\{N_T=k\}=0. \quad (4.30)$$

Hence, by equation (4.27) we obtain

$$\begin{aligned} E\left[\frac{1}{N_T+K}\sum_{g=1}^{N_T+K} w_g f_g\right] \leqslant & E\left[\frac{1}{N_T}\sum_{g=1}^{N_T} w_g f_g\right] \\ & + \lim_{T\to+\infty}\frac{1}{K}\sum_{g=1}^{K} w_g E[f_g] P\{N_T=0\} \\ & + \lim_{T\to+\infty}\sum_{k=1}^{+\infty}\frac{KUV}{k}P\{N_T=k\} \\ = & E\left[\frac{1}{N_T}\sum_{g=1}^{N_T} w_g f_g\right] \quad \text{as } T\to+\infty. \end{aligned} \quad (4.31)$$

On the other hand, we have

$$\begin{aligned} & \sum_{k=1}^{+\infty}\frac{1}{k+K}\sum_{g=1}^{k+K} w_g E[f_g] P\{N_T=k\} \geqslant \sum_{k=1}^{+\infty}\frac{1}{k+K}\sum_{g=1}^{k} w_g E[f_g] P\{N_T=k\} \\ & = \sum_{k=1}^{+\infty}\left[\frac{1}{k}-\left(\frac{1}{k}-\frac{1}{k+K}\right)\right]\sum_{g=1}^{k} w_g E\left[f_g\right] P\{N_T=k\} \\ & = \sum_{k=1}^{+\infty}\frac{1}{k}\sum_{g=1}^{k} w_g E[f_g] P\{N_T=k\} - \sum_{k=1}^{+\infty}\frac{K}{k(k+K)}\sum_{g=1}^{k} w_g E[f_g] P\{N_T=k\}. \end{aligned} \quad (4.32)$$

By the definition of mathematical expectation we have

$$E\left[\frac{1}{N_T}\sum_{g=1}^{N_T} w_g f_g\right] = \sum_{k=1}^{+\infty}\frac{1}{k}\sum_{g=1}^{k} w_g E[f_g]P\{N_T = k\}\cdot \tag{4.33}$$

Hence, inequality (4.32) yields

$$\begin{aligned}&\sum_{k=1}^{+\infty}\frac{1}{k+K}\sum_{g=1}^{k+K} w_g E[f_g]P\{N_T = k\}\\ &\geqslant E\left[\frac{1}{N_T}\sum_{g=1}^{N_T} w_g f_g\right] - \sum_{k=1}^{+\infty}\frac{K}{k(k+K)}\sum_{g=1}^{k} w_g E[f_g]P\{N_T = k\}\cdot\end{aligned} \tag{4.34}$$

Thus, according to equation (4.21) we have

$$\begin{aligned}&E\left[\frac{1}{N_T+K}\sum_{g=1}^{N_T+K} w_g f_g\right] \geqslant \frac{1}{K}\sum_{g=1}^{K} w_g E[f_g]P\{N_T = 0\}\\ &+ E\left[\frac{1}{N_T}\sum_{g=1}^{N_T} w_g f_g\right] - \sum_{k=1}^{+\infty}\frac{K}{k(k+K)}\sum_{g=1}^{k} w_g E[f_g]P\{N_T = k\}.\end{aligned} \tag{4.35}$$

Since $\sum_{g=1}^{k} w_g E[f_g] \leqslant kUV$, it follows that

$$\begin{aligned}\sum_{k=1}^{+\infty}\frac{K}{k(k+K)}\sum_{g=1}^{k} w_g E[f_g]P\{N_T = k\} &\leqslant \sum_{k=1}^{\infty}\frac{KUV}{k+K}P\{N_T = k\}\\ &\leqslant E\left[\frac{KUV}{N_T+K}\right]\cdot\end{aligned} \tag{4.36}$$

Thus, by inequality (4.35) we obtain

$$\begin{aligned}E\left[\frac{1}{N_T+K}\sum_{g=1}^{N_T+K} w_g f_g\right] \geqslant &\frac{1}{K}\sum_{g=1}^{K} w_g E[f_g]P\{N_T = 0\}\\ &+ E\left[\frac{1}{N_T}\sum_{g=1}^{N_T} w_g f_g\right] - E\left[\frac{KUV}{N_T+K}\right]\cdot\end{aligned} \tag{4.37}$$

