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Differential Geometry

On extrinsic symmetric spaces with zero mean curvature in Minkowski space-time



Sur les espaces symétriques extrinsèques à courbure moyenne nulle dans l'espace-temps de Minkowski

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ABSTRACT

For an extrinsic symmetric space M in Minkowski space-time, we prove that if M is spacelike with zero mean curvature, then it is totally geodesic and if M is timelike with zero mean curvature, then it is totally geodesic or it is a flat hypersurface.

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RÉSUMÉ

Pour un espace symétrique extrinsèque M dans l'espace-temps de Minkowski, nous prouvons que, si M est de type espace et à courbure moyenne nulle, alors M est totalement géodésique, tandis que, si M est de type temps à courbure moyenne nulle, il s'agit alors d'une sous-variété totalement géodésique ou d'une hypersurface.

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1. Introduction

Let M be a non-degenerate submanifold in Minkowski space-time \mathbb{R}^n_1 . It is called a spacelike, timelike submanifold if its tangent space T_xM is spacelike, timelike for each $x \in M$, respectively. The reflection for the affine isometry $s_x : \mathbb{R}^n_1 \to \mathbb{R}^n_1$ is defined by:

$$s_x(x) = x$$
, $s_*|_{T_xM} = -id$, $s_*|_{(T_xM)^{\perp}} = id$.

We call M a symmetric submanifold or extrinsic symmetric space if it is invariant under the reflection at each affine normal space $(T_xM)^{\perp}$, that is, $s_x(M) = M$ for all $x \in M$. The covariant derivation of the second fundamental form α and the above reflection s_x give:

$$\left(\nabla_{u}^{\perp}\alpha\right)(v,w) = s_{*}\left(\nabla_{u}^{\perp}\alpha\right)(v,w) = \left(\nabla_{s_{*}u}^{\perp}\alpha\right)(s_{*}v,s_{*}w) = -\left(\nabla_{u}^{\perp}\alpha\right)(v,w)$$

for all $x \in M$ and $u, v, w \in T_xM$. Thus an extrinsic symmetric space has the parallel second fundamental form. The converse holds due to Strübing in [8].

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In [3] Ferus showed that if an extrinsic symmetric space M in Euclidean space \mathbb{R}^n has zero mean curvature, then M is totally geodesic. Here we consider an extrinsic symmetric space M in \mathbb{R}^n_1 whose mean curvature is zero. And we show the following theorem:

Theorem. Let M be an extrinsic symmetric space in \mathbb{R}^n_1 . If M is spacelike with zero mean curvature, then it is totally geodesic. And if M is timelike with zero mean curvature, then it is totally geodesic or it is a flat hypersurface.

2. Proof of the theorem

The Lie algebra of an indefinite extrinsic symmetric space is constructed in [6]. We recall some necessary notations in [6] for the proof of the theorem. Let $M \subset V = \mathbb{R}^n_1$ be an extrinsic symmetric space and $\hat{K} = \langle s_X; \ X \in M \rangle \subset O(V)$ be the group generated by all reflection s_X . A one-parameter subgroup of the group \hat{K} determined by a geodesic γ , $t \longmapsto p_t(\gamma) := s_{\gamma(t/2)} \circ s_{\gamma(0)} = s_{\exp(v/2)} \circ s_{\gamma(0)}$ with $v = \gamma'(0)$ is called a transvection. We denote by K the identity component of the Lie group generated by transvections [6, Lemma 3.1] and let $M = K/K_X$, where K_X is the isotropy group for some fixed $X \in M$. Let \mathfrak{k} be the Lie algebra of the group K. Then we have a Cartan decomposition $\mathfrak{k} = \mathfrak{k}_+ + \mathfrak{k}_-$ with respect to the involution σ given by the conjugation of the reflection s_X , which satisfies [1,6]:

$$[\mathfrak{k}_{-},\mathfrak{k}_{-}] = \mathfrak{k}_{+}. \tag{1}$$

The infinitesimal transvection $t_v \in \mathfrak{k}_-$ is given by its differential $dp_s(\gamma)|_{s=0}$. We identify \mathfrak{k}_- with the tangent space T_xM and define a metric on \mathfrak{k}_- such that $\langle t_v, t_w \rangle_{\mathfrak{k}_-} = \langle v, w \rangle$ for any $v, w \in T_xM$. The isotropy action on the tangent space is assumed to be effective in order to get the non-degenerate metric on \mathfrak{k}_- .

