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FLEXIBLE MEASURES IN PRODUCTION PROCESS: A DEA-BASED APPROACH

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Abstract. Data envelopment analysis (DEA) has been proven as an excellent data-oriented efficiency analysis method for comparing decision making units (*DMUs*) with multiple inputs and multiple outputs. In conventional DEA, it is assumed that the status of each measure is clearly known as either input or output. However, in some situations, a performance measure can play input role for some *DMUs* and output role for others. Cook and Zhu [*Eur. J. Oper. Res.* **180** (2007) 692–699] referred to these variables as flexible measures. The paper proposes an alternative model in which each flexible measure is treated as either input or output variable to maximize the technical efficiency of the *DMU* under evaluation. The main focus of this paper is on the impact that the flexible measures has on the definition of the PPS and the assessment of technical efficiency. An example in UK higher education intuitions shows applicability of the proposed approach.

Keywords. Data envelopment analysis, efficiency, flexible measure.

Mathematics Subject Classification. 90B030, 90B060.

1. Introduction

Data envelopment analysis (DEA) is concerned with comparative assessment of the efficiency of decision making units (*DMUs*). In the classical DEA models, the efficiency of a *DMU* is obtained by maximizing ratio of the weighted sum of its

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outputs to the weighted sum of its inputs, subject to the condition that this ratio does not exceed one for any *DMU*. Since the pioneering work of Charnes *et al.* [4], DEA has demonstrated to be an effective technique for measuring the relative efficiency of a set of homogeneous *DMUs* which utilize the same inputs to produce the same outputs. DEA has been used in many contexts including education systems, health care units, agricultural productions, military logistics and many other applications (see Charnes *et al.* [5], Cooper *et al.* [10] and Emrouznejad *et al.* [11]). In conventional DEA applications, given a set of available measures, it is assumed that the status of each measure is clearly stated as an input or an output variable in the production process prior to using DEA. For example, in a conventional study of bank branch activities, one can clearly specify that staff is an input, and profit earned is an output. However, in higher education application, for instance, there is always a question that in assessing universities activities, whether a *research income* is an input or an output? In the literature, many authors suggested that it should be considered as an input, because it is the money earned by university and it is used in the same period. Others said that it is the income that universities earned so it should be considered as an output. However in the interest of gaining a higher efficiency score some universities may keep the *research income* in the input side and some other universities in the output side. The main question is "how to decide about the role of *research income* for each university?".

Cook and Bala [6] and Cook and Zhu [8] have pointed a very similar question in the context of DEA. Cook and Bala [6] examined the problem of deciding the appropriate status of flexible measures when additional information is present. Specifically, they investigate the situation where bank branch consultants provide additional "classification data" specifying quality of each branch; good versus poor branch. Their idea was to assign a status to each flexible measure such as to provide efficiency scores that are in best agreement with expert opinion. Cook and Zhu [8] considered variables whose status are flexible and proposed a different method for classifying these variables by introducing a fractional programming problem to accommodate flexible measures. Their model is later transformed to a linear form using the Charnes and Cooper [3] transformation procedure (see also [12]).

One main drawback in the model proposed by Cook and Bala [6] is the requirement to enter extra information to decide about the status of each variable and the main weakness in the model proposed by Cook and Zhu [8] is that their model overestimates the efficiency. The current paper, suggests a different approach in which each flexible measure of the model is treated as either input or output to maximize the technical efficiency of the *DMU* under evaluation. The main focus of this paper is on the impact that the flexible measures have on the definition of the PPS and the assessment of technical efficiency.

The paper is organized as follows: Section 2 provides the basic DEA model. The third section introduces a DEA-based approach for modeling production processes in the presence of flexible measures. A comparison with the Cook and Zhu [8] model is given in Section 4. Section 5 illustrates the usefulness and applicability of the proposed model in assessing higher education institutions in UK. Conclusions and further remarks are given in Section 5.