It follows from equations (4.6) and (4.29) that

$$\lim_{T\to+\infty} E\left[\frac{KUV}{N_T+K}\right] = 0. \tag{4.38}$$

Hence, according to inequality (4.37),

$$E\left[\frac{1}{N_T+K}\sum_{g=1}^{N_T+K} w_g f_g\right] \geqslant E\left[\frac{1}{N_T}\sum_{g=1}^{N_T} w_g f_g\right] \quad \text{as} \quad T\to+\infty. \tag{4.39}$$

Hence, by inequalities (4.31) and (4.39) the result follows.

Lemma 4.4. *Let f_g denote the flow time of the gth arriving customer. Suppose $\lambda = \sum_{j=1}^{n} \lambda_j$. If there exists a positive number U such that $E[f_g] \leqslant U$ for arbitrary g ($g \in Z^+$), then*

$$E\left[\frac{1}{T}\sum_{g=1}^{N_T} w_g f_g\right] = \lambda E\left[\frac{1}{N_T}\sum_{g=1}^{N_T} w_g f_g\right] \quad \text{as} \quad T \to +\infty. \tag{4.40}$$

Proof. By the definition of mathematical expectation we have

$$\begin{aligned} E\left[\frac{1}{T}\sum_{g=1}^{N_T} w_g f_g\right] &= \sum_{k=1}^{+\infty} \frac{1}{T} \sum_{g=1}^{k} w_g E[f_g] P\{N_T = k\} \\ &= \lambda \sum_{k=1}^{+\infty} \frac{1}{\lambda T} \sum_{g=1}^{k} w_g E[f_g] \frac{\mathrm{e}^{-\lambda T}(\lambda T)^k}{k!} \\ &= \lambda \sum_{k=1}^{+\infty} \sum_{g=1}^{k} w_g E[f_g] \frac{\mathrm{e}^{-\lambda T}(\lambda T)^{k-1}}{k!}. \end{aligned} \tag{4.41}$$

Let $i = k - 1$, then equation (4.41) yields

$$\begin{aligned} E\left[\frac{1}{T}\sum_{g=1}^{N_T} w_g f_g\right] &= \lambda \sum_{i=0}^{+\infty} \left\{ \sum_{g=1}^{i+1} w_g E[f_g] \frac{\mathrm{e}^{-\lambda T}(\lambda T)^i}{i!} \frac{1}{i+1} \right\} \\ &= \lambda \sum_{i=0}^{+\infty} \frac{1}{i+1} \sum_{g=1}^{i+1} w_g E[f_g] P\{N_T = i\} \\ &= \lambda E\left[\frac{1}{N_T+1} \sum_{g=1}^{N_T+1} w_g f_g\right]. \end{aligned} \tag{4.42}$$

It follows from Lemma 4.3 that

$$E\left[\frac{1}{N_T+1}\sum_{g=1}^{N_T+1} w_g f_g\right] = E\left[\frac{1}{N_T}\sum_{g=1}^{N_T} w_g f_g\right] \text{ as } T \to +\infty. \tag{4.43}$$

Hence, by equations (4.42) and (4.43) the result follows.

Theorem 4.5. *Let N_T^d denote the number of decision-making states by time T, d_g denote the arrival time of the gth arriving customer, and c_g denote the completion time of the gth arriving customer. Suppose there exists a positive number U such that $E[f_g] \leqslant U$ for arbitrary g ($g \in Z^+$). Let $\overline{r_T^t}$ denote the time-average reward by time T, $\overline{f_T}$ denote the average flow time of the customers served by time T, which are defined as*

$$\overline{r_T^t} = \frac{1}{T}\sum_{k=1}^{N_T} r_k, \tag{4.44}$$

$$\overline{f_T} = \frac{1}{N_T}\sum_{g=1}^{N_T} w_g f_g, \tag{4.45}$$

where $f_g = c_g - d_g$*. Let* $\delta_g(t)$ *denote an indicator function defined as*

$$\delta_g(t) = \begin{cases} 0 \ \textit{if} \ \text{the gth customer has not arrived or has been completedly served by time } t \\ 1 \ \textit{if} \ \text{the gth customer is waiting or being served at time } t \end{cases} \tag{4.46}$$

