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p-Adic Hodge theory for open varieties

Théorie de Hodge p-adique pour les variétés ouvertes

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ABSTRACT

This is an announcement of results whose proofs will be published elsewhere: We establish forms of the C_{st} conjecture of Fontaine–Jannsen for proper semistable pairs over a complete discrete valuation ring R of mixed characteristic $(0, p)$ with perfect residue field, and partially properly supported cohomology. We derive the conjecture C_{pst} for separated K-schemes of finite type, where K is the fraction field of R. The proof is based on the method of syntomic complexes and p-adic vanishing cycles. A new ingredient is the use of hollow log schemes à la Ogus to provide tubular neighborhoods of intersections of components of divisors with normal crossings.

RÉSUMÉ

Cette Note annonce des résultats dont les démonstrations seront publiées ailleurs. Ils concernent des formes de la conjecture C_{st} de Fontaine–Jannsen pour les paires semistables propres sur un anneau de valuation discrète complet R de caractéristique mixte $(0, p)$ à corps résiduel parfait et des groupes de cohomologie partiellement à support propre. On en déduit la conjecture C_{pst} pour les K-schémas séparés de type fini, où K est le corps des fractions de R. La méthode de démonstration est celle des complexes syntomiques et des cycles évanescents p-adiques. Un nouvel ingrédient est l'utilisation de log schémas creux à la Ogus, qui fournissent des voisinages tubulaires d'intersections de composantes de diviseurs à croisements normaux.

1. Statements of results

Let K be a complete discrete valuation field of mixed characteristic $(0, p)$ whose residue field k is perfect. Let W be the ring of Witt vectors with coefficients in k, and K_0 be its fractional field. Let O_K denote the valuation ring of K, $\bar{K}$ an algebraic closure of K, and $G_K = \mathrm{Gal}(\bar{K}/K)$ the absolute Galois group of K. Let (X, D) be a semistable pair over O_K. (Here we replace "strict semistable" and "strict normal crossings" in the definition of a strict semistable pair in [2, Section 6.3] by "semistable" and "normal crossings" respectively.) Let Y and C be the special fibers of X and D respectively. Let $D = D^1 \cup D^2$ be a decomposition of D such that D^1 and D^2 do not contain common irreducible components. Put $C^i := D^i \otimes_{O_K} k$ for $i = 1, 2$. For $i = 1, 2$, let $M \supset M(D^i)$ be the log-structures on X (in the sense of Fontaine–Illusie–Kato [6]) defined by the

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divisor $Y \cup D$ and $Y \cup D^i$ respectively, and M_{X_K} be the one defined by the divisor D_K, where the subscripts $(-)_K$ denote the base change $K \otimes_{O_K}$. For $i = 1, 2$, let $M_Y \supset M_Y(C^i)$ be the pull-back log-structures on Y from the log-structure M_X and $M_X(D^i)$ respectively. Let N be the log-structure on $S := \operatorname{Spec} O_K$ defined by the maximal ideal, and N_n^0 be the log-structure on $\operatorname{Spec} W_n$ associated to $\Gamma(\operatorname{Spec} k, N_1^0) \to k \stackrel{[\ ]}{\to} W_n$, where N_1^0 is the log-structure on $\operatorname{Spec} k$ pulled-back from (S, N), and $[-]$ is the Teichmüller map.

We define "partially properly supported cohomologies" in the étale context, the de Rham one, and the crystalline one. They have proper support on $D^1_{\bar{K}}$, D^1_K, and C^1 respectively. Let j_1, j_2 be the open immersions $(X \setminus D^1)_{\bar{K}} \hookrightarrow X_{\bar{K}}$, and $(X \setminus D)_{\bar{K}} \hookrightarrow (X \setminus D^1)_{\bar{K}}$, respectively. Then, we define $H^m_{\text{ét}}(X_{\bar{K}}, D^1_{\bar{K}!}, D^2_{\bar{K}*}, \mathbb{Q}_p) := H^m_{\text{ét}}(X_{\bar{K}}, j_{1!}Rj_{2*}\mathbb{Q}_p)$. Here, we have a continuous action of G_K on this group. Let j'_1, j'_2 denote the open immersions $(X \setminus D)_{\bar{K}} \hookrightarrow (X \setminus D^2)_{\bar{K}}$ and $(X \setminus D^2)_{\bar{K}} \hookrightarrow X_{\bar{K}}$ respectively. The morphism $j_{1!}Rj_{2*}\mathbb{Q}_p \otimes^{\mathbb{L}} j'_{2!}Rj'_{1*}\mathbb{Q}_p \to j_{1!}j_{2!}\mathbb{Q}_p$ induces a product structure