2. DEA efficiency analysis

To describe the DEA efficiency measurement, assume there are n *DMUs* and the performance of each DMU is characterized by a production process of m inputs (x_{ij}: $i = 1, \ldots, m$) to yields s outputs (y_{rj}: $r = 1, \ldots, s$). The ratio DEA model also known as the CCR model, measures the efficiency of DMU_o by maximizing the ratio of its weighted sum of outputs to its weighted sum of inputs, *i.e.*

$$\theta_o = \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}}$$

where the maximum is sought subject to the conditions that this ratio does not exceed one for any DMU_j and all the input and output weights are positive. Hence the following fractional model should be solved to obtain the efficiency score of DMU_o (Charnes *et al.* [4]):

$$\begin{aligned} &\text{Max } \theta_o = \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}} \\ &s.t. \ \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leqslant 1, \qquad j = 1, \ldots, n, \\ &\quad u_r, v_i \geqslant \varepsilon, \quad \text{for all } r,\ i, \end{aligned} \tag{2.1}$$

where $\varepsilon > 0$ is a non-archimedean constraint. This linear fractional programming problem can be reduced to a linear programming model (2.2) by using Charnes and Cooper [3] transformation.

$$\begin{aligned} &\text{Max } \theta_o = \sum_{r=1}^{s} \bar{u}_r y_{ro} \\ &s.t. \ \sum_{i=1}^{m} \bar{v}_i x_{io} = 1, \\ &\quad \sum_{r=1}^{s} \bar{u}_r y_{rj} - \sum_{i=1}^{m} \bar{v}_i x_{ij} \leqslant 0, \quad j = 1, \ldots, n, \\ &\quad \bar{u}_r, \bar{v}_i \geqslant \bar{\varepsilon}, \quad \text{for all } r,\ i. \end{aligned} \tag{2.2}$$

This model is a constant returns to scale (CRS) program and it assumes that the status of all input/output variables are known prior to solving the model. The efficiency ratio θ_o ranges between zero and one, with DMU_o being considered relatively efficient if it receives a score of one. From a managerial perspective, this model delivers assessments and targets with an output maximization orientation.

3. Flexible measures in production process

3.1. An axiomatic foundation

Suppose we have $nDMUs$, and that each DMU_j: $j = 1, \ldots, n$ uses m inputs x_{ij}: $i = 1, \ldots, m$ to produce s outputs y_{rj}: $r = 1, \ldots, s$. Suppose also that there exist t flexible measures z_{kj}: $k = 1, \ldots, t$, whose input/output statuses are not known, some $DMUs$ may use these measures, or some of these measures, as inputs and other $DMUs$ may use them as outputs. The unit under evaluation is denoted by $DMU_o : (x_o\ ,\ y_o)$.

Without loss of generality assume there are only three variables; x, y and z; in the assessment model. Let T be the production possibility set of technology under consideration. We postulate the following:

A1 *Feasibility of observed data.* $(x_j,\ y_j,\ z_j) \in T$ for any $j = 1,\ 2,\ \ldots,\ n$.
A2 *Unbounded ray.* $(x,\ y,\ z) \in T$ implies $\alpha\ (x,\ y,\ z) \in T$ for any $\alpha \geqslant 0$.
A3 *Convexity.* Let $(x',\ y',\ z') \in T$ and $(x'',\ y'',\ z'') \in T$. Then, for any $\lambda \in [\ 0\ ,\ 1\ ]$ the unit $\lambda(x',\ y',\ z') + (1-\lambda)(x'',\ y'',\ z'') \in T$.
A4 *Free disposability.* $(x,\ y,\ z) \in T$, $x' \geqslant x$ and $y' \leqslant y$, and (either $z' \geqslant z$ *or* $z' \leqslant z$), implies $(x',\ y',\ z') \in T$.
A5 *Minimal extrapolation.* For each T satisfying in the axioms A1–A4, we have $T \subseteq T'$.

It is noted that A5 is different with standard axioms presented in Banker *et al.* [1]. Now, an Algebraic representation of the PPS of the technology T, which satisfying the axioms A1–A5, is given.

Theorem 3.1. *The PPS T, which satisfies the axioms* A1−A5*, is defined as*

$$T = \left\{ (x, y, z) : x \geqslant \sum_{j=1}^{n} \lambda_j x_j, y \leqslant \sum_{j=1}^{n} \lambda_j y_j, \left(either\ z \leqslant \sum_{j=1}^{n} \lambda_j z_j\ or\ z \geqslant \sum_{j=1}^{n} \lambda_j z_j \right), \lambda_j \geqslant 0, \quad j = 1, 2, \ldots, n \right\}.$$

