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PRODUCTION PLANNING IN DATA ENVELOPMENT ANALYSIS WITHOUT EXPLICIT INPUTS

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Abstract. In the performance measurement using tools such as data envelopment analysis (DEA), data without explicit inputs has attracted considerable attention among researchers. In such studies the problem of production planning in the next production season is an important and interesting subject. Because of the uncertain nature of the future, decision makers need to provide robust procedures in order to examine alternative courses of action and their implications. The purpose of this paper is to develop an approach to production planning problem in production processes without explicit inputs that typically appears in centralized decision making environment. Application of the proposed approach is illustrated empirically using a real case.

Keywords. Data envelopment analysis, production planning, efficiency, output.

Mathematics Subject Classification. 90B30.

1. Introduction

Data Envelopment Analysis (DEA) was initially introduced by Charnes *et al.* [6] [CCR approach] and by Banker *et al.* [4] [BCC approach] for the purpose of measuring the relative performance of similar economic production systems. After the seminal work of Charnes *et al.* [6], several extensions of DEA on performance measurement in real life problems using DEA have been published. See references [1,8,9]. Over the last two decades, an important and interesting application

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of DEA has been on production planning in a centralized decision making environment, where a central decision maker has the power to control decision parameters and the production planning problem involves the participation of all units, each contributing in part to the total production. There are a considerable number of works that concentrate on production planning in manufacturing systems. In what follows, some of these studies are introduced.

To set up goals for desired outputs, Golany [11] presented an interactive linear programming procedure which was based on the empirical production functions generated by DEA and then adjusted by new information of the decision maker in each iteration. Athanassopoulos *et al.* [3] demonstrated how DEA could be used to develop policy making scenarios that would enable managers to identify the response of productive units such as power plants to different priorities regarding demand of services, costs and pollution emissions.

Beasley proposed nonlinear resource allocation models to jointly decide the input and output amounts of each decision making unit (DMU) for the next period while maximizing the average efficiency of all DMUs. Lozano and Villa [16] studied the decision making in a centralized environment and considered systems with multiple inputs and outputs and addressed an intra-organizational scenario in which all units fell under the supervision of a centralized decision maker. In the parallel research, Korhonen and Syrjanen [13] developed a DEA-based interactive approach to a resource allocation problem that typically appeared in a centralized decision making environment. From the productivity and efficiency perspective, Du *et al.* [7] looked in to the production planning problem by using DEA. They proposed two production planning ideas in a centralized decision making environment where demand changes could be forecasted. Amirteimoori and Kordrostami [2] provided an alternative production planning model based on DEA.

All of the preceding studies have taken inputs and outputs of the DMUs into consideration and they provided a production planning approach by considering the observed inputs and outputs. In some practical applications of DEA, we confront cases in which data sets are sometimes given without inputs. Lovell and Pastor [15] were the first researchers that studied the DEA models without inputs or without outputs. Although they demonstrated that CCR models without inputs or without outputs were meaningless, Liu *et al.* [14] carried out a study on the construction of DEA models without explicit inputs and provided a case study on 15 basic research institutes in Chinese Academy of Science.

Having taken the importance of systems without inputs into consideration, we will consider the problem of production planning in a centralized decision making environment. It has been assumed that the demands for the outputs can be forecasted in the next production season and the paper develops a DEA-based production planning model to determine the most favorable output levels for each operational unit in the next production season.

The rest of the paper is organized as follows: The following section provides a background on DEA. In Section 3, the DEA model without inputs is constructed axiomatically. The proposed production planning model is given in Section 4.

Section 5 applies the approach to a real data set consisting of 11 universities. Finally, we end up with the result.

2. Preliminaries

DEA is a mathematical programming model that measures the relative efficiency of operational units with multiple inputs and outputs but no obvious production function to aggregate the data in it entirely. Assume there are n DMUs and the performance of each DMU is characterized by a production process of m inputs $(x_{ij}: \ i = 1, \ldots, m)$ to yield s outputs $(y_{rj}: \ r = 1, \ldots, s)$. Relative efficiency is defined as the ratio of weighted sum of outputs to the weighted sum of inputs. Charnes *et al.* [6] proposed the following LP problem to obtain the efficiency score of DMU_o:

$$
\begin{aligned}
&\text{Max} \ \ e_o = \textstyle\sum_{r=1}^{s} u_r y_{ro} \\
&s.t. \\
&\qquad \textstyle\sum_{i=1}^{m} v_i x_{io} = 1, \\
&\qquad \textstyle\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \ j = 1, \ \ldots, \ \ n, \\
&\qquad u_r \geq \epsilon, \quad r = 1, \ \ldots, \ s, \\
&\qquad v_i \geq \epsilon, \quad i = 1, \ \ldots, \ m.
\end{aligned}
\tag{2.1}
$$