$$H^i_{\text{ét}}\big(X_{\bar{K}}, D^1_{\bar{K}!}, D^2_{\bar{K}*}, \mathbb{Q}_p\big) \otimes H^j_{\text{ét}}\big(X_{\bar{K}}, D^2_{\bar{K}!}, D^1_{\bar{K}*}, \mathbb{Q}_p\big) \to H^{i+j}_{\text{ét},c}\big((X \setminus D)_{\bar{K}}, \mathbb{Q}_p\big).$$

Next, let $\omega^\bullet_{X_K/K} = \Omega^\bullet_{X_K/K}(\log D_K)$ be the de Rham complex with log poles along D_K, and $I(D^1_K)$ be the defining ideal of D^1_K in $\mathcal{O}_{X_K}$. Then, we define $H^m_{\text{dR}}(X_K, D^1_{K!}, D^2_{K*}/K) := H^m(X_K, I(D^1_K)\omega^\bullet_{X_K/K})$. Here, we have a filtration on it by the image of $H^m(X_K, I(D^1_K)\omega^{\bullet \geqslant i}_{X_K/K})$. The morphism $I(D^1_K)\omega^\bullet_{X_K/K} \otimes I(D^2_K)\omega^\bullet_{X_K/K} \to I(D_K)\omega^\bullet_{X_K/K}$ induces a product structure

$$H^i_{\text{dR}}\big(X_K, D^1_{K!}, D^2_{K*}/K\big) \otimes H^j_{\text{dR}}\big(X_K, D^1_{K!}, D^2_{K*}/K\big) \to H^{i+j}_{\text{dR},c}\big((X \setminus D)_K/K\big),$$

where we put $H^{i+j}_{\text{dR},c}((X \setminus D)_K/K) := H^{i+j}_{\text{dR}}(X_K, D_{K!}, \emptyset_*/K)$.

Finally, to define the crystalline cohomology with partial proper support, we define some crystalline sheaves (see also [9, Sections 2 and 5]). We define a sheaf of monoids M_{Y/W_n} by $\Gamma((U, T, M_T, \delta), M_{Y/W_n}) := \Gamma(T, M_T)$ for $(U, T, M_T, \delta) \in ((Y, M_Y)/(W_n, N^0_n, pW_n, \gamma))^{\log}_{\text{crys}}$, and an ideal sheaf $I_{Y/W_n}(C^1)$ of M_{Y/W_n} by setting $\Gamma((U, T, M_T, \delta), I_{Y/W_n}(C^1))$ to consist of $a \in \Gamma((U, T, M_T, \delta), M_{Y/W_n})$ such that the image of a in $M_{T,\bar{x}}/\mathcal{O}^\times_{T,\bar{x}} \cong M_{Y,\bar{x}}/\mathcal{O}^\times_{Y,\bar{x}}$ is contained in $\mathfrak{p}$ at all points $x \in T$ and all primes $\mathfrak{p} \in \operatorname{Spec}(M_{Y,\bar{x}}(C^1)/\mathcal{O}^\times_{Y,\bar{x}})$ of height 1 horizontal with respect to $N^0_{n,\overline{f(x)}}/\mathcal{O}^\times_{W_n,\overline{f(x)}} \to M_{Y,\bar{x}}(C^1)/\mathcal{O}^\times_{Y,\bar{x}}$, i.e., such that the image of $N^0_{n,\overline{f(x)}}/\mathcal{O}^\times_{W_n,\overline{f(x)}}$ is contained in $(M_{Y,\bar{x}}(C^1)/\mathcal{O}^\times_{Y,\bar{x}}) \setminus \mathfrak{p}$. We define an ideal $K_{Y/W_n}(C^1)$ of $\mathcal{O}_{Y/W_n}$ by $I_{Y/W_n}(C^1)\mathcal{O}_{Y/W_n}$. Then, we define

$$H^m_{\text{log-crys}}\big(Y, C^1_!, C^2_*\big) := \varprojlim_n H^m_{\text{crys}}\big((Y, M_Y)/\big(W_n, N^0_n\big), K_{Y/W_n}\big(C^1\big)\big) \otimes \mathbb{Q}_p.$$