Proof. It is obvious that the set T satisfies A1–A4. To see that T is the minimal set, assume that T' also satisfies A1–A4. We need to show that $(x,\ y,\ z) \in T$ implies that $(x,\ y,\ z) \in T'$. Consider the following representation of the unit $(x,\ y,\ z)$.

$$x \geqslant \sum_{j=1}^{n} \lambda_j x_j$$

$$y \leqslant \sum_{j=1}^{n} \lambda_j y_j$$

$$\text{either} \quad z \leqslant \sum_{j=1}^{n} \lambda_j z_j$$

$$\text{or} \quad z \geqslant \sum_{j=1}^{n} \lambda_j z_j.$$

For the vector $\lambda = (\lambda_1,\ \lambda_2,\ \ldots,\ \lambda_n)$ from this representation, define

$$(x_\lambda, y_\lambda, z_\lambda) = \left(\sum_{j=1}^{n} \lambda_j x_j, \sum_{j=1}^{n} \lambda_j y_j, \sum_{j=1}^{n} \lambda_j z_j \right).$$

It is clear that $(x_\lambda,\ y_\lambda,\ z_\lambda) \in T'$, and this unit dominates $(x,\ y,\ z)$ in the Pareto sense. So, we conclude that $(x,\ y,\ z) \in T'$. This completes the proof. □

3.2. A DEA model with flexible measure: an input oriented approach

Based on the proposed PPS and as results of Theorem 3.1 we propose the following model for measuring efficiency of DMU_o. This is an input oriented model, each DMU will set the status of variable z on the interest of their efficiency level.

$$\text{Min } \theta$$

$$s.t.$$

$$\sum_{j=1}^{n} \lambda_j x_j \leqslant \theta\ x_o, \tag{3.1}$$

$$\sum_{j=1}^{n} \lambda_j y_j \ \geqslant\ y_o, \tag{3.2}$$

$$\begin{cases} \text{either} \\ \displaystyle\sum_{j=1}^{n} \lambda_j z_j \leqslant \theta z_o, & (3.3) \\ \text{or} \\ \displaystyle\sum_{j=1}^{n} \lambda_j z_j \geqslant z_o, & (3.4) \end{cases}$$

$$\lambda_j \geqslant 0, \quad j = 1,\ 2,\ \ldots,\ n.$$

Obviously this is not a straightforward linear programming. Hence the following procedure explains the transformation of the above model to a mixed integer linear program.

It is to be noted that one and only one of the constraints (3.3) or (3.4) must be hold for variable z. Suppose M be a large positive number. Consider the following constraints:

$$\sum_{j=1}^{n} \lambda_j z_j \leqslant \theta z_o + M\delta_1 \tag{3.5}$$

$$-\sum_{j=1}^{n} \lambda_j z_j \leqslant -z_o + M\delta_2 \tag{3.6}$$

$$\delta_1 + \delta_2 = 1, \tag{3.7}$$

$$\delta_1\ ,\ \delta_2 \in \{\ 0\ ,\ 1\ \}. \tag{3.8}$$

Clearly selecting $\delta_1 = 0$ forces $\delta_2 = 1$, hence, the constrain (3.6) is redundant and (3.5) holds. This means that z_o is selected as an input for DMU_o. On the other hand, if we let $\delta_1 = 1$, then $\delta_2 = 0$, hence (3.5) is redundant and (3.6) holds. In this case, z_o is selected as an output for DMU_o. Model (3.1)–(3.4) can now be restated in the following mixed-integer linear program:

$$\begin{aligned}
&\text{Min } \theta \\
&s.t. \\
&\qquad \sum_{j=1}^{n} \lambda_j x_j \leqslant \theta x_o \\
&\qquad \sum_{j=1}^{n} \lambda_j y_j \geqslant y_o \\
&\qquad \sum_{j=1}^{n} \lambda_j z_j \leqslant \theta z_o + M\delta_1 \\
&\qquad -\sum_{j=1}^{n} \lambda_j z_j \leqslant -z_o + M\delta_2 \\
&\qquad \delta_1 + \delta_2 = 1 \\
&\qquad \delta_1\,,\, \delta_2 \in \{\,0\,,\,1\,\} \\
&\qquad \lambda_j \geqslant 0, \quad j = 1,\ 2,\ \ldots,\ n.
\end{aligned} \tag{3.9}$$