Put $\langle \mathfrak{k}_+, \mathfrak{k}_- \rangle = 0$ and define the metric on \mathfrak{k}_+ as:

$$\langle A, [t_{\nu}, t_{w}] \rangle_{\mathfrak{k}_{\perp}} = \langle [A, t_{\nu}], t_{w} \rangle_{\mathfrak{k}_{\perp}} \tag{2}$$

for all $A \in \mathfrak{k}_+$ and $v, w \in T_x M$. This is well defined by the property of the curvature tensor $R(v, w)u = -[[t_v, t_w], t_u]$ of an intrinsically symmetric space M and the effective isotropy action. Then we get an $\mathrm{ad}(\mathfrak{k})$ -invariant metric on \mathfrak{k} . Put $\mathfrak{p}_- = T_x M$ and $\mathfrak{p}_+ = T_x M^{\perp}$. A Lie algebra \mathfrak{k} is extended to a Lie algebra $\mathfrak{g} = \mathfrak{k} + \mathfrak{p}$ [2] by defining a skew symmetric product $[\,,\,]$ on $\mathfrak{g} = \mathfrak{k} + \mathfrak{p}$:

$$[A, v] = A_* v, \qquad \langle A, [v, w] \rangle_{\mathfrak{p}} = \langle A_* v, w \rangle, \tag{3}$$

where * denotes the linearized action of A. Note that there is another Cartan decomposition $\mathfrak{g} = \mathfrak{g}_+ + \mathfrak{g}_-$ by putting $\mathfrak{g}_+ = \mathfrak{k}_+ + \mathfrak{p}_+$ and $\mathfrak{g}_- = \mathfrak{k}_- + \mathfrak{p}_-$.

The following bracket relations of an extrinsic symmetric space will be used to calculate the Killing form $B^{\mathfrak{g}}$:

$$[t_{\nu}, w] = \alpha(\nu, w), \qquad [t_{\nu}, \eta] = -S_{\eta}\nu, \tag{4}$$

where α is the second fundamental form and S is the Shape operator [2,4,5]. And since the linear maps $t:T_xM\to\mathfrak{k}_-$, $v\mapsto t_v$ and $S:(T_xM)^\perp\to S(T_xM)$, $\eta\mapsto S_\eta$ are equivariant with respect to the action of \mathfrak{k}_+ , we see that:

$$[A, t_{\nu}] = t_{A_*\nu}, \qquad [A_*, S_{\eta}] = S_{A_*\eta}$$
 (5)

for all $A \in \mathfrak{k}_+$ [2].

The Killing form of a Lie algebra \mathfrak{g} is defined by $B^{\mathfrak{g}}(v,w)=\operatorname{trace}(\operatorname{ad}(v)\circ\operatorname{ad}(w))$ for all $v,w\in\mathfrak{g}$. Let us denote by $B^U(v,w)=\sum_i\epsilon_i\langle\operatorname{ad}(v)\operatorname{ad}(w)u_i,u_i\rangle$ the partial trace of a non-degenerate subspace U in \mathfrak{g} , where $\{u_i\}$ is an orthonormal basis of U with $\langle u_i,u_j\rangle=\epsilon_i\delta_{ij}$. The Killing form of a symmetric space is well known. Let \mathfrak{k} be a Lie algebra of a symmetric space with a Cartan decomposition $\mathfrak{k}=\mathfrak{k}_++\mathfrak{k}_-$ satisfying $[\mathfrak{k}_-,\mathfrak{k}_-]=\mathfrak{k}_+$. Since $\operatorname{ad}(t_v)$ is skew symmetric for $t_v\in\mathfrak{k}_-$ and maps \mathfrak{k}_- to \mathfrak{k}_+ and vice versa, we have $B^{\mathfrak{k}_-}=B^{\mathfrak{k}_+}$ on \mathfrak{k}_- . Hence we get