The Frobenius morphism induces a semilinear automorphism φ on $H^m_{\text{log-crys}}(Y, C^1_!, C^2_*)$. We define a monodromy operator $\mathcal{N}$ on it as follows. Take an embedding system $\{(U^\bullet, M) \hookrightarrow (Z^\bullet, M_{Z^\bullet}), \{F_{Z_n}\}_n\}$ of $(Y, M_Y) \to (\operatorname{Spec} W_n[T], M)$. Then, we have a distinguished triangle (see also [5, Section 3.6])

$$R\theta_*\big(K_{\mathcal{D}^{\bullet\prime}} \otimes \omega^\bullet_{(Z^{\bullet\prime}, M_{Z^{\bullet\prime}})/(W_n, N^0_n)}\big)[-1] \to W_n \otimes^L_{W_n\langle T\rangle} R\theta_*\big(K_{\mathcal{D}^{\bullet\prime}} \otimes \omega^\bullet_{Z^{\bullet\prime}/W_n}\big)$$
$$\to R\theta_*\big(K_{\mathcal{D}^{\bullet\prime}} \otimes \omega^\bullet_{(Z^{\bullet\prime}, M_{Z^{\bullet\prime}})/(W_n, N^0_n)}\big) \stackrel{[+1]}{\to},$$

where $(\mathcal{D}^\bullet, \delta_{\mathcal{D}^\bullet})$ is the PD-envelope of $U^\bullet \hookrightarrow Z^\bullet$, $K_{\mathcal{D}^\bullet} := K_{Y/W_n}(C^1)_{(U^\bullet \hookrightarrow \mathcal{D}^\bullet, \delta_{\mathcal{D}^\bullet})}$, $\theta = (\theta_*, \theta^*) : (Y^\bullet)^\sim_{\text{ét}} \to Y^\sim_{\text{ét}}$ is the canonical morphism of topoi. The boundary map of the exact sequence after taking $H^*(Y, -)$ of the above distinguished triangle induces a monodromy operator $\mathcal{N}$ on $H^m_{\text{log-crys}}(Y, C^1_!, C^2_*)$ satisfying $\mathcal{N}\varphi = p\varphi\mathcal{N}$ after taking $\mathbb{Q}_p \otimes \varprojlim_n$. (We can also define the monodromy operator by the method of [8, Section 4.3].) We then get a structure of $(\varphi, \mathcal{N})$-module on $H^m_{\text{log-crys}}(Y, C^1_!, C^2_*)$. The morphism $K(C^1)_{Y/W_n} \otimes K(C^2)_{Y/W_n} \to K(C)_{Y/W_n}$ induces a product structure

$$H^i_{\text{log-crys}}\big(Y, C^1_!, C^2_*\big) \otimes H^j_{\text{log-crys}}\big(Y, C^2_!, C^1_*\big) \to H^{i+j}_{\text{log-crys},c}(Y \setminus C),$$

where $H^{i+j}_{\text{log-crys},c}(Y \setminus C) := H^{i+j}_{\text{log-crys}}(Y, C_!, \emptyset_*)$.

Note that $H^m(X, \emptyset_!, D_*) = H^m(X \setminus D)$, and $H^m(X, D_!, \emptyset_*) = H^m_c(X \setminus D)$ for each cohomology where $X = X_{\bar{K}}, X_K, Y$, and $D = D_{\bar{K}}, D_K, C$ respectively.

We construct a Hyodo–Kato isomorphism and a comparison isomorphism for partially properly supported cohomologies:

Proposition 1.1 *(Hyodo–Kato isomorphism for partially properly supported cohomology). Take a uniformizer $\pi \in K$. Then, we have an isomorphism*:

$$\rho_\pi : K \otimes_{K_0} H^m_{\text{log-crys}}\big(Y, C^1_!, C^2_*\big) \stackrel{\sim}{\to} H^m_{\text{dR}}\big(X_K, D^1_{K!}, D^2_{K*}/K\big).$$

Remark. For a unit $u \in O^\times_K$, we have $\rho_{\pi u} = \rho_\pi \exp(\log(u)\mathcal{N})$. See also [5, Theorem (5.1)] and [8, Remark 4.4.18].