3.3. Generalization of the proposed model

In model (3.9) we assumed there are only 3 variables; x, y and z; where the status of variable z to be determined by DEA model. Consider now that there are multi-inputs, x_{ij}: $i = 1, \ldots, m$, multi-outputs, y_{rj}: $r = 1, \ldots, s$, and multi-flexible measures, z_{kj}: $k = 1, \ldots, t$. To generalize the proposed model we should allow each DMU to select the status of each flexible measure, so each DMU sets some of the flexible measures to input variables and some others to output variables to secure the best possible efficiency score. In this case model (3.10) is proposed.

$$\begin{aligned}
&\text{Min } \theta \\
&s.t. \\
&\qquad \sum_{j=1}^{n} \lambda_j x_{ij} \leqslant \theta x_{io}, && i = 1,\ 2,\ \ldots,\ m, \\
&\qquad \sum_{j=1}^{n} \lambda_j y_{rj} \geqslant y_{ro}, && r = 1,\ 2,\ \ldots,\ s, \\
&\qquad \sum_{j=1}^{n} \lambda_j z_{kj} \leqslant \theta z_{ko} + M\delta_{1k}, && k = 1,\ 2,\ \ldots,\ t, \\
&\qquad -\sum_{j=1}^{n} \lambda_j z_{kj} \leqslant -z_{ko} + M\delta_{2k}, && k = 1,\ 2,\ \ldots,\ t, \\
&\qquad \delta_{1k} + \delta_{2k} = 1, && k = 1,\ 2,\ \ldots,\ t, \\
&\qquad \delta_{1k}\,,\, \delta_{2k} \in \{\,0\,,\,1\,\}, && k = 1,\ 2,\ \ldots,\ t, \\
&\qquad \lambda_j \geqslant 0, \quad j = 1,\ 2,\ \ldots,\ n.
\end{aligned} \tag{3.10}$$

The above model is an input oriented measure. A similar model, see model (3.11), can be formulated to present output orientation DEA with flexible measures.

$$
\begin{aligned}
&\text{Max } \phi \\
&s.t. \\
&\quad \sum_{j=1}^{n} \lambda_j x_{ij} \leqslant x_{io}, && i = 1, 2, \ldots, m, \\
&\quad \sum_{j=1}^{n} \lambda_j y_{rj} \geqslant \phi y_{ro}, && r = 1, 2, \ldots, s, \\
&\quad \sum_{j=1}^{n} \lambda_j z_{kj} \leqslant z_{ko} + M\delta_{1k}, && k = 1, 2, \ldots, t, \\
&\quad \sum_{j=1}^{n} \lambda_j z_{kj} \leqslant -\phi z_{ko} + M\delta_{2k}, && k = 1, 2, \ldots, t, \\
&\quad \delta_{1k} + \delta_{2k} = 1, \\
&\quad \delta_{1k}, \delta_{2k} \in \{0, 1\}, && k = 1, 2, \ldots, t, \\
&\quad \lambda_j \geqslant 0, \quad j = 1, 2, \ldots, n.
\end{aligned}
\tag{3.11}
$$

One important conclusion is that unlike standard DEA, even under constant returns to scale, the input-oriented model (3.10) and the output oriented model (3.11) may produce different efficiency scores. This is obviously expected under flexible measures since one *DMU* may consider one of its flexible measures as an input variable in one model but an output variable in the other model.

4. Comparison with the Cook and Zhu [8] model

In this section, we compare the model developed in this paper with the following model proposed by Cook and Zhu [8].

$$
\begin{aligned}
&\text{Max } \bar{e}_o = \sum_{r=1}^{s} \mu_r y_{ro} + \sum_{k=1}^{t} \delta_k z_{ko} \\
&s.t. \sum_{i=1}^{m} v_i x_{io} + \sum_{k=1}^{t} \gamma_k z_{ko} - \sum_{k=1}^{t} \delta_k z_{ko} = 1, \\
&\quad \sum_{r=1}^{s} \mu_r y_{rj} + 2\sum_{k=1}^{t} \delta_k z_{kj} - \sum_{i=1}^{m} v_i x_{ij} - \sum_{k=1}^{t} \gamma_k z_{kj} \leqslant 0,\ j = 1, 2, \ldots, n, \\
&\quad 0 \leqslant \delta_k \leqslant M d_k, \quad k = 1, 2, \ldots, t, \\
&\quad \delta_k \leqslant \gamma_k \leqslant \delta_k + M(1 - d_k), \quad k = 1, 2, \ldots, t, \\
&\quad \mu_r, v_i, \gamma_k, \delta_k \geqslant 0, \quad \text{for all } i, r, k, \\
&\quad d_k \in \{0, 1\}, \quad k = 1, 2, \ldots, t.
\end{aligned}
\tag{4.1}
$$

