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Partial Differential Equations

# Lipschitz stability estimate in the inverse Robin problem for the Stokes system



*Estimation de stabilité lipschitzienne de coefficients de Robin pour le système de Stokes*

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## ABSTRACT

We are interested in the inverse problem of recovering a Robin coefficient defined on some non-accessible part of the boundary from available data on another part of the boundary in the non-stationary Stokes system. We prove a Lipschitz stability estimate under the *a priori* assumption that the Robin coefficient lives in some compact and convex subset of a finite dimensional vectorial subspace of the set of continuous functions. To do so, we use a theorem proved by L. Bourgeois and which establishes Lipschitz stability estimates for a class of inverse problems in an abstract framework.

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## R É S U M É

Nous nous intéressons à l'identification d'un coefficient de Robin défini sur une partie non accessible du bord, à partir de mesures disponibles sur une autre partie de celui-ci, dans le système de Stokes non stationnaire. Nous prouvons une estimation de stabilité lipschitzienne sous l'hypothèse *a priori* que le coefficient de Robin est défini dans un sous-ensemble compact et convexe d'un sous-espace vectoriel de dimension finie de l'espace des fonctions continues. Pour ce faire, nous utilisons un théorème prouvé par L. Bourgeois permettant d'établir des inégalités de stabilité lipschitzienne pour une classe de problèmes inverses dans un cadre abstrait.

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## Version française abrégée

## Introduction

Soit  $T > 0$ ,  $\Omega \subset \mathbb{R}^d$ , avec  $d \in \mathbb{N}^*$ , un ouvert borné lipschitzien et connexe tel que  $\partial\Omega = \Gamma_1 \cup \Gamma_0 \cup \Gamma_{\text{out}}$  avec  $\Gamma_{\text{out}} = \bigcup_{i=1}^N \Gamma_i$  et  $\nu$  est la normale extérieure à  $\Omega$ . On considère le système de Stokes suivant :

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$$\begin{cases} \partial_t u - \Delta u + \nabla p = 0, & \text{dans } (0, T) \times \Omega, \\ \operatorname{div} u = 0, & \text{dans } (0, T) \times \Omega, \\ u = 0, & \text{dans } (0, T) \times \Gamma_l, \\ \partial_\nu u - p\nu = g, & \text{sur } (0, T) \times \Gamma_0, \\ \partial_\nu u - p\nu + qu = 0, & \text{sur } (0, T) \times \Gamma_{\text{out}}, \\ u(0, \cdot) = u_0, & \text{dans } \Omega. \end{cases} \quad (1)$$

Le problème inverse qui nous intéresse est le suivant : on cherche à identifier le coefficient de Robin  $q$  défini sur la partie non accessible du bord  $\Gamma_{\text{out}}$  à partir de mesures disponibles sur  $\Gamma_0$  pour  $(u, p)$  solution du système (1). La résolution de ce type de problème inverse dans le cas stationnaire a déjà été étudiée dans [3], [4] et [7]. Dans les deux premiers travaux, une inégalité de stabilité logarithmique est obtenue, alors qu'une inégalité de stabilité lipschitzienne est établie dans [7] sous l'hypothèse *a priori* que le coefficient de Robin est constant par morceaux sur  $\Gamma_{\text{out}}$ . Dans chacun de ces papiers, les mesures intervenant dans les inégalités de stabilité sont la vitesse  $u$ , la pression  $p$  et la dérivée normale de la pression  $\partial p / \partial n$  sur  $\Gamma \subseteq \Gamma_0$ . Le cas du système de Stokes non stationnaire a été abordé dans [3], dans le cas particulier où le coefficient de Robin ne dépend pas du temps. L'idée, introduite dans [2] dans le cas de l'équation de Laplace, consiste à étendre l'inégalité de stabilité valable pour le problème stationnaire au problème non stationnaire en utilisant une inégalité provenant de la théorie des semigroupes analytiques. Cela conduit à faire des mesures en temps infini.

