Homological algebra/Algebraic geometry

Singular Hochschild cohomology via the singularity category

La cohomologie de Hochschild singulière via la catégorie des singularités

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ABSTRACT

We show that the singular Hochschild cohomology (= Tate–Hochschild cohomology) of an algebra $A$ is isomorphic, as a graded algebra, to the Hochschild cohomology of the differential graded enhancement of the singularity category of $A$. The existence of such an isomorphism is suggested by recent work by Zhengfang Wang.

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RÉSUMÉ

Nous montrons que la cohomologie de Hochschild singulière (cohomologie de Tate–Hochschild) d’une algèbre $A$ est isomorphe, en tant qu’algèbre graduée, à la cohomologie de Hochschild de l’enrichissement différentiel gradué de la catégorie des singularités de $A$. L’existence d’un tel isomorphisme est suggérée par des travaux récents de Zhengfang Wang.

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1. Introduction

Let $k$ be a commutative ring. We write $\otimes$ for $\otimes_k$. Let $A$ be a right Noetherian (non-commutative) $k$-algebra projective as a $k$-module. The stable derived category or singularity category of $A$ is defined as the Verdier quotient

$$Sg(A) = D^b(\text{mod }A) / \text{per}(A)$$

of the bounded derived category of finitely generated (right) $A$-modules by the perfect derived category $\text{per}(A)$, i.e. the full subcategory of complexes quasi-isomorphic to bounded complexes of finitely generated projective modules. It was introduced by Buchweitz in an unpublished manuscript [4] in 1986 and rediscovered, in its scheme-theoretic variant, by Orlov in 2003 [24]. Notice that it vanishes when $A$ is of finite global dimension and thus measures the degree to which $A$ is ‘singular’, a view confirmed by the results of [24].

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Let us suppose that the enveloping algebra $A^e = A \otimes A^{\text{op}}$ is also right Noetherian. In analogy with Hochschild cohomology, in view of Buchweitz’ theory, it is natural to define the Tate–Hochschild cohomology or singular Hochschild cohomology of $A$ to be the graded algebra with components

$$HH^n_{\text{sg}}(A, A) = \text{Hom}_{\text{sg}(A^e)}(A, \Sigma^n A), \ n \in \mathbb{Z},$$

where $\Sigma$ denotes the suspension (=shift) functor. It was studied, for example, in [10,2,23] and more recently in [29,30,28,31,27,5]. Wang showed in [29] that, like Hochschild cohomology [11], singular Hochschild cohomology carries a structure of Gerstenhaber algebra. Now recall that the Gerstenhaber algebra structure on Hochschild cohomology is a small part of much richer higher structure on the Hochschild cochain complex $C(A, A)$ itself, namely the structure of a $B_\infty$-algebra in the sense of Getzler–Jones [12, 5.2] given by the brace operations [1,16]. In [27], Wang improves on [29] by defining a singular Hochschild cochain complex $C_{\text{sg}}(A, A)$ and endowing it with a $B_\infty$-structure, which in particular yields the Gerstenhaber algebra structure on $HH^n_{\text{sg}}(A, A)$.

Using [17], Lowen–Van den Bergh showed in [21, Theorem 4.4.1] that the Hochschild cohomology of $A$ is isomorphic to the Hochschild cohomology of the canonical differential graded (= dg) enhancement of the (bounded or unbounded) derived category of $A$ and that the isomorphism lifts to the $B_\infty$-level. Together with the complete structural analogy between Hochschild and singular Hochschild cohomology described above, this suggests the question whether the singular Hochschild cohomology of $A$ is isomorphic to the Hochschild cohomology of the canonical dg enhancement $\text{Sg}_{\text{dg}}(A)$ of the singularity category $\text{Sg}(A)$ (note that such an enhancement exists by the construction of $\text{Sg}(A)$ as a Verdier quotient [19, 6]). Chen–Li–Wang show in [5] that this does hold at the level of Gerstenhaber algebras when $A$ is the radical square zero algebra associated with a finite quiver without sources or sinks. Our main result is the following.