This is a mixed integer linear programming in which DMU_j produces s different outputs y_{rj}: $r = 1, \ldots, s$ using m different inputs x_{ij}: $i = 1, \ldots, m$. Also, the values assumed by flexible measures are denoted asz_{kj}: $k = 1, \ldots, t$.

One main drawback from Cook and Zhu [8] model is that it is a very optimistic model and the following two theorems show that model (4.1) is always overestimating the efficiency.

First consider the following two models. Model (4.2) is a CCR model assuming all flexible measures as input variables and model (4.3) is a CCR model assuming all flexible measure as output variables.

$$\begin{aligned}
&\text{Max } \breve{e}_o = \sum_{r=1}^{s} \mu_r y_{ro} \\
&\quad s.t. \sum_{i=1}^{m} v_i x_{io} + \sum_{k=1}^{t} \gamma_k z_{ko} = 1, \\
&\qquad \sum_{r=1}^{s} \mu_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} - \sum_{k=1}^{t} \gamma_k z_{kj} \leqslant 0, \quad j = 1, 2, \ldots, n, \\
&\qquad \mu_r, v_i, \gamma_k \geqslant 0, \quad \text{for all } i, r, k.
\end{aligned} \tag{4.2}$$

$$\begin{aligned}
&\text{Max } \widehat{e}_o = \sum_{r=1}^{s} \mu_r y_{ro} + \sum_{k=1}^{t} \delta_k z_{ko} \\
&\quad s.t. \sum_{i=1}^{m} v_i x_{io} = 1, \\
&\qquad \sum_{r=1}^{s} \mu_r y_{rj} + \sum_{k=1}^{t} \delta_k z_{kj} - \sum_{i=1}^{m} v_i x_{ij} \leqslant 0, \quad j = 1, 2, \ldots, n, \\
&\qquad \mu_r, v_i, \delta_k \geqslant 0, \quad \text{for all } i, r, k.
\end{aligned} \tag{4.3}$$

Theorem 4.1. *The optimal objective value to* (4.2) *and* (4.3) *are not greater than the optimal objective value to* (4.1).

Proof. We first show that $\breve{e}_o \leqslant \bar{e}_o$. Let (μ^*, v^*, γ^*) be an optimal solution to (4.2). It is easy to see that ($\mu^*, v^*, \gamma^*, \delta = 0, d = 0$)is a feasible solution to (4.1), hence, we must have $\breve{e}_o \leqslant \bar{e}_o$. Similarly, we can show that $\widehat{e}_o \leqslant \bar{e}_o$, in which $\widehat{e}_o$ is the optimal objective value to (4.3). □

Theorem 4.2. *The optimal objective value to* (3.10) *is not greater than the optimal objective value to* (4.1).

Proof. Consider the dual formulation of the continuous relaxation of (3.10) as follows:

$$\begin{aligned}
&\text{Max } e_o^* = \sum_{r=1}^{s} \mu_r y_{ro} + \sum_{k=1}^{t} \rho_k z_{ko} + \sum_{k=1}^{t} \pi_k \\
&\quad s.t. \sum_{i=1}^{m} v_i x_{io} + \sum_{k=1}^{t} \gamma_k z_{ko} = 1, \\
&\qquad \sum_{r=1}^{s} \mu_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} - \sum_{k=1}^{t} \gamma_k z_{kj} + \sum_{k=1}^{t} \rho_k z_{kj} \leqslant 0, \ j = 1, 2, \ldots, n, \\
&\qquad \pi_k + M\gamma_k \leqslant 0, \quad k = 1, 2, \ldots, t, \\
&\qquad \pi_k + M\rho_k \leqslant 0, \quad k = 1, 2, \ldots, t, \\
&\qquad \mu_r, v_i, \gamma_k, \rho_k \geqslant 0, \quad \text{for all } i, r, k.
\end{aligned} \tag{4.4}$$