This model is a constant returns to scale program and it assumes that the status of all input/output variables is known prior to solving the model. The efficiency ratio e_o ranges between zero and one, with DMU_o being considered relatively efficient if it receives a score of one. From a managerial perspective, this model delivers assessments and targets with an output maximization orientation. The result of the DEA model (1) is the determination of the hyperplanes that define an envelop surface or Pareto frontier. DMUs that lie on the surface determine the envelope and are deemed to be efficient, whilst those that do not are deemed inefficient. The foregoing model is an input-orientation model while another DEA-model is output-orientation model. Additive model combines both orientations in a single model as follows:

$$
\begin{aligned}
&\text{Max} \ \ e_o = \textstyle\sum_{i=1}^{m} {s_i}^{-} + \sum_{r=1}^{s} {s_r}^{+} \\
&s.t. \\
&\qquad \textstyle\sum_{j=1}^{n} \lambda_j x_{ij} + {s_i}^{-} = x_{io}, i = 1, \ \ldots, \ m, \\
&\qquad \textstyle\sum_{j=1}^{n} \lambda_j y_{rj} - {s_r}^{+} = y_{ro}, r = 1, \ \ldots, \ s, \\
&\qquad {s_i}^{-}, \ {s_i}^{+}, \ \lambda_j \geq 0, \quad \text{for all } i, \ j, \ r.
\end{aligned}
\tag{2.2}
$$

DMU_o is said to be efficient in additive sense if and only if $e_o = 1$. In the following section, a DEA model without explicit inputs is introduced.

3. Data envelopment analysis without inputs

In the performance measurement using DEA, data set are sometimes given without inputs. In spite of the fact that Li *et al.* studied the problem of DEA

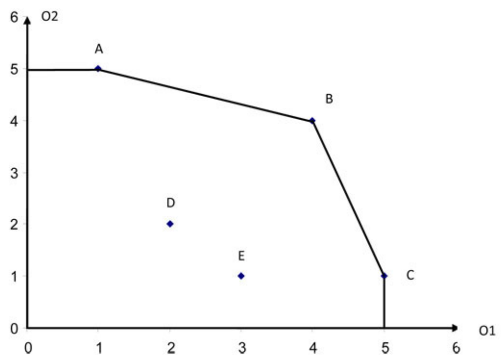

FIGURE 1. Production possibility set in the two outputs case.

TABLE 1. Data for a simple example.

DMU	A	B	C	D	E
O_1	1	4	5	2	3
O_2	5	4	1	2	1

models without explicit inputs, in what follows, we will axiomatically construct the production possibility set in this situation. Having considered a production process without explicit inputs, we suppose that there are n DMUs with each DMU_j : $(j = 1, \ldots, n)$ that has s outputs y_{rj} : $r = 1, \ldots, s$. Let T be the production possibility set (PPS) of technology under consideration. To construct T, we postulate the following axioms:

A1- Feasibility: $y_j = (y_{1j},\ y_{2j},\ \ldots,\ y_{sj}) \in T$ for any $j = 1, \ldots, n$.

A2- Convexity: Let y' and $y'' \in T$. Then, for any $\lambda \in [0, 1]$, the unit $\lambda y' + (1 - \lambda)y'' \in T$.

A3- Free disposability: $y \in T$ and $y' \leq y$ imply $y' \in T$.

A4- Minimal extrapolation: For each T' satisfying in the axioms A1–A3, we have $T \subseteq T'$.

Figure 1 shows a typical PPS in two-dimension in the two outputs case. Suppose there are five DMUs with two outputs as shown in Table 1.

The line connecting A, B and C identifies the efficient frontier and the production set is the region bounded by the axes and the frontier line.

Now, an algebraic representation of the technology T, which satisfies the axioms A1−A4 is given.