L'originalité de l'inégalité de stabilité lipschitzienne présentée dans cette Note est multiple : d'une part, nous obtenons une inégalité de stabilité valable pour le système de Stokes non stationnaire en temps fini avec un coefficient de Robin dépendant du temps et, d'autre part, l'unique mesure intervenant dans l'inégalité de stabilité est la vitesse  $u$  sur  $(0, T) \times \Gamma$ , avec  $\Gamma \subseteq \Gamma_0$ . De plus, l'ensemble des coefficients de Robin pour lequel l'inégalité de stabilité lipschitzienne est valide est un peu plus général que dans [7] : les coefficients de Robin ne sont plus nécessairement constants par morceaux, mais appartiennent à un sous-ensemble compact et convexe d'un sous-espace vectoriel de dimension finie de l'ensemble des fonctions continues. Enfin, nous avons besoin d'hypothèses de régularité moins fortes sur le bord du domaine  $\Omega$  et sur le flux  $g$ .

### Résultat principal

Afin d'être plus précis, nous introduisons quelques notations.

**Notation 0.1.** On note

$$L_+^\infty((0, T) \times \Gamma_{\text{out}}) = \{q \in L^\infty((0, T) \times \Gamma_{\text{out}}); \exists m > 0, q \geq m \text{ p. p. sur } (0, T) \times \Gamma_{\text{out}}\},$$

et

$$C^0(0, T; C_{pc}^0(\Gamma_{\text{out}})) = \{q : (0, T) \times \Gamma_{\text{out}} \rightarrow \mathbb{R}; q|_{(0, T) \times \Gamma_i} \in C^0((0, T) \times \Gamma_i) \text{ pour } 1 \leq i \leq N\}.$$

Le résultat principal de cette Note est résumé dans le théorème suivant :

**Théorème 0.1.** Soit  $M \in \mathbb{N}^*$ . On considère  $V_M$  un sous-espace vectoriel de  $C^0(0, T; C_{pc}^0(\Gamma_{\text{out}}))$  engendré par  $M$  fonctions linéairement indépendantes et  $K_M$  une partie convexe et compacte de  $V_M \cap L_+^\infty((0, T) \times \Gamma_{\text{out}})$ . Soit  $\Gamma \subseteq \Gamma_0$  une partie ouverte non vide du bord du domaine,  $u_0 \in H^1(\Omega)$  tel que  $\operatorname{div} u_0 = 0$  dans  $\Omega$  et  $g \in H^1(0, T; L^2(\Gamma_0))$  tel que  $g(t)$  est non identiquement zéro pour tout  $t \in (0, T)$ . Soit  $(u_k, p_k)$  la solution faible du système (1) avec  $q = q_k \in K_M$  pour  $k = 1, 2$ . Alors, il existe une constante  $C > 0$  telle que

$$\|q_1 - q_2\|_{L^\infty((0, T) \times \Gamma_{\text{out}})} \leq C \|u_1 - u_2\|_{L^2((0, T) \times \Gamma)}.$$

La preuve du **Théorème 0.1** est basée sur un théorème abstrait prouvé par L. Bourgeois dans [5] et repose sur le fait que l'application :

$$\begin{aligned} T : L_+^\infty((0, T) \times \Gamma_{\text{out}}) &\rightarrow L^2((0, T) \times \Gamma) \\ q &\rightarrow u|_\Gamma \end{aligned}$$

où  $(u, p)$  est solution du système (1) et avec  $\Gamma \subseteq \Gamma_0$ , est injective, de classe  $C^1$ , et sa dérivée est également injective. Notons que le résultat énoncé dans [5] permet d'établir des inégalités de stabilité lipschitzienne pour une classe de problèmes inverses. Il permet notamment de retrouver les résultats de stabilité développés dans [9] et [1], sans avoir recours à des arguments de quantification de résultats de continuation unique. L'auteur précise que l'on peut trouver l'idée originale développée dans [6] dans le cas particulier de la détection d'un obstacle se déplaçant dans un fluide à partir de mesures disponibles sur le bord du domaine. De plus, des théorèmes abstraits du même type, mais avec des hypothèses différentes, peuvent être trouvés dans [10].