It suffices to show that $e_o^* \leqslant \bar{e}_o$, in which $\bar{e}_o$ and e_o^* are the optimal objective values to (4.1) and (4.4), respectively. Let $(\mu^*,\ \rho^*,\ v^*,\ \gamma^*,\ \pi^*)$ be an optimal solution to (4.4). It is easy to see that $(\bar{\mu} = \mu^*,\ \bar{\rho} = \rho^*,\ \bar{v} = v^*,\ \bar{\gamma} = \gamma^* - \rho^*)$ is a feasible solution to (4.1), hence, we have

$$\begin{aligned} e_o^* = & \sum_{r=1}^{s} \mu_r^* y_{ro} + \sum_{k=1}^{t} \rho_k^* z_{ko} + \sum_{k=1}^{t} \pi_k^* \leqslant \sum_{r=1}^{s} \mu_r^* y_{ro} + \sum_{k=1}^{t} \rho_k^* z_{ko} \\ = & \sum_{r=1}^{s} \bar{\mu}_r y_{ro} + \sum_{k=1}^{t} \bar{\rho}_k z_{ko} \leqslant \bar{e}_o. \end{aligned}$$

This completes the proof. □

5. Illustration of the proposed model: an application in UK higher education institutions

This section illustrates the proposed model in assessing UK higher education institutions. We apply our model to the data used by Cook and Zhu [8] (see also Beasley [2], Cook and Zhu [7] and Cook *et al.* [9]). Two factors are selected as inputs: general expenditure (x_1) and equipment expenditure (x_2), and three factors as outputs: undergraduate students (y_1), postgraduate research (y_2) and postgraduate teaching (y_3). The flexible measure here is the research income (z_1). The status of research income for each university will be determined by the model. The data set consists of 50 universities as shown in appendix.

The results of model (3.10) are reported under heading "efficiency with flexible variable" in Table 1. The optimal values to δ_1 and δ_2 indicate that either research income was considered as an input or as an output variable in the assessment model. As can be seen 25 (out of the 50) universities treat the research income as an output measure, and 25 universities treat it as an input measure in the model.

Cook and Zhu identified 21 efficient universities, our model found that there are only 7 efficient universities, over all we think that the Cook and Zhu model overestimated the efficiency across all other universities. Theoretically this was proved in the previous section. Table 1 clearly shows that the Cook and Zhu [8] efficiency is always higher than the efficiency obtained as compare to our model. Further, we investigated the efficiency of higher education institution in two cases when we used simple CCR model one with research income as an input and another model with research income as an output. These are also reported in Table 1. As an example take university 12, where our efficiency is 58%, the two efficiency when considering research income as input and as an output are 67% and 58% while the cook and Zhu efficiency is 71%, it is well overestimated, it can be seen that it is outside the interval [58%, 67%]. As another example take university 26, with efficiency of 43% and 57%, respectively, when we use CCR model and consider research income as an output or as an input, respectively. We doubt that this university, which is within lowest efficient universities, become efficient, as calculated in Cook and Zhu, just by adjusting its research income. It is very clear

Table 1. Efficiency assessment of higher education institutions, a comparative study.