**Theorem 1.1.** There is a canonical isomorphism of graded algebras between the singular Hochschild cohomology of $A$ and the Hochschild cohomology of the dg singularity category $\text{Sg}_{\text{dg}}(A)$.

**Conjecture 1.2.** The isomorphism of the theorem lifts to an isomorphism

$$C_{\text{sg}}(A, A) \sim C(\text{Sg}_{\text{dg}}(A), \text{Sg}_{\text{dg}}(A))$$

in the homotopy category of $B_\infty$-algebras.

Let us mention an application of Theorem 1.1 obtained in joint work with Zheng Hua. Let $k$ be an algebraically closed field of characteristic 0 and $P$ the power series algebra $k[[x_1, \ldots, x_n]]$.

**Theorem 1.3 ([15]).** Suppose that $Q \in P$ has an isolated singularity at the origin and $A = P/(Q)$. Then $A$ is determined up to isomorphism by its dimension and the dg singularity category $\text{Sg}_{\text{dg}}(A)$.

In [8, Theorem 8.1], Efimov proves a related but different reconstruction theorem: he shows that if $Q$ is a polynomial, it is determined, up to a formal change of variables, by the differential $\mathbb{Z}/2$-graded endomorphism algebra $E$ of the residue field in the differential $\mathbb{Z}/2$-graded singularity category together with a fixed isomorphism between $H^*B$ and the exterior algebra $\Lambda k^n$.

In section 2, we generalize Theorem 1.1 to the non-Noetherian setting and prove the generalized statement. We comment on a possible lift of this proof to the $B_\infty$-level in section 3. We prove Theorem 1.3 in section 4.

2. Generalization and proof

2.1. Generalization to the non-Noetherian case

We assume that $A$ is an arbitrary $k$-algebra projective as a $k$-module. Its singularity category $\text{Sg}(A)$ is defined as the Verdier quotient $\mathcal{T}^{\sim -b}(\text{proj} A)/\mathcal{T}^{b}(\text{proj} A)$ of the homotopy category of right bounded complexes of finitely generated projective $A$-modules by its full subcategory of bounded complexes of finitely generated projective $A$-modules. Notice that when $A$ is right-Noetherian, this is equivalent to the definition given in the introduction.

The (partially) completed singularity category $\tilde{\text{Sg}}(A)$ is defined as the Verdier quotient of the right bounded derived category $\mathcal{D}^- A$ by its full subcategory consisting of all complexes quasi-isomorphic to bounded complexes of arbitrary projective modules.

**Lemma 2.2.** The canonical functor $\text{Sg}(A) \to \tilde{\text{Sg}}(A)$ is fully faithful.

**Proof.** Let $M$ be a right-bounded complex of finitely generated projective modules with bounded homology and $P$ a bounded complex of arbitrary projective modules. Since the components of $M$ are finitely generated, each morphism $M \to P$ in the derived category factors through a bounded complex $P'$ with finitely generated projective components. This yields the claim. \[\square\]
Since we do not assume that $A^\ell$ is Noetherian, the $A$-bimodule $A$ will not, in general, belong to the singularity category $Sg(A^\ell)$. But it always belongs to the completed singularity category $\hat{Sg}(A^\ell)$. We define the singular Hochschild cohomology of $A$ to be the graded algebra with components

$$HH^n_{\hat{Sg}}(A, A) = \text{Hom}_{\hat{Sg}(A^\ell)}(A, \Sigma^n A), \ n \in \mathbb{Z}.$$ 

**Theorem 2.3.** Even if $A^\ell$ is non-Noetherian, there is a canonical isomorphism of graded algebras between the singular Hochschild cohomology of $A$ and the Hochschild cohomology of the dg singularity category $Sg_{\text{dg}}(A)$.