Theorem 3.1. *The PPS T, which satisfies the axioms A1−A4 is defined as*

$$T = \left\{ y : \sum_{j=1}^{n} \lambda_j y_j \geq y,\ \sum_{j=1}^{n} \lambda_j = 1,\ \forall \lambda_j \geq 0,\ j = 1, \ldots, n \right\}$$

Proof. It is obvious that the set T satisfies axioms A1−A3. Now, we show that T is the minimal set. Assume that T' also satisfies A1−A3. We need to show that $y \in T$ implies $y \in T'$. Consider the representation $\sum_{j=1}^{n} \lambda_j y_j \geq y'$ of the unit y'. For the vector $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_n)$ from this representation, define $y_\lambda = \sum_{j=1}^{n} \lambda_j y_j$. It is clear that $y_\lambda \in T'$, and this unit dominates y in the Pareto sense. So, we conclude that $y \in T'$. The proof is completed.

Taking Theorem 1 into consideration, technical efficiency of DMU_o is determined to be related to other similar units and can focus on augmentation of the outputs as follows:

$$\begin{aligned} & e_o = \text{Max} \ \ \theta_o. \\ & s.t. \\ & \qquad \theta_o y_o \in T. \end{aligned} \tag{3.1}$$

Based on the axioms A1−A4 and the definition of T, model (3) is transformed in to the following form:

$$\begin{aligned} & e_o = \text{Max} \ \ \theta_o \\ & s.t. \\ & \qquad \theta_o y_o \leq \textstyle\sum_{j=1}^{n} \lambda_j y_{rj}, \ r = 1, \ \ldots, \ s, \\ & \qquad \textstyle\sum_{j=1}^{n} \lambda_j = 1, \\ & \qquad \lambda_j \geq 0, \ \text{for all } j. \end{aligned} \tag{3.2}$$

This problem has a feasible solution $\theta_o = 1, \ \lambda_o = 1, \ \lambda_j = 0; \ j = 1, \ldots, n, j \neq o$. Hence, the optimal value of θ_o, which is denoted by ${\theta_o}^*$, is not less than 1 and DMU_o is full-efficient if and only if $\theta_o = 1$. The dual formulation of the LP model (4) is expressed as follows:

$$\begin{aligned} & e_o = \text{Min} \ \rho \\ & s.t. \\ & \qquad \textstyle\sum_{r=1}^{s} u_r y_{rj} \leq \rho, \ j = 1, \ \ldots, \ n, \\ & \qquad \textstyle\sum_{r=1}^{s} u_r y_{ro} \geq 1, \\ & \qquad u_r \geq 0, \ r = 1, \ \ldots, \ s. \end{aligned} \tag{3.3}$$

In what follows, we show that the additive model with multiple inputs and outputs is a simple result of the additive model without inputs. The relative efficiency of DMU_o without inputs in additive sense is determined as follows:

$$\begin{aligned} & \text{Max } 1W \\ & s.t. \\ & \qquad y_o + W \in T, \\ & \qquad W \geq 0, \end{aligned} \tag{3.4}$$

in which $W = (w_1, \ w_2, \ \ldots, \ w_s)$ and 1 is a s-dimensional vector as $1 = (1, \ 1, \ \ldots, \ 1)$. This model considers the maximum output shortfalls in arriving at a point on the efficient frontier. Based on the definition of T, the following linear programming problem is used to evaluate the relative performance of DMU_o:

$$\begin{aligned} &\text{Max} \quad \textstyle\sum_{r=1}^{s} w_r \\ &s.t. \\ &\quad \textstyle\sum_{j=1}^{n} \lambda_j y_{rj} = y_{ro} + w_r, \ r = 1, \ \ldots, \ s, \\ &\quad \textstyle\sum_{j=1}^{n} \lambda_j = 1, \\ &\quad w_r \text{ and } \lambda_j \geq 0, \ \text{for all } r \text{ and } j. \end{aligned} \tag{3.5}$$

The DMU_o is additive efficient if and only if $\sum_{r=1}^{s} w_r = 0$.

Now, we show that the additive model (2) is equivalent to the LP model (7). To this end, let $x_{ij} = -z_{ij}$. Multiply the first m constraints of (2) by (–1). Then, the result immediately follows.

So far, a DEA model is introduced to evaluate the relative performance of the DMUs without inputs. In the next section, we will use this model to the production planning problem.