$$B^{\mathfrak{k}}(t_{v},t_{w})=2B^{\mathfrak{k}_{-}}(t_{v},t_{w}).$$

For an orthonormal basis $\{t_{e_i}\}$ of \mathfrak{k}_- with sign $\epsilon_i = \langle t_{e_i}, t_{e_i} \rangle$, the Killing form $B^{\mathfrak{k}}$ on \mathfrak{k}_- is calculated by:

$$\begin{split} B^{\ell}(t_{v},t_{w}) &= 2\sum \epsilon_{i} \langle \operatorname{ad}(t_{v}) \operatorname{ad}(t_{w}) t_{e_{i}}, t_{e_{i}} \rangle \\ &= 2\sum \epsilon_{i} \langle \left[t_{v}, \left[t_{w}, t_{e_{i}} \right] \right], t_{e_{i}} \rangle = -2\sum \epsilon_{i} \langle \left[\left[t_{w}, t_{e_{i}} \right], t_{v} \right], t_{e_{i}} \rangle = 2\sum \epsilon_{i} \langle R(w,e_{i})v, e_{i} \rangle. \end{split}$$

So we obtain:

$$B^{\ell}(t_{v}, t_{w}) = -2\operatorname{Ric}(t_{v}, t_{w}). \tag{6}$$

Let us denote by H the mean curvature of an extrinsic symmetric space M in \mathbb{R}^n_1 . The Killing form $B^{\mathfrak{g}}$ of an extrinsic symmetric space is obtained (see [6]):

$$B^{\mathfrak{g}}(v,w) = -\langle \alpha(v,w), 2mH \rangle = B^{\mathfrak{g}}(t_{v},t_{w}) \tag{7}$$

for all $v, w \in \mathfrak{p}_-$ and $m = \dim M$.

Remark 1. (See [7].) Let $\mathfrak{z}(\mathfrak{k})$ be the center of a Lie algebra \mathfrak{k} . Then:

$$X \in \mathfrak{z}(\mathfrak{k}) \Leftrightarrow 0 = \langle [X, U], V \rangle = \langle X, [U, V] \rangle$$
 for all $U, V \in \mathfrak{k}$,

hence $X \in [\mathfrak{k}, \mathfrak{k}]^{\perp}$. Thus if we assume $[\mathfrak{k}_-, \mathfrak{k}_-] = \mathfrak{k}_+$, then $X \in [\mathfrak{k}_+, \mathfrak{k}_-]^{\perp}$. Suppose that \mathfrak{k} is solvable and indecomposable. Then $X \in \mathfrak{z}(\mathfrak{k})$ must be lightlike, otherwise we get holonomy invariant $\mathrm{ad}(\mathfrak{k}_+)X = 0$ and non-degenerate subspace $\mathbb{R} \cdot X$. For the selfadjoint endomorphism Ric: $\mathfrak{k}_- \to \mathfrak{k}_-$, we get by (6):

$$-2\langle \operatorname{Ric}(t_{\nu}), \mathfrak{z}(\mathfrak{k})^{\perp} \rangle = -2 \operatorname{Ric}(t_{\nu}, \mathfrak{z}(\mathfrak{k})^{\perp}) = B^{\mathfrak{k}}(t_{\nu}, \mathfrak{z}(\mathfrak{k})^{\perp}) \subseteq B^{\mathfrak{k}}(t_{\nu}, [A, t_{w}]) = 0$$

for all $t_{\nu}, t_{w} \in \mathfrak{k}_{-}$ and $A \in \mathfrak{k}_{+}$. Hence $\text{Ric}(t_{\nu}) \subset (\mathfrak{z}(\mathfrak{k})^{\perp})^{\perp} = \mathfrak{z}(\mathfrak{k})$ for all $t_{\nu} \in \mathfrak{k}_{-}$. Since $\mathfrak{z}(\mathfrak{k})$ are totally isotropic and $\text{Ric} : \mathfrak{k}_{-} \to \mathfrak{k}_{-}$ is selfadjoint, we get:

$$0 = \langle \operatorname{Ric}(t_v), \operatorname{Ric}(t_w) \rangle = \langle \operatorname{Ric}^2(t_v), t_w \rangle$$

for all $t_v, t_w \in k_-$. Thus we obtain $Ric^2 = 0$.