Let $P$ be a right bounded complex of projective $A^\ell$-modules. For $q \in \mathbb{Z}$, let $\sigma_{>q} P$ and $\sigma_{\leq q} P$ denote its stupid truncations:

$$\sigma_{>q} P : \ldots \to 0 \to p^{q+1} \to p^q \to \ldots$$

$$\sigma_{\leq q} P : \ldots \to p^{q-1} \to p^q \to 0 \to \ldots$$

so that we have a triangle

$$\sigma_{>q} P \to P \to \sigma_{\leq q} P \to \Sigma \sigma_{>q} P.$$ 

We have a direct system

$$P = \sigma_{\leq 0} P \to \sigma_{\leq -1} P \to \sigma_{\leq -2} P \to \ldots \to P_{=q} \to \ldots.$$ 

**Lemma 2.4.** Let $L \in \mathcal{D}^-(A^\ell)$. We have a canonical isomorphism

$$\text{colim} \text{Hom}_{\mathcal{DA}^\ell}(L, \sigma_{\leq q} P) \to \text{Hom}_{\hat{Sg}(A^\ell)}(L, P), \ n \in \mathbb{Z}.$$ 

In particular, if $P$ is a projective resolution of $A$ over $A^\ell$, we have

$$\text{colim} \text{Hom}_{\mathcal{DA}^\ell}(A, \Sigma^n \sigma_{\leq q} P) \to \text{Hom}_{\hat{Sg}(A^\ell)}(A, \Sigma^n A), \ n \in \mathbb{Z}.$$ 

**Proof.** Clearly, if $Q$ is a bounded complex of projective modules, each morphism $Q \to P$ in the derived category $\mathcal{DA}^\ell$ factors through $\sigma_{<q} P \to P$ for some $q \leq -Q$. This shows that the morphisms $P \to \sigma_{\leq q} P$ form a cofinal subcategory in the category of morphisms $P \to P'$, whose cylinder is a bounded complex of projective modules. Whence the claim. 

2.5. **Proof of Theorem 2.3**

We refer to [18,20,25] for foundational material on dg categories. We will follow the terminology of [20]. Let $\mathcal{M} = \mathcal{C}^{\text{dg}}(\text{proj} A)$ denote the dg category of right-bounded complexes of finitely generated projective $A$-modules with bounded homology. Let $S$ denote the dg quotient of $\mathcal{M}$ by its full dg subcategory $\mathcal{P} = \mathcal{C}^{\text{dg}}(\text{proj} A)$ of bounded complexes of finitely generated projective $A$-modules. In the homotopy category of dg categories, we have an isomorphism between $S_{\text{dg}}(A)$ and $\mathcal{S} = \mathcal{M}/\mathcal{P}$. We view $A$ as a dg category with one object whose endomorphism algebra is $A$. We have the obvious inclusion and projection dg functors

$$A \xrightarrow{i} \mathcal{M} \xrightarrow{p} S.$$ 

For a dg category $\mathcal{A}$, denote by $\mathcal{DA}$ its derived category, by $\mathcal{A}^\ell$ the enveloping dg category $A \otimes A^{\text{op}}$ and by $I_A$ the identity bimodule

$$I_A : (A, B) \mapsto \mathcal{A}(A, B).$$ 

In the case of the algebra $A$, the identity bimodule is the $A$-bimodule $A$. The dg functors $i$ and $p$ induce dg functors in the enveloping dg categories, which we will denote by the same symbols

$$\mathcal{A}^\ell \xrightarrow{i} \mathcal{M}^\ell \xrightarrow{p} S^\ell.$$ 

The restriction along $i$ has the fully faithful left adjoint $i^* : \mathcal{DA}^\ell \to \mathcal{DM}^\ell$. We claim that it takes the identity bimodule $A$ to $I_{\mathcal{M}}$. For this, we use the bar resolution of $A$ as a bimodule

$$\ldots \to A \otimes A^{\otimes p} \otimes A \to \ldots \to A \otimes A \otimes A \to A \otimes A.$$ 