4. Production planning model

In organizations with centralized decision making environment, production usually involves the participation of all individual units, each contributing in part to the total production. The production planning problem involves determining the number of products produced by all individual units in the next season when demand changes can be predicted. We assume that there are n DMUs indexed by $DMU_j : \ (j = 1, \ldots, n)$. The rth output of DMU_j is symbolized by $y_{rj} : \ (r = 1, \ldots, s)$. Suppose that the demand change for output $r : \ (r = 1, \ldots, s)$ in the next production season can be forecasted as D_r. There are no restrictions on D_r that can be positive, negative or zero. To meet the demand changes, the central unit will determine the most favorable output plan for all DMUs.

We introduce the variables $d_{rj} : \ (r = 1, \ldots, s, \ j = 1, \ldots, n)$ to represent the demand changes of output r for DMU_j in the next season. Clearly, we must have $\sum_{j=1}^{n} d_{rj} = D_r$ for all r. In the proposed approach to production planning, we assume that the outputs in the next season should be changed in such a way that each DMU_j has efficiency score greater than or equal to e_j (e_j is the relative efficiency of DMU_j in the current season obtained from model (5)). So, we must have:

$$\begin{aligned} &\textstyle\sum_{r=1}^{s} u_r(y_{rj} + d_{rj}) \geq e_j, \ j = 1, \ \ldots, \ n, \\ &\textstyle\sum_{j=1}^{n} d_{rj} = D_r, \ r = 1, \ \ldots, \ s, \\ &u_r \geq 0, \ \textit{for all } r, \\ &d_{rj} \geq 0 \quad \text{when} \quad D_r \geq 0, \\ &d_{rj} \leq 0 \quad \text{when} \quad D_r \leq 0. \end{aligned} \tag{4.1}$$

Since u_r and d_{rj} are decision variables, this system of equations is clearly nonlinear. We make the change of variables $u_r d_{rj} = \bar{d}_{rj}$, and then, the system (8) is reduced to the following form:

$$\begin{aligned}
&\textstyle\sum_{r=1}^{s} u_r y_{rj} + \sum_{r=1}^{s} \bar{d}_{rj} \geq e_j,\ j = 1,\ \ldots,\ n,\\
&\textstyle\sum_{j=1}^{n} \bar{d}_{rj} = u_r D_r,\ r = 1,\ \ldots,\ s,\\
&u_r \geq 0,\ \text{for all } r,\\
&\bar{d}_{rj} \geq 0 \quad \text{when} \quad D_r \geq 0,\\
&\bar{d}_{rj} \leq 0 \quad \text{when} \quad D_r \leq 0.
\end{aligned} \tag{4.2}$$

There are s equations and n inequalities with $s(n+1)$ variables, and there is some flexibility in solutions. Suppose a target set for the rth output of DMU_j is $\bar{d}_{rj} = \alpha_j u_r D_r$. The scalar α_j will be selected proportionately to the performance of DMU_j.

For each DMU_j we let $\alpha_j = \frac{e_j}{\sum_{p=1}^{n} e_p}$ with $\sum_{j=1}^{n} \alpha_j = 1$. With these proportions, we take the performance of all DMUs into consideration. The difficulty with these values to $\bar{d}_{rj}$ is that there is no guarantee that they satisfy (9). In the absence of such a production plan, a rational objective is to introduce goal achievement variables for the efficiency and outputs levels.

Let $\bar{d}_{rj} - \alpha_j u_r D_r = {b_j}^+ - {b_j}^-$ and $\sum_{r=1}^{s} u_r y_{rj} + \sum_{r=1}^{s} \bar{d}_{rj} - 1 = {s_j}^+ - {s_j}^-$. The nonnegative variables ${b_j}^+$, ${b_j}^-$, ${s_j}^+$ and ${s_j}^-$ are deviational variables that represent the deviations above and below of the goals. To guarantee the feasibility and to ensure that each DMU_j can preserve its efficiency level, we consider the following constraints:

$$\begin{aligned}
&\textstyle\sum_{r=1}^{s} u_r y_{rj} + \sum_{r=1}^{s} \bar{d}_{rj} \geq e_j,\ j = 1,\ \ldots,\ n,\\
&\textstyle\sum_{r=1}^{s} u_r y_{rj} + \sum_{r=1}^{s} \bar{d}_{rj} - 1 = {s_j}^+ - {s_j}^-,\\
&\bar{d}_{rj} - \alpha_j u_r D_r = {b_j}^+ - {b_j}^-,\\
&\textstyle\sum_{j=1}^{n} \bar{d}_{rj} = u_r D_r,\ r = 1,\ \ldots,\ s,\\
&{b_j}^+,\ {b_j}^-,\ {s_j}^+,\ {s_j}^-,\ u_r \geq 0,\ \text{for all } r \text{ and } j,\\
&\bar{d}_{rj} \geq 0 \quad \text{when} \quad D_r \geq 0,\\
&\bar{d}_{rj} \leq 0 \quad \text{when} \quad D_r \leq 0.
\end{aligned} \tag{4.3}$$

Property 4.1. *The system including the whole inequalities in (10) is feasible.*

Proof. Clearly

$$\begin{aligned}
&u_r = 0,\ r = 1, \ldots, s,\\
&\bar{d}_{rj} = 0,\ r = 1, \ldots, s \text{ and } j = 1, \ldots, n,\\
&{b_j}^+ = {b_j}^-,\ j = 1, \ldots, n,\\
&{s_j}^+ = 0,\ j = 1, \ldots, n,\\
&{s_j}^- = 1,\ j = 1, \ldots, n,
\end{aligned}$$

are feasible solution to (10).

Now, a multi-objective linear programming (MOLP) model is developed to determine a production plan. To this end, we solve the following MOLP model:

$$
\begin{aligned}
&\text{Min} \quad \textstyle\sum_{j=1}^{n}[s_j{}^{+} + s_j{}^{-}]\\
&\text{Min} \quad \textstyle\sum_{j=1}^{n}[b_j{}^{+} + b_j{}^{-}]\\
&s.t.\\
&\textstyle\sum_{r=1}^{s} u_r y_{rj} + \sum_{r=1}^{s} \bar{d}_{rj} \geq e_j,\ j = 1,\ \ldots,\ n,\\
&\textstyle\sum_{r=1}^{s} u_r y_{rj} + \sum_{r=1}^{s} \bar{d}_{rj} - 1 = s_j{}^{+} - s_j{}^{-},\ j = 1,\ \ldots,\ n,\\
&\bar{d}_{rj} - \alpha_j u_r D_r = b_j{}^{+} - b_j{}^{-},\ j = 1,\ \ldots,\ n,\\
&\textstyle\sum_{j=1}^{n} \bar{d}_{rj} = u_r D_r,\ r = 1,\ \ldots,\ s,\\
&b_j{}^{+},\ b_j{}^{-},\ s_j{}^{+},\ s_j{}^{-},\ u_r \geq 0,\ \text{for all } r \text{ and } j,\\
&\bar{d}_{rj} \geq 0 \quad \text{when} \quad D_r \geq 0,\\
&\bar{d}_{rj} \leq 0 \quad \text{when} \quad D_r \leq 0.
\end{aligned}
\tag{4.4}
$$

Minimizing the sum of $b_j{}^{+}$ and $b_j{}^{-}$ means that we minimize the deviation of $\bar{d}_{rj}$ and $\alpha_j u_r D_r$. Simultaneously, minimizing $\sum_{j=1}^{n}[s_j{}^{+}+s_j{}^{-}]$ means that we minimize the deviation between $\sum_{r=1}^{s} u_r y_{rj}+\sum_{r=1}^{s} \bar{d}_{rj}$ and the benchmark level one. Viewing the first minimizing objective prior to the second one in the MOLP model (11), we solve the following linear programming problem:

$$
\begin{aligned}
&\text{Min} \quad \epsilon_1 \textstyle\sum_{j=1}^{n}[s_j{}^{+} + s_j{}^{-}] + \epsilon_2 \sum_{j=1}^{n}[b_j{}^{+} + b_j{}^{-}]\\
&s.t.\\
&\textstyle\sum_{r=1}^{s} u_r y_{rj} + \sum_{r=1}^{s} \bar{d}_{rj} \geq e_j,\ j = 1,\ \ldots,\ n,\\
&\textstyle\sum_{r=1}^{s} u_r y_{rj} + \sum_{r=1}^{s} \bar{d}_{rj} - 1 = s_j{}^{+} - s_j{}^{-},\ j = 1,\ \ldots,\ n,\\
&\bar{d}_{rj} - \alpha_j u_r D_r = b_j{}^{+} - b_j{}^{-},\ j = 1,\ \ldots,\ n,\\
&\textstyle\sum_{j=1}^{n} \bar{d}_{rj} = u_r D_r,\ r = 1,\ \ldots,\ s,\\
&b_j{}^{+},\ b_j{}^{-},\ s_j{}^{+},\ s_j{}^{-},\ u_r \geq 0,\ \text{for all } r \text{ and } j,\\
&\bar{d}_{rj} \geq 0 \quad \text{when} \quad D_r \geq 0,\\
&\bar{d}_{rj} \leq 0 \quad \text{when} \quad D_r \leq 0,\\
&(\bar{d}_{1j},\ \bar{d}_{2j},\ \ldots,\ \bar{d}_{sj}) \in \Omega_j,\ j = 1,\ \ldots,\ n,
\end{aligned}
\tag{4.5}
$$

where ϵ_1 and ϵ_2 are user-defined values that reflect the importance of the objectives and represent positive values with $\epsilon_1 + \epsilon_2 = 1$. The set Ω_j is assurance region defined by any user-defined restrictions imposed on the variable $\bar{d}_{rj}$.

Theorem 4.2. *Planned by planning model (12), the new efficiency scores of all DMUs are not decreased.*

Proof. Let $e_o{}^*$ denote current efficiency score of DMU_o. Consider the assessment of DMU_o in the next season as follows:

$$\begin{aligned} &\theta_o = \text{Min } \rho \\ &s.t. \\ &\quad \textstyle\sum_{r=1}^{s} u_r(y_{rj} + d_{rj}{}^*) \le \rho, \ j = 1, \ \ldots, \ n, \\ &\quad \textstyle\sum_{r=1}^{s} u_r(y_{ro} + d_{ro}{}^*) \ge 1, \\ &\quad u_r \ge 0, \ r = 1, \ \ldots, \ s, \end{aligned} \tag{4.6}$$

in which $d_{rj}{}^*$ are optimal production plan for DMU_o obtained from model (12). Suppose in contrary that $\sum_{r=1}^{s} u_r{}^*(y_{ro} + d_{ro}{}^*) > e_o{}^* \ge 1$ (in which $u_r{}^*$ are the optimal weights obtained from model (5)). So, there exists $\epsilon > 0$ such that

$$\sum_{r=1}^{s} u_r{}^*(y_{ro} + d_{ro}{}^*) - \epsilon \ge e_o{}^*.$$

Now

$$\begin{aligned} &u_r{}^* : r = 1, \ \ldots, \ s, \\ &\bar{d}_{rj} = d_{rj}{}^* : r = 1, \ \ldots, \ s, \ j \ne o, \\ &\bar{d}_{ro} = d_{ro}{}^* + \epsilon : r = 1, \ \ldots, \ s \end{aligned}$$

is a feasible solution to (12), and

$$\sum_{r=1}^{s} u_r{}^*(y_{ro} + d_{ro}{}^*) < \sum_{r=1}^{s} u_r{}^*(y_{ro} + \bar{d}_{ro})$$

This is a contradiction and this completes the proof.

In the next section an application of the proposed approach is given on university branches.

5. An application

This section illustrates the production planning approach discussed in this paper by applying it to a real world data of 11 universities in Iran. Islamic Azad University (IAU) of Iran has 17 regions in 29 provinces and in each region a number of university branches are working under the supervision of central organization in Tehran. These universities do business separately in their regions. Although the universities fall under the supervision of central organization of IAU, they don't receive any governmental or nongovernmental subsidies.

The main functions of IAU are education and research. From looking at the operation contents of the university branches, we realize that the usage of inputs is not the major consideration of the central organization. Emphasis of the central

Table 2. Data for IAU branches.

DMU_j	y_1	y_2	y_3	y_4	Original efficiency
#1	1.0000	0.9919	1.0000	0.9504	1.0000
#2	0.9286	0.9352	0.2000	0.8727	1.0634
#3	0.3961	0.4534	0.0300	0.2562	2.2395
#4	0.2468	0.3927	0.1500	0.2273	2.5877
#5	0.3701	0.4251	0.0300	0.3554	2.3898
#6	0.461	0.7368	0.3100	0.7678	1.3024
#7	0.7662	0.9190	0.7000	0.8430	1.0923
#8	0.9545	1.0000	0.7600	1.0000	1.0000
#9	0.8831	1.0162	0.5200	0.9587	1.0000
#10	0.6558	0.8947	0.2100	0.8554	1.1317
#11	0.4351	0.4534	0.1100	0.5372	1.8615

management is to increase the educational and scientific research services. Therefore, we concentrate on the outputs produced by the universities. Regarding the outputs in assessing university branches, we have considered four output variables in our analysis that includes: assessment score (y_1), scientific publications (y_2), external research funding obtained (y_3) and the number of students (y_4). In what follows, we give a brief discussion to these variables.