Theorem 1.2 *(C_{st} for partially properly supported cohomology). There is a B_{st}-linear isomorphism*

$$B_{\mathrm{st}} \otimes_{\mathbb{Q}_p} H^m_{\text{ét}}\big(X_{\bar{K}}, D^1_{\bar{K}!}, D^2_{\bar{K}*}, \mathbb{Q}_p\big) \cong B_{\mathrm{st}} \otimes_{K_0} H^m_{\text{log-crys}}\big(Y, C^1_!, C^2_*\big),$$

which preserves the action of G_K ($g \otimes g$ on LHS, $g \otimes 1$ on RHS), the Frobenius ($\varphi \otimes 1$ on LHS, $\varphi \otimes \varphi$ on RHS), and the monodromy operator ($\mathcal{N} \otimes 1$ on LHS, $\mathcal{N} \otimes 1 + 1 \otimes \mathcal{N}$ on RHS). It also preserves the Hodge filtration after tensoring with B_{dR} over B_{st} ($\mathrm{Fil} \otimes H^m_{\text{ét}}$ on LHS, $\sum \mathrm{Fil} \otimes \mathrm{Fil}$ on RHS under the above Hyodo–Kato isomorphism after choosing a uniformizer $\pi \in K$). Moreover, these are compatible with product structures.

By standard arguments using de Jong's alterations (cf. [10, appendix]) we derive the de Rham conjecture and the potentially semistable conjecture for separated K-schemes of finite type, as well as variants for partially properly supported cohomologies. For de Rham cohomology of non-smooth varieties, we use the algebraic de Rham cohomology defined by Hartshorne (cf. [3,4]).

Theorem 1.3 *(C_{dR} for partially properly supported cohomology). Let X be a proper smooth K-scheme, and D be a normal crossing divisor on X. Let D be $D^1 \cup D^2$, where D^1 and D^2 have no common irreducible components. Then, there exists a canonical B_{dR}-isomorphism*

$$B_{\mathrm{dR}} \otimes_{\mathbb{Q}_p} H^m_{\text{ét}}\big(X_{\bar{K}}, D^1_{\bar{K}!}, D^2_{\bar{K}*}, \mathbb{Q}_p\big) \cong B_{\mathrm{dR}} \otimes_K H^m_{\mathrm{dR}}\big(X, D^1_!, D^2_*/K\big),$$

which preserves the actions of G_K and the Hodge filtrations. Moreover, it is compatible with the product structures.

Remark. By passing to the associated graded, we get a Hodge–Tate decomposition:

$$\hat{\bar{K}} \otimes_{\mathbb{Q}_p} H^m_{\text{ét}}\big(X_{\bar{K}}, D^1_{\bar{K}!}, D^2_{\bar{K}*}, \mathbb{Q}_p\big) \cong \bigoplus_{0 \leqslant j \leqslant m} \hat{\bar{K}}(-j) \otimes_K H^{m-j}\big(X, I\big(D^1\big)\Omega^j_{X/K}(\log D)\big).$$

Theorem 1.4 *(C_{dR}). Let U be a separated K-scheme of finite type. Then, there exist canonical B_{dR}-isomorphisms*

$$B_{\mathrm{dR}} \otimes_{\mathbb{Q}_p} H^m_{\text{ét}}(U_{\bar{K}}, \mathbb{Q}_p) \cong B_{\mathrm{dR}} \otimes_K H^m_{\mathrm{dR}}(U/K), \qquad B_{\mathrm{dR}} \otimes_{\mathbb{Q}_p} H^m_{\text{ét},c}(U_{\bar{K}}, \mathbb{Q}_p) \cong B_{\mathrm{dR}} \otimes_K H^m_{\mathrm{dR},c}(U/K),$$

which preserve the actions of G_K and the Hodge filtrations. Here, H_{dR} and $H_{\mathrm{dR},c}$ mean Hartshorne's algebraic de Rham cohomology.