Its image under $i^*$ is the sum total dg module of the complex
\[ \cdots \to \mathcal{M}(A, \sim) \otimes A \otimes \mathcal{M}(\sigma, A) \to \mathcal{M}(A, \sim) \otimes \mathcal{M}(A, \sigma) \]

with $p$th term $M(A, \sim) \otimes A^{\otimes p} \otimes M(\sigma, A)$. We have to show that the sum total dg module of the augmented complex

\[ \cdots \to \mathcal{M}(A, \sim) \otimes A \otimes \mathcal{M}(\sigma, A) \to \mathcal{M}(A, \sim) \otimes \mathcal{M}(\sigma, A) \to \mathcal{M}(\sigma, \sim) \to 0 \]

is acyclic. Denote this augmented complex by $C(\sigma, \sim)$. Let $P$ and $Q$ in $\mathcal{M}$ be given. We have to show that $C(P, Q)$ is acyclic, i.e. that the sum total dg module of

\[ \cdots \to Q \otimes A^{\otimes p} \otimes \mathcal{M}(P, A) \to \cdots \to Q \otimes \mathcal{M}(P, A) \to \mathcal{M}(P, Q) \to 0 \]

is acyclic. This is clear if $P = A$, since then $C(Q, A)$ is just the bar resolution of the right dg module $Q$. Thus, it also holds if $P$ is a bounded complex of finitely generated projective modules. In the general case, we consider the filtration of $P$ by the stupid truncations $\sigma_{\leq q}P$, $q \leq 0$. Clearly, $C(P, Q)$ is the inverse limit of the acyclic complexes $C(\sigma_{\leq q}P, Q)$ and the transition maps in this inverse system are componentwise surjective. It follows that $C(P, Q)$ is acyclic as was to be shown.

Now fix a projective resolution $P$ of $A$ as a bimodule. Denote by $\sigma_{\leq q}P$ its stupid truncations, $q \leq 0$. We have a direct system

\[ P \to \sigma_{\leq -1}P \to \sigma_{\leq -2}P \to \cdots \to P_{\leq q} \to \cdots \]

By Lemma 2.4, we have a canonical isomorphism

\[ \colim \text{Hom}_{\mathbb{D}(A^e)}(A, \Sigma^n \sigma_{\leq q}P) \xrightarrow{\sim} \text{Hom}_{\mathbb{D} S_{\mathbb{S}}(A^e)}(A, \Sigma^n A). \]

Since $i^*$ is fully faithful and $i^*(A) = I_N$, we have

\[ \colim \text{Hom}_{\mathbb{D}(A^e)}(A, \Sigma^n \sigma_{\leq q}P) = \colim \text{Hom}_{\mathbb{D}S^e_{\mathbb{S}}}(I_N, \Sigma^n i^*(\sigma_{\leq q}P)). \]

Since $p : N \to \mathbb{S}$ is a localization, we have $p^*(I_N) = I_{\mathbb{S}}$. Thus, we get a map

\[ \colim \text{Hom}_{\mathbb{D}S^e_{\mathbb{S}}}(I_N, \Sigma^n i^*(\sigma_{\leq q}P)) \to \colim \text{Hom}_{\mathbb{D}S^e_{\mathbb{S}}}(I_{\mathbb{S}}, \Sigma^n p^*(\sigma_{\leq q}P)). \]

We claim that it is a bijection for all $n$. For this, we first reinterpret the left-hand side. Since $i^* : \mathbb{D}A^e \to \mathbb{D}M^e$ is fully faithful, by Lemma 2.4, it is isomorphic to

\[ \text{Hom}_{\mathbb{D}(M^e)/N}(I_N, \Sigma^n I_N), \]

where $N$ is the image under $i^*$ of the full subcategory of $\mathbb{D}A^e$ formed by the complexes quasi-isomorphic to bounded complexes of arbitrary projective $A^e$-modules. Let us now consider the right-hand side. The cones over the morphisms $i^*(P) \to i^*(\sigma_{\leq q}P)$ are finite extensions of shifts of arbitrary coproducts of objects $Y(P', P'')$, where $P'$ and $P''$ are finitely generated projective $A$-modules. The functor $p^* : \mathbb{D}M^e \to \mathbb{D}S^e$ commutes with arbitrary coproducts and vanishes on the $Y(P', P'')$. Thus the images under $p^*$ of the morphisms $i^*(P) \to i^*(\sigma_{\leq q}P)$ are all invertible so that the right-hand side is isomorphic to

\[ \text{Hom}_{\mathbb{D}S^e_{\mathbb{S}}}(I_{\mathbb{S}}, \Sigma^n I_{\mathbb{S}}) = \text{Hom}_{\mathbb{D}M^e}(p^*(I_N), \Sigma^n p^*(I_N)). \]