Then we have

$$E\left[\overline{r_T^t}\right] = -\lambda E[\overline{f_T}] \quad \textit{as} \quad T \to +\infty. \tag{4.47}$$

Proof. By equations (3.34) and (4.44) we obtain

$$\begin{aligned}
\overline{r_T^t} &= \frac{1}{T}\sum_{k=1}^{N_T^d} -(\tau_{k+1} - \tau_k)\sum_{j=1}^{n} w_j q_{j,k}^* \\
&= -\frac{1}{T}\sum_{k=1}^{N_T^d}\int_{t=\tau_k}^{\tau_{k+1}}\sum_{g=1}^{N_T} w_g \delta_g(t)\mathrm{d}t \\
&= -\frac{1}{T}\sum_{k=1}^{N_T^d}\sum_{g=1}^{N_T}\int_{t=\tau_k}^{\tau_{k+1}} w_g \delta_g(t)\mathrm{d}t \\
&= -\frac{1}{T}\sum_{g=1}^{N_T}\sum_{k=1}^{N_T^d}\int_{t=\tau_k}^{\tau_{k+1}} w_g \delta_g(t)\mathrm{d}t \\
&= -\frac{1}{T}\sum_{g=1}^{N_T}\int_{t=0}^{T} w_g \delta_g(t)\mathrm{d}t \\
&= -\frac{1}{T}\sum_{g=1}^{N_T} w_g f_g.
\end{aligned} \tag{4.48}$$

Hence,

$$E[\overline{r_T^t}] = -E\left[\frac{1}{T}\sum_{g=1}^{N_T} w_g f_g\right]. \tag{4.49}$$

It follows from Lemma 4.4 that

$$E\left[\overline{r_T^t}\right] = -\lambda E\left[\frac{1}{N_T}\sum_{g=1}^{N_T} w_g f_g\right] \ \text{as } T \to \infty. \tag{4.50}$$

According to equations (4.45) and (4.50) the result follows.

By Theorem 4.5 the reward function has the following property:

Proposition 4.6. *Minimizing all the customers' expected weighted mean flow time defined as equation* (2.1) *is equivalent to maximizing the expected time-average reward in the SMDP model when the queuing system runs infinite time.*

From the above proposition, we can see that solving the discussed queueing control problem is equivalent to solving the SMDP with average reward objective formulated in Section 3. In other words, to find an optimal policy for the queueing control problem is equivalent to find an optimal policy for the SMDP.

5. A numerical example

In this section we present a numerical example to illustrate the computation of transition probabilities and the probability distribution of transition times and demonstrate the property of the multi-server queueing control problem. In this queueing control problem, there are $3(n = 3)$ classes of customers to be served at a service station containing $3(m = 3)$ unrelated parallel servers. The rates of the Poission processes for customers' arrival are $\lambda_1 = 0.1$, $\lambda_2 = 0.05$, and $\lambda_3 = 0.04$. The mean matrix of the normal distributions for service times is

$$\{\mu^p_{i,j}\}_{(1\leqslant i\leqslant 3, 1\leqslant j\leqslant 3)} = \begin{bmatrix} 10.47 & 5.98 & 8.12 \\ 5.10 & 13.52 & 16.91 \\ 11.36 & 6.68 & 13.92 \end{bmatrix}.$$

The standard deviation matrix of the normal distributions for service times is

$$\{\sigma^p_{i,j}\}_{(1\leqslant i\leqslant 3, 1\leqslant j\leqslant 3)} = \begin{bmatrix} 2.99 & 1.17 & 1.87 \\ 1.31 & 1.57 & 2.88 \\ 3.05 & 2.16 & 3.32 \end{bmatrix}.$$

The mean matrix of the normal distributions for conversion times is

$$\{\mu^s_{j,l}\}_{(1\leqslant j\leqslant 3, 1\leqslant l\leqslant 3)} = \begin{bmatrix} 0 & 21.40 & 16.10 \\ 23.30 & 0 & 29.90 \\ 25.90 & 21.90 & 0 \end{bmatrix}.$$