Remark. We define the Hodge filtration on Hartshorne's algebraic de Rham cohomology in the following way: Let $U^\bullet \to U$ be a hypercovering over K, and $U^\bullet \hookrightarrow Z^\bullet$ be an embedding, where $U^\nu \hookrightarrow Z^\nu$ is a closed immersion and Z^ν is smooth over K for each ν. Put $\hat{Z}^\bullet$ and $\hat{\Omega}^{\bullet'}_{Z^\bullet/K}$ to be the formal completions of $Z^\bullet$ and $\Omega^{\bullet'}_{Z^\bullet/K}$ along the ideal of $U^\bullet$ respectively. Then, we define i-th Hodge filtration to be the image of the homomorphism $H^m(\hat{Z}^\bullet, \hat{\Omega}^{\bullet' \geqslant i}_{Z^\bullet/K}) \to H^m(\hat{Z}^\bullet, \hat{\Omega}^{\bullet'}_{Z^\bullet/K}) = H^m_{\mathrm{dR}}(U/K)$.

Theorem 1.5 *(C_{pst}). (See also [2, Section 1].) Let U be a scheme of finite type over $\operatorname{Spec} K$. Then, $H^m_{\text{ét}}(U_{\bar{K}}, \mathbb{Q}_p)$, $H^m_{\text{ét},c}(U_{\bar{K}}, \mathbb{Q}_p)$ are potentially semistable p-adic representations of G_K.*

Remark. By a theorem of Berger and André–Kedlaya–Mebkhout ("de Rham representations are potentially semistable representations", see [1]), Theorem 1.4 implies Theorem 1.5. However, we show it without using the (φ, Γ)-theory, the rigid analytic method, or the p-adic differential equation theory.

2. Outline of proof

We use the method of syntomic cohomology. Product structures play a key role in the final step of the proof. The crucial point is to show the compatibility of comparison maps with the product structures. We do this by introducing "hollow log schemes" (à la Ogus [7]).

Let us come back to the situation of Theorem 1.2. For simplicity we assume that $D^2 = \emptyset$ and D has simple normal crossings. Put $D^{(c)} := \coprod_{\# J = c,\ J \subset I} \bigcap_{j \in J} D_j$, where $D = \bigcup_j D_j$. We construct comparison maps on the intersections of the divisors $D^{(c)}$'s, and put them together to get a comparison map about the properly supported cohomology H_c (and the partially properly supported cohomology in the general situation).

Let $M_{D^{(c)}}$ be the pull-back to $D^{(c)}$ of the log-structure M. The non-trivial part of the log-structure $M_{D^{(c)}}/\mathcal{O}^\times_{D^{(c)}}$ "spreads over" $D^{(c)}$, so we call these log schemes *hollow log schemes* (the notion of "hollow log schemes" first appeared in [7]). Note that their reduction modulo p^n is not log smooth over $(\operatorname{Spec} W_n, N^0_n)$ for any n in general.

Let $i^{(c)} : (D^{(c)}, M_{D^{(c)}}) \to (X, M)$ to be the canonical morphism. Then, for the étale side, we have a resolution

$$0 \to j_! \mathbb{Z}/p^n\mathbb{Z}_{(X \setminus D)_{\bar{K}}} \to \mathbb{Z}/p^n\mathbb{Z}_{(X_{\bar{K}}, M_{X_{\bar{K}}})} \to i^{(1)}_* \mathbb{Z}/p^n\mathbb{Z}_{(D^{(1)}_{\bar{K}}, M_{D^{(1)}_{\bar{K}}})} \to i^{(2)}_* \mathbb{Z}/p^n\mathbb{Z}_{(D^{(2)}_{\bar{K}}, M_{D^{(2)}_{\bar{K}}})} \to \cdots$$

in the log étale site. For the de Rham side, we also have a similar resolution

$$0 \to I(D_K)\omega^{\bullet}_{(X_K, M_{X_K})} \to \omega^{\bullet}_{(X_K, M_{X_K})} \to i^{(1)}_{*}\omega^{\bullet}_{(D^{(1)}_K, M_{D^{(1)}_K})} \to i^{(2)}_{*}\omega^{\bullet}_{(D^{(2)}_K, M_{D^{(2)}_K})} \to \cdots.$$