### 1. Introduction

Let  $T > 0$ ,  $\Omega \subset \mathbb{R}^d$ , with  $d \in \mathbb{N}^*$ , be a Lipschitz bounded connected open set such that  $\partial\Omega = \Gamma_i \cup \Gamma_0 \cup \Gamma_{\text{out}}$  and  $\Gamma_{\text{out}} = \bigcup_{i=1}^N \Gamma_i$ . We are interested in the inverse problem of identifying the Robin coefficient  $q$  defined on some non-accessible part of the boundary  $\Gamma_{\text{out}}$  from available data on  $\Gamma_0$  for  $(u, p)$ , solution of the Stokes system (1). Similar inverse problems have already been studied in the stationary case in [3], [4] and [7]. In [3] and [4], a logarithmic stability estimate is obtained, whereas a Lipschitz stability estimate is established in [7] under the *a priori* assumption that the Robin coefficient is piecewise constant on  $\Gamma_{\text{out}}$ . In each cases, the measurements involved in the stability estimates are the velocity  $u$ , the pressure  $p$  and the normal derivative of the pressure  $\partial p / \partial n$  on  $\Gamma \subseteq \Gamma_0$ . The case of the non-stationary Stokes system has been addressed in [3] in the particular case where the Robin coefficient does not depend on time. The idea, introduced in [2] in the case of the Laplace equation, is to extend the stability estimate valid for the stationary problem to the non-stationary problem by using an inequality coming from the theory of analytic semigroups. This leads to infinite time measurements.

The originality of the Lipschitz stability estimate presented in this Note is multiple: on the one hand, we obtain a stability estimate valid for the non-stationary Stokes system in finite time with a time-dependent Robin coefficient and, on the other hand, the only measurement involved in the stability estimate is the velocity  $u$  on  $(0, T) \times \Gamma$ , with  $\Gamma \subseteq \Gamma_0$ . In addition, the set of admissible Robin coefficients is more general than in [7]: Robin coefficients are not necessarily piecewise constant, but belong to some compact and convex subset of a finite dimensional vectorial subspace of the set of continuous functions. Finally, we relax the regularity assumptions needed both on the boundary of the domain  $\Omega$  and on the flux  $g$ .

### 2. Main result

To be more precise, we introduce some notations.

**Notation 2.1.** We denote by:

$$L_+^\infty((0, T) \times \Gamma_{\text{out}}) = \{q \in L^\infty((0, T) \times \Gamma_{\text{out}}); \exists m > 0, q \geq m \text{ a.e. on } (0, T) \times \Gamma_{\text{out}}\},$$

and

$$C^0(0, T; C_{pc}^0(\Gamma_{\text{out}})) = \{q : (0, T) \times \Gamma_{\text{out}} \rightarrow \mathbb{R}; q|_{(0, T) \times \Gamma_i} \in C^0((0, T) \times \Gamma_i) \text{ for } 1 \leq i \leq N\}.$$

The main result of this Note is summarized in the following theorem.

**Theorem 2.1.** *Let  $M \in \mathbb{N}^*$ . Let  $V_M$  be a subspace of  $C^0(0, T; C_{pc}^0(\Gamma_{\text{out}}))$  spanned by some  $M$  linearly independent functions and  $K_M$  be any compact and convex subset of  $V_M \cap L_+^\infty((0, T) \times \Gamma_{\text{out}})$ . Let  $\Gamma \subseteq \Gamma_0$  be a nonempty open subset of the boundary,  $u_0 \in H^1(\Omega)$  be such that  $\text{div } u_0 = 0$  in  $\Omega$  and  $g \in H^1(0, T; L^2(\Gamma_0))$  be such that  $g(t)$  is not identically zero for all  $t \in (0, T)$ . Let  $(u_k, p_k)$  be the weak solution of system (1) with  $q = q_k \in K_M$  for  $k = 1, 2$ . Then, there exists a constant  $C > 0$  such that:*

$$\|q_1 - q_2\|_{L^\infty((0, T) \times \Gamma_{\text{out}})} \leq C \|u_1 - u_2\|_{L^2((0, T) \times \Gamma)}.$$