The standard deviation matrix of the normal distributions for conversion times is

$$\{\sigma^s_{j,l}\}_{(1\leqslant j\leqslant 3, 1\leqslant l\leqslant 3)} = \begin{bmatrix} 0 & 2.64 & 3.59 \\ 3.14 & 0 & 4.19 \\ 2.75 & 3.21 & 0 \end{bmatrix}.$$

Suppose the kth decision-making state is represented as

$$s_k = [2, 3, 4; 1, 2, 3; 0, 1, 2; 0, 4.6, 5.8].$$

Thus the first server is idle and the other servers are busy at this decision-making epoch. Suppose a_k selects a customer of the second class to serve on the first server. Then the system immediately transfers into the kth interim state s_k^* represented as

$$s_k^* = [2,2,4;1,2,3;2,1,2;0,4.6,5.8]$$

By equations (3.16) and (3.17) the probability of that the triggering event of the state transition from s_k^* to s_{k+1} is arrival of a customer of the first class is computed as

$$\begin{aligned} &P_F\left(s_k^*, a_k, s_{k+1}^1\right) = F_1\left(+\infty\right) \\ &= \frac{0.1\int_0^{+\infty} \mathrm{e}^{-0.19x} \int_x^{+\infty} \frac{\exp\left\{-\frac{(y-21.40-5.98)^2}{2(2.64^2+1.17^2)}\right\}}{\sqrt{2\pi(2.64^2+1.17^2)}}\mathrm{d}y \int_{4.6+x}^{+\infty} \frac{\exp\left\{-\frac{(y-23.30-5.10)^2}{2(3.14^2+1.31^2)}\right\}}{\sqrt{2\pi(3.14^2+1.31^2)}}\mathrm{d}y}{\int_0^{+\infty} \frac{\exp\left\{-\frac{(y-21.40-5.98)^2}{2(2.64^2+1.17^2)}\right\}}{\sqrt{2\pi(2.64^2+1.17^2)}}\mathrm{d}y \int_{4.6}^{+\infty} \frac{\exp\left\{-\frac{(y-23.30-5.10)^2}{2(3.14^2+1.31^2)}\right\}}{\sqrt{2\pi(3.14^2+1.31^2)}}\mathrm{d}y} \\ &\times \frac{\int_{5.8+x}^{+\infty} \frac{\exp\left\{-\frac{(y-21.90-6.68)^2}{2(3.21^2+2.16^2)}\right\}}{\sqrt{2\pi(3.21^2+2.16^2)}}\mathrm{d}y\mathrm{d}x}{\int_{5.8}^{+\infty} \frac{\exp\left\{-\frac{(y-21.90-6.68)^2}{2(3.21^2+2.16^2)}\right\}}{\sqrt{2\pi(3.21^2+2.16^2)}}\mathrm{d}y} \\ &= 0.515. \end{aligned}$$

Similarly, the probabilities of that the triggering event of the state transition from s_k^* to s_{k+1} is arrival of a customer of the second class and of the third class are respectively computed as

$$P_F\left(s_k^*, a_k, s_{k+1}^2\right) = F_2(+\infty) = 0.256,$$

$$P_F\left(s_k^*, a_k, s_{k+1}^3\right) = F_3(+\infty) = 0.205.$$

By equations (3.30), (3.31) and (3.32), the probability of that the triggering event of the state transition from s_k^* to s_{k+1} is completion of serving a customer on the first server is computed as

$$\begin{aligned} &P_G\left(s_k^*, a_k, s_{k+1}^1\right) = G_1\left(+\infty\right) \\ &= \frac{\int_0^{+\infty} \mathrm{e}^{0.19(5.8-x)} \int_{x+4.6}^{+\infty} \frac{\exp\left\{-\frac{(y-23.30-5.10)^2}{2(3.14^2+1.31^2)}\right\}}{\sqrt{2\pi(3.14^2+1.31^2)}}\mathrm{d}y \int_{x+5.8}^{+\infty} \frac{\exp\left\{-\frac{(y-21.90-6.68)^2}{2(3.21^2+2.16^2)}\right\}}{\sqrt{2\pi(3.21^2+2.16^2)}}\mathrm{d}y}{\int_0^{+\infty} \frac{\exp\left\{-\frac{(y-21.40-5.98)^2}{2(2.64^2+1.17^2)}\right\}}{\sqrt{2\pi(2.64^2+1.17^2)}}\mathrm{d}y \int_{4.6}^{+\infty} \frac{\exp\left\{-\frac{(y-23.30-5.10)^2}{2(3.14^2+1.31^2)}\right\}}{\sqrt{2\pi(3.14^2+1.31^2)}}\mathrm{d}y} \\ &\times \frac{\frac{\exp\left\{-\frac{(x-21.40-5.98)^2}{2(2.64^2+1.17^2)}\right\}}{\sqrt{2\pi(2.64^2+1.17^2)}}\mathrm{d}x}{\int_{5.8}^{+\infty} \frac{\exp\left\{-\frac{(y-21.90-6.68)^2}{2(3.21^2+2.16^2)}\right\}}{\sqrt{2\pi(3.21^2+2.16^2)}}\mathrm{d}y} \\ &= 0.003. \end{aligned}$$