DMU	Efficiency, research income as input variable	Efficiency, research income as output variable	Efficiency, research income as flexible variable	δ_1	δ_2	Cook and Zhu (2007)	d
1	1	1	1	1	0	1	1
2	0.64	0.61	0.61	0	1	1	0
3	0.66	0.84	0.66	1	0	0.84	0
4	0.69	0.65	0.65	0	1	0.69	1
5	0.89	1	0.89	1	0	1	0
6	1	0.79	0.79	0	1	1	0
7	1	1	1	1	0	1	1
8	0.81	0.75	0.75	0	1	0.81	1
9	0.53	1	0.53	1	0	1	0
10	0.91	0.89	0.89	0	1	0.91	1
11	0.75	0.89	0.75	1	0	0.89	0
12	0.67	0.58	0.58	0	1	0.71	1
13	0.77	0.80	0.77	1	0	0.80	0
14	0.70	0.75	0.70	1	0	0.77	0
15	0.69	0.70	0.69	1	0	0.70	0
16	0.52	0.53	0.52	1	0	0.54	0
17	0.82	0.52	0.52	0	1	0.82	1
18	0.63	0.59	0.59	0	1	0.63	1
19	1	1	1	1	0	1	1
20	0.87	0.73	0.73	0	1	1	0
21	0.62	0.59	0.59	0	1	0.70	0
22	0.72	0.66	0.66	0	1	0.72	1
23	0.55	0.60	0.55	1	0	0.62	0
24	1	0.46	0.46	0	1	1	0
25	1	0.95	0.95	0	1	1	1
26	0.57	0.43	0.43	0	1	1	0
27	0.78	0.70	0.70	0	1	0.86	1
28	0.81	1	0.81	1	0	1	0
29	0.83	0.78	0.78	0	1	0.83	1
30	0.89	0.63	0.63	0	1	1	0
31	0.78	0.73	0.73	0	1	0.78	1
32	0.84	0.89	0.84	1	0	0.90	0
33	1	1	1	1	0	1	1
34	1	0.93	0.93	0	1	1	0
35	1	1	1	1	0	1	1
36	0.73	0.81	0.73	1	0	0.84	0
37	0.83	0.78	0.78	0	1	0.83	1
38	0.81	0.83	0.81	1	0	0.83	0
39	0.67	0.62	0.62	0	1	0.79	0
40	0.74	0.74	0.74	0	1	0.74	1
41	1	1	1	1	0	1	1
42	0.82	0.80	0.80	0	1	0.85	0
43	0.64	0.92	0.64	1	0	0.92	0
44	1	1	1	1	0	1	0
45	0.89	0.89	0.88	0	1	1	0
46	0.85	0.85	0.85	0	1	1	0
47	0.69	0.65	0.69	1	0	0.69	1
48	0.79	0.84	0.79	1	0	0.94	0
49	0.47	0.80	0.47	1	0	1	0
50	0.84	0.84	0.84	1	0	0.84	0

that Cook and Zhu estimation is not correct, in fact that it might be due to nature of the model they proposed.

6. Conclusion and further remarks

In this paper we developed a DEA model to calculate the technical efficiency of *DMUs* with flexible measures. For these types of production systems, the conventional DEA model is modified to incorporate flexible measures. The proposed approach is potentially useful in many applications including efficiency assessment of manufacturing, health care systems, educational authorities where generally a variable could be considered as an input or as an output. The paper started with an axiomatic study of the proposed model. Finally an application in higher education used to show the usefulness of the model. Further research can be done to transform other DEA models, *e.g.* slack-based model, using similar concept.

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Appendix. Data set for higher education institutions

DMU	G. expenditure	E. expenditure	UG students	PG research	PG teaching	Research income
1	528	64	145	26	0	254
2	2605	301	381	70	16	1485
3	304	23	44	6	3	45
4	1620	485	287	48	0	940
5	490	90	91	30	8	106
6	2675	767	352	170	4	2967
7	422	0	70	31	12	298
8	986	126	203	32	0	776
9	523	32	60	17	0	39
10	585	87	80	44	17	353
11	931	161	191	20	0	293
12	1060	91	139	37	0	781
13	500	109	104	19	0	215
14	714	77	132	24	0	269
15	923	121	135	41	10	392
16	1267	128	169	31	0	546
17	891	116	125	24	0	925
18	1395	571	176	41	14	764
19	990	83	28	93	36	615
20	3512	267	511	176	23	3182
21	1451	226	198	53	0	791
22	1018	81	161	34	5	741
23	1115	450	148	36	4	347
24	2055	112	207	48	1	2945
25	440	74	115	9	0	453
26	3897	841	353	93	28	2331
27	836	81	129	37	0	695
28	1007	50	174	30	7	98
29	1188	170	253	38	0	879
30	4630	628	544	217	0	4838
31	977	77	94	52	26	490
32	829	61	128	42	17	291
33	898	39	190	19	1	327
34	901	131	168	59	9	956
35	924	119	119	85	37	512
36	1251	62	193	56	13	563
37	1011	235	217	36	0	714
38	732	94	151	26	3	297
39	444	46	49	21	2	277
40	308	28	57	7	0	154
41	483	40	117	23	0	531
42	515	68	79	30	7	305
43	593	82	101	10	1	85
44	570	26	71	31	20	130
45	1317	123	293	40	1	1043
46	2013	149	403	53	2	1523
47	992	89	161	31	1	743
48	1038	82	151	60	13	513
49	206	1	16	6	0	72
50	1193	95	240	32	0	485

Source: Beasley [2]