Each year, assessment teams are sent to the universities and the results of these assessments are a score to each university. We have used this score in our analysis as the first output. Scientific publications include domestic and international papers and published books. This variable is the second output in this application. External research funding obtained is considered as the third output. Finally, the number of graduate and undergraduate students is considered as the fourth output.

Because of some limitations, the data are normalized by dividing into the maximum number of each output. Table 2 contains a list of the full data. The LP model (5) is used to determine the efficiency of the universities in the current season. The original efficiency scores of all universities are listed in the last column of Table 2. As the table indicates, three universities, #1, #8 and #9 are efficient.

The central organization forecasts the demand changes for assessment score, scientific publications, external research funding obtained and number of students as $D_1 = 1.3,\ D_2 = 1.1,\ D_3 = 1.4$ and $D_4 = 1.2$, respectively.

In this application, it is assumed that the importance of the output levels and efficiency conservation are equal, therefore, we let $\epsilon_1 = \epsilon_2 = 0.5$. In applying the model described herein, ratio constraints of the form $\dot{\beta}_r \leq \bar{d}_{rj} \leq \ddot{\beta}_r$ on the variables $\bar{d}_{rj}$ are imposed. By applying the proposed model (12), new plans for all 11 universities are listed in Table 3. The last column shows the new efficiency scores. As can be seen, the number of efficient universities increases from three to four. What we find in this application is that the proposed approach leads to efficiency score greater than the original efficiency of each university. The optimal weights obtained from model (12) are listed at the bottom of Table 3.

TABLE 3. New plans and efficiency scores.

DMU_j	y_1	y_2	y_3	y_4	s_j^-	s_j^+	b_j^-	b_j^+	New efficiency
#1	0.0310	0.0128	0.0401	0.0219	0	0	0	0	1.0000
#2	0.1043	0.0861	0.1134	0.0952	0	0	0.0733	0	1.0000
#3	0.1989	0.1807	0.2080	0.1898	0	0	0.1679	0	1.6601
#4	0.2026	0.1844	0.2117	0.1935	0	0	0.1716	0	1.8427
#5	0.1229	0.1047	0.1320	0.1138	0	0.2945	0.0919	0	1.9935
#6	0.1379	0.1197	0.1470	0.1288	0	0	0.1069	0	1.1611
#7	0.0776	0.0594	0.0867	0.0685	0	0	0.0466	0	1.0600
#8	0.0501	0.0319	0.0592	0.0410	0	0	0.0191	0	1.0000
#9	0.0752	0.0570	0.0843	0.0661	0	0	0.0442	0	1.0000
#10	0.1236	0.1054	0.1326	0.1145	0	0	0.0926	0	1.0614
#11	0.1760	0.1578	0.1850	0.1669	0	0	0.1450	0	1.4785

$u_1 = 0.2621,\ u_2 = 0.1279,\ u_3 = 0.3148,\ u_4 = 0.2006.$

The interpretation of our model can be illustrated by considering a specific university, say University #2. The original efficiency of this university is 1.0634 and with new outputs in the next year, new efficiency score will increase to 1.0000. Moreover, all output data to this university should increase in the next season.

We used GAMS software on a machine with the following specifications: CPU: Intel Pentium 4 at 2GHz, RAM: 512 MB.

6. Conclusion

In real applications of DEA, we confront systems in which the input consumption is not important and regardless of how many inputs are consumed, the focus is on the output productions. This paper is concerned with the production planning problem in a centralized decision making environment in which all operational units fall under the supervision of a central decision maker. We assume that the central decision maker has the power to forecast the demand changes for outputs in the next seasons. The proposed approach in this paper solves a linear programming problem and takes the efficiency of the units into consideration so that the planned production for each unit becomes proportionate to the ability of the units.

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