The proof of Theorem 2.1 is based on an abstract theorem proved by L. Bourgeois in [5] and relies on the fact that the application that associates with a Robin coefficient the velocity on  $(0, T) \times \Gamma$  is injective, of class  $\mathcal{C}^1$ , and its derivative is also injective. The result stated in [5] establishes Lipschitz stability estimates for a class of inverse problems. For instance, it allows us to find again the stability results developed in [9] and [1] without resorting to quantification of unique continuation results. The author points out that one can find the original idea developed in [6] in the particular case of the detection of a moving obstacle in a fluid from measurements available on the boundary of the domain. Moreover, abstract theorems of the same type, but with different assumptions, can be found in [10].

The sequel of this paper is organized as follows. We present in Section 3 some preliminary results that will be useful to prove Theorem 2.1. Then the proof of Theorem 2.1 is given in Section 4.

### 3. Preliminary results

In the section, we state results that will be useful in the proof of Theorem 2.1. We begin by stating regularity result for a slightly more general Stokes system than system (1) (we add non-homogeneous Robin boundary condition on  $(0, T) \times \Gamma_{\text{out}}$ ):

$$\begin{cases} \partial_t u - \Delta u + \nabla p = 0, & \text{in } (0, T) \times \Omega, \\ \text{div } u = 0, & \text{in } (0, T) \times \Omega, \\ u = 0, & \text{in } (0, T) \times \Gamma_i, \\ \partial_\nu u - p\nu = g, & \text{on } (0, T) \times \Gamma_0, \\ \partial_\nu u - p\nu + qu = \kappa, & \text{on } (0, T) \times \Gamma_{\text{out}}, \\ u(0, \cdot) = u_0, & \text{in } \Omega. \end{cases} \tag{2}$$

**Proposition 3.1.** Let  $g \in H^1(0, T; L^2(\Gamma_0))$ ,  $\kappa \in H^1(0, T; L^2(\Gamma_{\text{out}}))$ ,  $u_0 \in H^1(\Omega)$  be such that  $\text{div } u_0 = 0$  in  $\Omega$  and let  $q \in L^\infty_+(\!(0, T) \times \Gamma_{\text{out}})$ . Then, system (2) has a unique solution  $(u, p)$  in  $L^2(0, T; H^1(\Omega)) \cap H^1(0, T; L^2(\Omega)) \times L^2(0, T; L^2(\Omega))$ . Moreover, there exists a constant  $C > 0$ , independent of  $q$ , such that the following inequality holds:

$$\|u\|_{L^2(0, T; H^1(\Omega))} \leq C(\|u_0\|_{L^2(\Omega)} + \|g\|_{L^2(0, T; L^2(\Gamma_0))} + \|\kappa\|_{L^2(0, T; L^2(\Gamma_{\text{out}}))}).$$

**Proof of Proposition 3.1.** The proof is mainly contained in the appendix of [3]. The main difference here is that we work with non-homogeneous Robin boundary conditions, which leads to slight modifications. □

The following Proposition 3.2 concerns the identifiability of the inverse problem we are interested in.

**Proposition 3.2.** Let  $\Gamma \subseteq \Gamma_0$  be a nonempty open subset of the boundary,  $u_0 \in H^1(\Omega)$  be such that  $\text{div } u_0 = 0$  in  $\Omega$ . Assume that  $g \in H^1(0, T; L^2(\Gamma_0))$  is such that  $g(t)$  is not identically zero for all  $t \in (0, T)$ . Let  $(u_k, p_k)$  be the weak solutions of system (1) with  $q = q_k \in C^0(0, T; C^0_{pc}(\Gamma_{\text{out}}))$  for  $k = 1, 2$ . We assume that  $u_1 = u_2$  on  $(0, T) \times \Gamma$ . Then  $q_1 = q_2$  on  $(0, T) \times \Gamma_{\text{out}}$ .