We denote by \mathbb{R}^n_{μ} an *n*-dimensional pseudo-Euclidean space whose metric is given by:

$$\langle v, w \rangle = -v_1 w_1 - \dots - v_{\mu} w_{\mu} + v_{\mu+1} w_{\mu+1} + \dots + v_n w_n$$

with $0 \le \mu \le n - 1$.

Lemma 1. Let M be an extrinsic symmetric space in \mathbb{R}^n_μ . If the mean curvature of M is zero everywhere, then the Lie algebra \mathfrak{g} of M is solvable.

Proof. If the mean curvature H is zero, then $B^{\mathfrak{g}}(t_{\nu}, t_{w}) = -B^{\mathfrak{g}}(\nu, w) = 0$ for all $\nu, w \in \mathfrak{p}_{-}$ by (7). It follows from $[\mathfrak{k}_{-}, \mathfrak{k}_{-}] = \mathfrak{k}_{+}$ (1) and the $ad(\mathfrak{g})$ -invariant Killing form that:

$$B^{\mathfrak{g}}(\mathfrak{k}_{+},\mathfrak{k}_{+}) = B^{\mathfrak{g}}(\mathfrak{k}_{+},[\mathfrak{k}_{-},\mathfrak{k}_{-}]) = B^{\mathfrak{g}}([\mathfrak{k}_{+},\mathfrak{k}_{-}],\mathfrak{k}_{-}) \subseteq B^{\mathfrak{g}}(\mathfrak{k}_{-},\mathfrak{k}_{-}) = 0$$

and $B^{\mathfrak{g}}(\mathfrak{p}_+,\mathfrak{p}_+)=0$ since:

$$B^{\mathfrak{g}}(\mathfrak{p}_+,[\mathfrak{k}_+,\mathfrak{p}_+]) = B^{\mathfrak{g}}(\mathfrak{k}_+,[\mathfrak{p}_+,\mathfrak{p}_+]) \subseteq B^{\mathfrak{g}}(\mathfrak{k}_+,\mathfrak{k}_+) = 0,$$

$$B^{\mathfrak{g}}([\mathfrak{k}_{-},\mathfrak{p}_{-}],\mathfrak{p}_{+}) = B^{\mathfrak{g}}(\mathfrak{k}_{-},[\mathfrak{p}_{-},\mathfrak{p}_{+}]) \subset B^{\mathfrak{g}}(\mathfrak{k}_{-},\mathfrak{k}_{-}) = 0.$$

Therefore $B^{\mathfrak{g}}$ is solvable by the Cartan Criterion for solvability, that is, $B^{\mathfrak{g}}(X,Y)=0$ for all $X\in\mathfrak{g}$ and $Y\in[\mathfrak{g},\mathfrak{g}]$. \square

To show Lemma 2, the following bracket relations in [2] of an extrinsic symmetric space are used:

$$[v, w] = [t_v, t_w], \qquad [v, \eta] = t_{S_n v}, \qquad [\eta, \xi] v = -[S_{\eta}, S_{\xi}] v \tag{8}$$

for any $v, w \in \mathfrak{p}_-$ and $\eta, \xi \in \mathfrak{p}_+$.

Lemma 2. If $X + t_Y \in \mathfrak{g}_- \cap \mathfrak{z}(\mathfrak{g})$ for $t_Y \in \mathfrak{k}_-$ and $X \in \mathfrak{p}_-$, then $\mathrm{Ric}(X, v) = \mathrm{Ric}(Y, v) = 0$, AX = AY = 0 and $\alpha(X, v) = \alpha(Y, v) = 0$ for all $A \in \mathfrak{k}_+$ and $v \in \mathfrak{p}_-$.

Proof. Let $X + t_Y \in \mathfrak{g}_- \cap \mathfrak{z}(\mathfrak{g})$ for $t_Y \in \mathfrak{k}_-$ and $X \in \mathfrak{p}_-$. Then by the relations (3), (4), (5) and (8), we get:

$$[A, X] = 0,$$
 $[t_w, X] = \alpha(w, X) = 0,$ $[w, X] = 0,$ $[\eta, X] = t_{S_n X} = 0,$

$$[A,t_Y] = t_{A_*Y} = 0, \qquad [t_w,t_Y] = [w,Y] = 0, \qquad [w,t_Y] = -\alpha(w,Y) = 0, \qquad [\eta,t_Y] = S_\eta Y = 0$$