Now notice that we have a Morita morphism of dg categories

\[ \mathbb{S}^e \xrightarrow{\sim} \frac{M \otimes M^\text{op}}{P \otimes M^\text{op} + M \otimes P^\text{op}}. \]

The functor $p^*$ induces the quotient functor

\[ \frac{\mathbb{D}(M \otimes M^\text{op})}{\mathbb{D}(P \otimes M^\text{op} + M \otimes P^\text{op})} \to \mathbb{D}(\mathbb{S}^e). \]

It suffices to show that $p^*$ induces bijections in the morphism spaces with target $I_M$

\[ \text{Hom}_{\mathbb{D}(M^e)/N}(\sigma, I_M) \to \text{Hom}_{\mathbb{D}S^e}(p^*(\sigma), p^*(I_M)). \]

For this, it suffices to show that $I_M$ is right orthogonal in $\mathbb{D}(M^e)/N$ on the images under the Yoneda functor of the objects in $P \otimes M^\text{op} + M \otimes P^\text{op}$. To show that $I_M$ is right orthogonal on $Y(M \otimes P^\text{op})$, it suffices to show that it is right orthogonal to an object $Y(M, A)$, $M, A \in M$. Now a morphism in $\mathbb{D}(M^e)/N$ is given by a diagram of $\mathbb{D}(M^e)$ representing a left fraction

\[ Y(M, A) \to I'_M \to I_M \]

where the cone over $I_M \to I'_M$ lies in $N$. For each object $X$ of $\mathbb{D}M^e$, we have canonical isomorphisms
\[ \text{Hom}_{DM}(Y(M, A), X) = H^0(X(M, A)) = \text{Hom}_{DM}(Y(M, A), X). \]

Thus, the given fraction corresponds to a diagram in \( DM \) of the form

\[ Y(M) \longrightarrow I_M^*(?, A) \leftarrow I_M(?, A) = M(?, A), \]

where the cone over \( I_M(?, A) \rightarrow I_M^*(?, A) \) is the image under \( i^*: DA \rightarrow DM \) of a bounded complex with projective components. Thus, we may assume that \( I_M^*(?, A) \) is a finite extension of shifts of arbitrary coproducts of objects \( Y(Q) \), where \( Q \) is a finitely generated projective \( A \)-module. Since \( M \) has finitely generated components, the given morphism \( Y(M) \rightarrow I_M^*(?, A) \) must then factor through \( Y(Q) \) for an object \( Q \) of \( P \). This means that the given morphism \( Y(M, A) \rightarrow I_M^*(?, A) \) factors through \( Y(Q, A) \), which lies in \( N \). Thus, the given fraction represents the zero morphism of \( DM/N \), as was to be shown. The case of an object in \( Y(P \otimes N) \) is analogous. In summary, we have shown that the canonical map

\[ \text{colim} \text{Hom}_{DM}(I_M, \Sigma^n i^*(\sigma_{\leq q} P)) \longrightarrow \text{colim} \text{Hom}_{DS}(I_S, \Sigma^n (pi)^*(\sigma_{\leq q} P)) \]

is bijective. As we have already observed, the direct system \((pi)^*(\sigma_{\leq q} P)\) is constant in \( DS \). Moreover, we know that \( i^*(P) = I_M \) and \( p^*(I_M) = I_S \). Thus the right-hand side is isomorphic to

\[ \text{Hom}_{DS}(I_S, \Sigma^n I_S), \]

which is the \( n \)th component of the Hochschild cohomology of the \( dg \) category \( Sg_{dg}(A) \).