**Proof of Proposition 3.2.** The proof is based on the unique continuation result for the Stokes system proved by C. Fabre and G. Lebeau in [8]. Thanks to the previous proposition,  $(u, p) \in L^2(0, T; H^1(\Omega)) \cap H^1(0, T; L^2(\Omega)) \times L^2(0, T; L^2(\Omega))$ , which gives enough regularity to prove a similar result to Corollary 3.2 in [3]. Then, we proceed exactly as in the proof of Proposition 3.3 in [3] where the proof is done in the particular case when the Robin coefficient does not depend on time, by arguing by contradiction. □

#### 4. Proof of the main result

In this section, we give the proof of Theorem 2.1. We consider the operator:

$$\begin{aligned} T : L^\infty_+(\!(0, T) \times \Gamma_{\text{out}}) &\rightarrow L^2(\!(0, T) \times \Gamma) \\ q &\rightarrow u|_\Gamma \end{aligned}$$

where  $(u, p)$  is solution of system (1). The proof consists in applying some abstract theorem proved in [5]. To do so, we have to verify that:

- (i)  $T|_{V_M \cap L^\infty_+(\!(0, T; \Gamma_{\text{out}})}$  is injective,
- (ii)  $T$  is differentiable at any point  $q \in L^\infty_+(\!(0, T) \times \Gamma_{\text{out}})$  and its Fréchet derivative is the operator

$$\begin{aligned} dT_q : L^\infty_+(\!(0, T) \times \Gamma_{\text{out}}) &\rightarrow L^2(\!(0, T) \times \Gamma) \\ h &\rightarrow v_h|_\Gamma, \end{aligned}$$

and where  $(v_h, \tau_h)$  is solution to:

$$\begin{cases} \partial_t v_h - \Delta v_h + \nabla \tau_h = 0, & \text{in } (0, T) \times \Omega, \\ \text{div } v_h = 0, & \text{in } (0, T) \times \Omega, \\ v_h = 0, & \text{in } (0, T) \times \Gamma_l, \\ \partial_\nu v_h - \tau_h \nu = 0, & \text{on } (0, T) \times \Gamma_0, \\ \partial_\nu v_h - \tau_h \nu + q v_h = -hu, & \text{on } (0, T) \times \Gamma_{\text{out}}, \\ v_h(0, \cdot) = 0, & \text{in } \Omega, \end{cases} \tag{3}$$

where  $u$  is solution to system (1). Moreover, the mapping:

$$\begin{aligned} dT : L^\infty_+(\!(0, T) \times \Gamma_{\text{out}}) &\rightarrow \mathcal{L}(L^\infty_+(\!(0, T) \times \Gamma_{\text{out}}), L^2(\!(0, T) \times \Gamma)) \\ q &\rightarrow dT_q, \end{aligned} \tag{4}$$

is continuous.

- (iii) For all  $q \in V_M \cap L^\infty_+(\!(0, T) \times \Gamma_{\text{out}})$ , the operator  $dT_q : V_M \rightarrow L^2(\!(0, T) \times \Gamma)$  is injective.

Then, we directly obtain from (i), (ii), (iii) and Theorem 2.1 in [5], the Lipschitz stability estimate stated in Theorem 2.1. Let us now prove (i), (ii) and (iii). Step (i) is a direct consequence of Proposition 3.2. Let us prove step (ii). Let  $q \in L^\infty_+(\!(0, T) \times \Gamma_{\text{out}})$  and  $h \in L^\infty_+(\!(0, T) \times \Gamma_{\text{out}})$  with  $q + h \in L^\infty_+(\!(0, T) \times \Gamma_{\text{out}})$ . We denote by  $(u, p)$  (resp.  $(u_h, p_h)$ ) be the solution of system (1) associated with  $q$  (resp. to  $q + h$ ). Note that  $(u_h - u, p_h - p)$  is the solution of the Stokes system (2) with  $g = 0$ ,  $u_0 = 0$  and  $\kappa = -hu_h$ . Let  $M_1 > 0$  be such that  $\|u_0\|_{L^2(\Omega)} + \|g\|_{L^2(0, T; L^2(\Gamma_0))} \leq M_1$ . Then, thanks to Proposition 3.1, there exists  $C > 0$  such that:

$$\|u_h - u\|_{L^2(0,T;H^1(\Omega))} \leq C \|h\|_{L^\infty((0,T) \times \Gamma_{\text{out}})} \|u_h\|_{L^2(0,T;H^1(\Omega))} \leq C(M_1) \|h\|_{L^\infty((0,T) \times \Gamma_{\text{out}})}. \quad (5)$$