for all $w \in \mathfrak{p}_-$ and $\eta \in \mathfrak{p}_+$. Equivalently we get

$$[A, t_X] = t_{A_*X} = 0,$$
 $[t_w, t_X] = [w, X] = 0,$

$$[w, t_X] = -\alpha(w, X) = 0,$$
 $[\eta, t_X] = S_{\eta}X = 0$

and

$$[A, Y] = 0,$$
 $[t_w, Y] = \alpha(w, Y) = 0,$ $[w, Y] = 0,$ $[\eta, Y] = t_{S_n Y} = 0.$

The curvature tensor of a symmetric space shows Ric(X, v) = Ric(Y, v) = 0. \square

Proof of the theorem. The Killing form $B^{\mathfrak{g}}$ of an extrinsic symmetric space shows that if the mean curvature of an extrinsic symmetric space in \mathbb{R}^n_1 is zero, then \mathfrak{g} is solvable by Lemma 1. A solvable Lie algebra has the decreasing sequence called commutator series whose end is Abelian ideal. Let us denote by $\mathfrak{z}(\mathfrak{g})$ the center of a Lie algebra \mathfrak{g} . If $\mathfrak{z}(\mathfrak{g}) \cap \mathfrak{p}_+$ is not empty, then we have $S_{\eta}v = 0$ by the bracket relation (4) for all $v \in \mathfrak{p}_-$ and $\eta \in \mathfrak{z}(\mathfrak{g}) \cap \mathfrak{p}_+$. It suffices to consider the quotient Lie algebra $\mathfrak{g}/(\mathfrak{z}(\mathfrak{g}) \cap \mathfrak{p}_+)$ which we denote again by \mathfrak{g} . So the elements of the center belong to \mathfrak{g}_- by Remark 1 and Lemma 2. For a solvable Lie algebra \mathfrak{g} , we get $\mathfrak{g} = 0$ in the case of $[\mathfrak{g},\mathfrak{g}] = \mathfrak{g}$. So M is trivially totally geodesic by (4). Thus we need to look at the nontrivial case of $[\mathfrak{g},\mathfrak{g}] \subsetneq \mathfrak{g}$. First, if M is spacelike, then there is a tangent vector $X \in \mathfrak{z}(\mathfrak{g})$ satisfying $\alpha(X,v) = 0$ for all $v \in \mathfrak{p}_-$ by Lemma 2. The quotient Lie algebra $\mathfrak{g}/\mathfrak{z}(\mathfrak{g})$ is also solvable. By the induction arguments, we see that M is totally geodesic. Second, in the case of timelike M with $[\mathfrak{g},\mathfrak{g}] \subsetneq \mathfrak{g}$, we may have a lightlike tangent vector $X \in \mathfrak{z}(\mathfrak{g})$ with $\alpha(X,v) = 0$ for all $v \in \mathfrak{p}_-$ by Lemma 2. Otherwise M is totally geodesic by the first induction arguments. So after factoring out (if necessary) a totally geodesic Riemannian extrinsic space by the first arguments, we get an indecomposable Lie algebra. Thus we need to look at the case where $\mathrm{Ric}^2 = 0$ by Remark 1. Hence we consider $\mathrm{Ric}(X,v) = 0$ for a lightlike $X \in \mathfrak{p}_- \cap \mathfrak{z}(\mathfrak{g})$ and all $v \in \mathfrak{p}_-$ by Lemma 2 and an indecomposable Lie algebra \mathfrak{g} . Given the above X, we take an orthonormal basis $\{e_i\}_{i=1}^m$ of \mathfrak{p}_- such that $X = e_1 + e_m$ and $N = -\frac{1}{2}e_1 + \frac{1}{2}e_m$ for timelike e_1 and spacelike e_m . Since the Ricci tensor is two-step nilpotent with $\mathrm{Ric}(X,v) = 0$ for all $v \in \mathfrak{p}_-$, we

$$Ric X = 0, Ric N = X, Ric(v, w) = 0, for all v, w \in Span\{X, N\}^{\perp}, (9)$$

where $\operatorname{Span}\{X, N\}^{\perp}$ is the orthogonal complement of $\operatorname{Span}\{X, N\}$. Thus it follows from:

$$Ric(X, e_1) = Ric(e_1 + e_m, e_1) = Ric(e_1, e_1) + Ric(e_1, e_m) = 0,$$

$$\operatorname{Ric}(X, e_m) = \operatorname{Ric}(e_1 + e_m, e_m) = \operatorname{Ric}(e_1, e_m) + \operatorname{Ric}(e_m, e_m) = 0$$

that:

$$Ric(e_1, e_1) = Ric(e_m, e_m) = -Ric(e_1, e_m). \tag{10}$$

Similarly from $\alpha(X, e_1) = 0$ and $\alpha(X, e_m) = 0$, we get:

$$\alpha(e_1, e_1) = \alpha(e_m, e_m) = -\alpha(e_1, e_m). \tag{11}$$

The Ricci equation:

$$\operatorname{Ric}(v, w) = \sum \epsilon_i \langle R(v, e_i)e_i, w \rangle = \langle \alpha(v, w), mH \rangle - \sum \epsilon_i \langle \alpha(v, e_i), \alpha(e_i, w) \rangle$$

shows that if H = 0, then we have by (11):

$$-\operatorname{Ric}(e_{1}, e_{m}) = -\langle \alpha(e_{1}, e_{1}), \alpha(e_{1}, e_{m}) \rangle + \langle \alpha(e_{1}, e_{m}), \alpha(e_{m}, e_{m}) \rangle + \sum_{i \neq 1, m} \langle \alpha(e_{1}, e_{i}), \alpha(e_{i}, e_{m}) \rangle$$

$$= \sum_{i \neq 1, m} \langle \alpha(e_{1}, e_{i}), \alpha(e_{i}, e_{m}) \rangle \geqslant 0,$$
(12)

since \mathfrak{p}_+ is positive definite. In the same way, we have:

$$-\operatorname{Ric}(e_1, e_1) = \sum_{i \neq 1, m} \langle \alpha(e_1, e_i), \alpha(e_i, e_1) \rangle \geqslant 0. \tag{13}$$

Therefore it follows from (10), (12) and (13) that:

$$Ric(e_1, e_1) = Ric(e_m, e_m) = -Ric(e_1, e_m) = 0,$$
 (14)

which leads to the flat Ricci tensor Ric = 0 together with (9).

Then by (13) and $\alpha(X, e_i) = 0$, we get:

$$\alpha(e_1, e_i) = 0, \qquad \alpha(e_m, e_i) = 0 \quad (i \neq 1, m). \tag{15}$$

Again the Ricci equation and (15):

$$0 = \text{Ric}(e_i, e_i) = \langle \alpha(e_i, e_i), mH \rangle - \sum_{k \in \mathbb{N}} \epsilon_k \langle \alpha(e_i, e_k), \alpha(e_k, e_i) \rangle$$

$$= -\langle \alpha(e_i, e_1), \alpha(e_1, e_i) \rangle + \langle \alpha(e_i, e_m), \alpha(e_m, e_i) \rangle + \sum_{k \neq 1, m} \langle \alpha(e_i, e_k), \alpha(e_k, e_i) \rangle$$

$$= \sum_{k \neq 1, m} \langle \alpha(e_i, e_k), \alpha(e_k, e_i) \rangle$$

imply:

$$\alpha(e_i, e_k) = 0 \quad (i, k \neq 1, m). \tag{16}$$

By the second fundamental form (11), (15), (16) and the Gauss equation, we get:

$$R(e_1, e_i, e_i, e_1) = \langle \alpha(e_i, e_i), \alpha(e_1, e_1) \rangle - \langle \alpha(e_1, e_i), \alpha(e_1, e_i) \rangle = 0,$$

$$R(e_m, e_i, e_i, e_m) = 0, \qquad R(e_i, e_i, e_i, e_i) = 0 \quad (i, j \neq 1, m).$$

Hence *M* is flat with one nonzero normal vector $\alpha(e_1, e_1)$ (11).

With the same arguments in the proof of theorem, we get:

Corollary. Let M be an extrinsic symmetric space in pseudo-Euclidean space \mathbb{R}^n_{μ} . If M is spacelike with zero mean curvature, then M is totally geodesic.

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