Similarly, the probabilities of that the triggering event of the state transition from s_k^* to s_{k+1} is completion of serving a customer on the second server and on the third class are respectively computed as

$$P_G(s_k^*, a_k, s_{k+1}^2) = G_2(+\infty) = 0.008,$$

$$P_G(s_k^*, a_k, s_{k+1}^3) = G_3(+\infty) = 0.013.$$

By equations (3.16), (3.31) and (3.33), the probability of that the transition time from state s_k to state s_{k+1} is less than or equal to 10 is computed as

$$\begin{aligned} F(10) &= \sum_{J=1}^{n} F_J(10) + \sum_{1\leqslant I\leqslant m, T_I^*\neq 0} G_I(10) \\ &= \sum_{J=1}^{3} F_J(10) + \sum_{I=1}^{3} G_I(10) \\ &= 0.850. \end{aligned} \tag{5.1}$$

To demonstrate the result of Theorem 4.5, we run the SMDP model of the above numerical example under a stochastic policy with 30 random instances. Define an index GAP as

$$\text{GAP} = |f_T - r_T/\lambda|, \tag{5.2}$$

where f_T denotes the average flow time of the customers served by time T averaged over 30 random instances and r_T denotes the time-average reward by time T averaged over 30 random instances. Figure 2 shows the variation of GAP value with respect to the number of customers served. As shown in Figure 2, GAP value asymptotically decreases to zero, which means f_T asymptotically approaches r_T/λ and demonstrates equation (4.47).

6. Concluding remarks

This paper studies a multi-server queueing control problem with customers of various classes considering service times and conversion times following normal distributions. We convert the queueing control problem into a particular SMDP model by constructing elaborate state representation, actions and the reward function, investigate the mechanism of state transitions by deriving the analytic expression of the transition probabilities and transition times, study the property of the queuing control system and show the equivalence of the queueing control objective and the solution to the modeled SMDP. Although the service times and conversion times follow normal distribution in this study, the method for developing the mechanism of state transitions is also applicable for service times and conversion times following general distribution functions. The technique for formulating queueing networks control problems with adaptive multi-state service stations is worthy of further investigation.

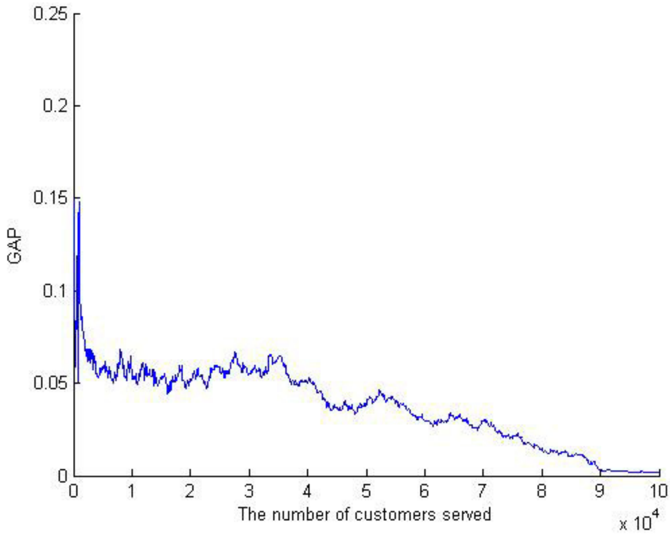

FIGURE 2. Variation of GAP value with respect to the number of customers served.