3. Remark on a possible lift to the \( B_\infty \)-level

The above proof produces in fact isomorphisms in the derived category of \( k \)-modules

\[ \text{colim} \text{RHom}_A(I_M, \sigma_{\leq q} P) \rightarrow \text{colim} \text{RHom}_{DM}(I_M, i^* \sigma_{\leq q} P) \]

\[ \rightarrow \text{colim} \text{RHom}_{DS}(I_S, p^* i^* \sigma_{\leq q} P) \]

\[ = \text{RHom}_{DS}(I_S, I_S). \]

If we choose for \( P \) the bar resolution of \( A \), then \( \sigma_{\leq q} P \) is canonically isomorphic to \( \Sigma^n \Omega^2 A \) so that the first complex carries a canonical \( B_\infty \)-structure constructed by Wang. As explained in the introduction, it is classical that the last complex carries a canonical \( B_\infty \)-structure. It turns out that when one makes the second complex explicit, it can be chosen identical to the first complex (essentially because \( i^* \) is fully faithful and \( i^* A = I_M \)). Only the interpretation changes. Thus, the problem is to construct a compatible \( B_\infty \)-structure on the third complex.

4. Proof of Theorem 1.3

By the Weierstrass preparation theorem, we may assume that \( Q \) is a polynomial. Put \( R = k[x_1, \ldots, x_n]/(Q) \) so that \( A \) is isomorphic to the completion \( \hat{R} \). By Theorem 3.2.7 of \([14]\), in sufficiently high even degrees, the Hochschild cohomology of \( R \) is isomorphic to

\[ T = k[x_1, \ldots, x_n]/(Q, \partial_1 Q, \ldots, \partial_n Q) \]

as an \( R \)-module. Since \( R \otimes R \) is Noetherian and Gorenstein (cf. Theorem 1.6 of \([26]\)), by Theorem 6.3.4 of \([4]\) the singular Hochschild cohomology of \( R \) coincides with Hochschild cohomology in sufficiently high degrees. By Theorem 1.1, the Hochschild cohomology of \( Sg_{dg}(R) \) is isomorphic to the singular Hochschild cohomology of \( R \) and thus isomorphic to \( T \) in high even degrees. Since \( R \) is a hypersurface, the \( dg \) category \( Sg_{dg}(R) \) is isomorphic, in the homotopy category of \( dg \) categories, to the underlying differential \( \mathbb{Z} \)-graded category of the differential \( \mathbb{Z}/2 \)-graded category of matrix factorizations of \( Q \), cf. \([9]\), \([24]\) and Theorem 2.49 of \([3]\). Thus, it is 2-periodic and so is its Hochschild cohomology. It follows that the zeroth Hochschild cohomology of \( Sg_{dg}(R) \) is isomorphic to \( T \) as an algebra. The completion functor \( \tau \otimes_k \hat{R} \) yields an embedding \( Sg(R) \rightarrow Sg(A) \) through which \( Sg(A) \) identifies with the idempotent completion of the triangulated category \( Sg(R) \), cf. Theorem 5.7 of \([7]\). Therefore, the corresponding \( dg \) functor \( Sg_{dg}(R) \rightarrow Sg_{dg}(A) \) induces an equivalence in the derived categories and an isomorphism in Hochschild cohomology. So we find an isomorphism

\[ HH^0(Sg_{dg}(A), Sg_{dg}(A)) \cong T. \]

Since \( Q \) has an isolated singularity at the origin, we have an isomorphism

\[ T \cong k[[x_1, \ldots, x_n]]/(Q, \partial_1 Q, \ldots, \partial_n Q) \]

with the Tyurina algebra of \( A = P/(Q) \). Now, by the Mather–Yau theorem \([22]\), more precisely by its formal version \([13, \text{Prop. 2.1}]\), in a fixed dimension, the Tyurina algebra determines \( A \) up to isomorphism.
Notice that the Hochschild cohomology of the dg category of matrix factorizations considered as a differential $\mathbb{Z}/2$-graded category is different: as shown by Dyckerhoff [7], it is isomorphic to the Milnor algebra $P/(\delta_1 Q, \ldots, \delta_{n} Q)$ in even degree and vanishes in odd degree.

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