Now, let us consider  $(e_h, \rho_h) = (u_h - u - v_h, p_h - p - \tau_h)$ , where  $(v_h, \tau_h)$  is solution to system (3). First, we readily check that the operator  $h \in L^\infty((0, T) \times \Gamma_{\text{out}}) \rightarrow v_h|_\Gamma \in L^2((0, T) \times \Gamma)$  is linear continuous. Secondly,  $(e_h, \rho_h)$  is solution to the Stokes system (2) with  $g = 0$ ,  $u_0 = 0$  and  $\kappa = -h(u_h - u)$ , which implies, thanks to Proposition 3.1 and inequality (5), that:

$$\|e_h\|_{L^2(0,T;H^1(\Omega))} \leq C \|h\|_{L^\infty((0,T) \times \Gamma_{\text{out}})} \|u_h - u\|_{L^2(0,T;H^1(\Omega))} \leq C(M_1) \|h\|_{L^\infty((0,T) \times \Gamma_{\text{out}})}^2,$$

which proves that  $T$  is Fréchet differentiable and  $dT_q(h) = v_h|_\Gamma$ .

Let us prove now the continuity of the mapping  $dT$  defined in (4). Let  $(v_h, \tau_h)$  (resp.  $(v_h^l, \tau_h^l)$ ) be the solution of system (3) associated with  $q$  (resp.  $q = q + l$ ) and where  $(u, p)$  (resp.  $(u, p) = (u_l, p_l)$ ) is the solution to system (1) associated with  $q$  (resp.  $q = q + l$ ). We have that  $(v_h^l - v_h, p_h^l - p_h)$  is the solution to the Stokes system (2) with  $g = 0$ ,  $u_0 = 0$  and  $\kappa = -lv_h^l - h(u_l - u)$ . This implies, thanks to Proposition 3.1,

$$\|v_h^l - v_h\|_{L^2(0,T;H^1(\Omega))} \leq C \|l\|_{L^\infty((0,T) \times \Gamma_{\text{out}})} \|v_h^l\|_{L^2(0,T;H^1(\Omega))} + C \|h\|_{L^\infty((0,T) \times \Gamma_{\text{out}})} \|u_l - u\|_{L^2(0,T;H^1(\Omega))},$$

which leads to, applying again Proposition 3.1 and inequality (5) with  $h = l$ :

$$\|v_h^l - v_h\|_{L^2(0,T;H^1(\Omega))} \leq C(M_1) \|l\|_{L^\infty((0,T) \times \Gamma_{\text{out}})} \|h\|_{L^\infty((0,T) \times \Gamma_{\text{out}})},$$

where  $C$  is uniform with respect to  $h$  and  $l$ . Therefore, we have proved that:

$$\|dT_{q+l} - dT_q\| \leq C \|l\|_{L^\infty((0,T) \times \Gamma_{\text{out}})},$$

where  $\| \cdot \|$  denotes the operator norm. Thus the mapping  $dT$  is continuous.

It remains to prove step (iii). Let  $q \in L_+^\infty((0, T) \times \Gamma_{\text{out}}) \cap V_M$ . Assume that  $h \in V_M$  is such that  $v_h|_{(0,T) \times \Gamma} = 0$ . Then, since  $(v_h|_{(0,T) \times \Gamma}, (\partial_\nu v_h - \tau_h \nu)|_{(0,T) \times \Gamma}) = (0, 0)$ , we obtain from the unique continuation result that  $v_h = 0$  in  $(0, T) \times \Omega$  and then  $hu = 0$  on  $(0, T) \times \Gamma_{\text{out}}$ . We conclude that  $h = 0$  by contradiction, exactly as for the injectivity of the mapping  $T$  (see [3]).

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