Acknowledgements. This project is supported by the National Natural Science Foundation of China (Grant No. 71201026), the Development Program for Excellent Young Teachers in Higher Education Institutions of Guangdong Province (No. Yq2013156), Humanities and Social Sciences Program of MOE, P.R.C. (No. 10YJC630405), Science and Technological Program for Dongguan's Higher Education, Science and Research, and Health Care Institutions (No. 2011108102017).

REFERENCES

[1] R. Andriansyah, T.V. Woensel and F.R.B. Cruz, Performance optimization of open zero-buffer multi-server queueing networks. *Comput. Oper. Res.* **37** (2010) 1472–1487.

[2] A. Azaron, H. Katagiri and K. Kato, Longest path analysis in networks of queues: Dynamic scheduling problems. *Eur. J. Oper. Res.* **174** (2006) 132–149.

[3] S. Borst, N. Hegde and A. Proutière, Interacting queues with server selection and coordinated scheduling-application to cellular data networks. *Annal. Oper. Res.* **170** (2009) 59–78.

[4] C.K. Chen, H.H. Kuo and J.J. Yan, GA-based PID active queue management control design for a class of TCP communication networks. *Exp. Syst. Appl.* **36** (2009) 1903–1913.

[5] D. Choi, T.S. Kim and S. Lee, Analysis of a queueing system with a general service scheduling function, with applications to telecommunication network traffic control. *Eur. J. Oper. Res.* **178** (2007) 463–471.

[6] A. Choudhury and M. Pallabi, Some aspects of balking and reneging in finite buffer queues. *RAIRO-Oper. Res.* **45** (2011) 223–240.

[7] J. Granger, *Performance improvement of queueing networks with synchronization stations*, Ph.D. dissertation, University of Wisconsin (2006).

[8] M. Harchol-Balter, T. Osogami and A. Scheller-Wolf, Multi-server queueing systems with multiple priority classes. *Queueing Syst.* **51** (2005) 331–360.

[9] M. Haridass and R. Arumuganathan, Analysis of a MX/G (a, b)/1 queueing system with vacation interruption. *RAIRO-Oper. Res.* **46** (2012) 305–334.

[10] C. Kim, S. Dudin and V. Klimenok, The MAP/PH/1/N queue with flows of customers as a model for traffic. *Perform. Eval.* **66** (2009) 564–579.

[11] Y.L. Koçağa and A.R. Ward, Admission control for a multi-server queue with abandonment. *Queueing Syst.* **65** (2010) 275–323.

[12] L.B. Le, E. Hossain, A.S. Alfa, Queueing analysis and admission control for multi-rate wireless networks with opportunistic scheduling and ARQ-based error control. *IEEE Int. Conf. Commun.* **5** (2005) 3329–3333.

[13] Y. Nazarathy and G. Weiss, Near optimal control of queueing networks over a finite time horizon. *Annal. Oper. Res.* **170** (2009) 233–249.

[14] V. Nitto Personè, Analysis of cyclic queueing networks with parallelism and vacation. *Annal. Oper. Res.* **170** (2009) 95–112.

[15] M. Pallabi and A. Choudhury, Aspects of impatience in a finite buffer queue. *RAIRO-Oper. Res.* **46** (2012) 189–209.

[16] I.C. Paschalidis, C. Su and M.C. Caramanis, New scheduling policies for multiclass queueing networks: Applications to peer-to-peer systems, in proc. of 42nd *IEEE Conf. Decision and Control*, Maui, Hawau (2003) 1604–1609.

[17] I.C. Paschalidis, C. Su and M.C. Caramanis, Target-pursuing scheduling and routing policies for multiclass queueing networks. *IEEE Trans. Automat. Contr.* **49** (2004) 1709–1722.

[18] B. Sikdar and D. Manjunath, Queueing analysis of scheduling policies in copy networks of space-based multicast packet switches. *IEEE/ACM Trans. Netw.* **8** (2000) 396–406.

[19] P. Tehrani and Q. Zhao, Multichannel scheduling and its connection to queueing network control problem, *in proc. of The 2010 Military Communications Conference*, San Jose, CA (2010) 482–486.

[20] X. Xu, and Z.G. Zhang, Analysis of multi-server queue with a single vacation (e, d)-policy. *Performance Evaluation* **63** (2006) 825–838.