By introducing a certain crystalline sheaf of rings $\mathcal{O}^{/\mathrm{hol}}$ we get a similar resolution for the crystalline side here. We define $\mathcal{O}^{/\mathrm{hol}}$ on $((D^{(c)}_n, M)/(W_n, N^0_n, pW_n, \gamma))^{\log}_{\mathrm{crys}}$ as follows. First, we define a subsheaf $M^{\mathrm{hol}}_{D^{(c)}}$ of $M_{D^{(c)}}$ for $c \geqslant 1$ as follows. Take an irreducible component V of $D^{(c)}$. Then, we define a log-structure M^{hol}_V on V by pulling-back the log-structure on X defined by the irreducible components of D in which the image of V in X is contained. Next, we define a crystalline sheaf M^{hol} on the log crystalline site $((D^{(c)}_n, M_{D^{(c)}_n})/(W_n, N^0_n, pW_n, \gamma))^{\log}_{\mathrm{crys}}$. For $((D^{(c)}_n, M_{D^{(c)}_n})/(W_n, N^0_n, pW_n, \gamma))^{\log}_{\mathrm{crys}} \ni (i : U \hookrightarrow T, M_T, \delta)$, let M^{hol}_T denote the subsheaf of M_T characterized by the isomorphism

$$i_* M^{\mathrm{hol}}_{D^{(c)}_n}|_U / \mathcal{O}^{\times}_U \cong M^{\mathrm{hol}}_T / \mathcal{O}^{\times}_T$$

under the isomorphism $i_* M|_U/\mathcal{O}^{\times}_U \cong M_T/\mathcal{O}^{\times}_T$. We define $M^{\mathrm{hol}}_{(i:U\hookrightarrow T, M_T, \delta)} := M^{\mathrm{hol}}_T$. Then, we define the crystalline sheaf $\mathcal{O}^{/\mathrm{hol}}$ on $((D^{(c)}_n, M)/(W_n, N^0_n, pW_n, \gamma))^{\log}_{\mathrm{crys}}$ to be

$$\mathcal{O}^{/\mathrm{hol}}_{(i:U\hookrightarrow T, M_T, \delta)} := \mathcal{O}_T / \bigl(\text{PD-ideal generated by the image of } M^{\mathrm{hol}}_T \setminus \mathcal{O}^{\times}_T \to \mathcal{O}_T\bigr)$$

for $((D^{(c)}_n, M_{D^{(c)}_n})/(W_n, N^0_n, pW_n, \gamma))^{\log}_{\mathrm{crys}} \ni (i : U \hookrightarrow T, M_T, \delta)$.

We have a product structure

$$\mathbb{Z}/p^n\mathbb{Z}_{(X_{\bar K}, M_{\bar K})} \otimes \bigl[i^{(\bullet)}_{*}\mathbb{Z}/p^n\mathbb{Z}_{(D^{(\bullet)}_{\bar K}, M_{D^{(\bullet)}_{\bar K}})}\bigr] \to \bigl[i^{(\bullet)}_{*}\mathbb{Z}/p^n\mathbb{Z}_{(D^{(\bullet)}_{\bar K}, M_{D^{(\bullet)}_{\bar K}})}\bigr],$$

which induces the product structure $H^i_{\text{ét}} \otimes H^j_{\text{ét},c} \to H^{i+j}_{\text{ét},c}$, and a product

$$\omega^{\bullet'}_{(X_K, M_{X_K})} \otimes \bigl[i^{(\bullet)}_{*}\omega^{\bullet'}_{(D^{(\bullet)}_K, M_{D^{(\bullet)}_K})}\bigr] \to \bigl[i^{(\bullet)}_{*}\omega^{\bullet'}_{(D^{(\bullet)}_K, M_{D^{(\bullet)}_K})}\bigr],$$

which induces the product structure $H^i_{\mathrm{dR}} \otimes H^j_{\mathrm{dR},c} \to H^{i+j}_{\mathrm{dR},c}$. We show the compatibility of the comparison maps with the product structures.

In the proof of the general C_{pst} conjecture we establish a comparison isomorphism

$$B_{\mathrm{st}} \otimes_{\mathbb{Q}_p} H^m_{\text{ét},(c)}\bigl(U^{\bullet}_{\bar K}, \mathbb{Q}_p\bigr) \cong B_{\mathrm{st}} \otimes_{W(k_L)} H^m_{\mathrm{crys},(c)}\bigl(\bigl(U^{\bullet}\bigr) \otimes_{O_L} k_L / W(k_L)\bigr),$$

for a truncated simplicial semistable pair $U^{\bullet}$ over O_L where $[L : K] < \infty$, and $H_{(c)} = H$ or H_c.

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