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The main result and its corollary need an additional hypothesis, as follows:

**Theorem 1.** Let $Q$ be a countably generated Archimedean quadratic module contained in the algebra $A = \mathbb{R}[x_1, \ldots, x_d, h_1, \ldots, h_m]$ spanned by the coordinate functions and by Borel measurable functions $h_1, \ldots, h_m$ on $\mathbb{R}^d$. Assume that $Q$ has the moment property, that is, every linear functional on $A$ which is non-negative on $Q$ is representable by a positive measure. If a function $f \in A$ is positive on $P(Q)$, then $f \in Q$.

Similarly,

**Corollary 2.** Let $q_1, \ldots, q_n$ be elements of the algebra $A = \mathbb{R}[x_1, \ldots, x_d, h_1, \ldots, h_m]$ generated by the coordinate functions and by Borel measurable functions $h_1, \ldots, h_m$ on $\mathbb{R}^d$. Let $\Sigma A^2$ denote the convex cone of sums of squares, and consider the Borel measurable set

$$P(q_0, q_1, \ldots, q_n) = \{x \in \mathbb{R}^d; q_i(x) \geq 0, \ 0 \leq i \leq n\},$$

where $q_0(x) = 1 - (x_1^2 + \cdots + x_d^2 + h_1^2 + \cdots + h_m^2)$.

If a function $f \in A$ is positive on $P(q_0, q_1, \ldots, q_n)$, then $f \in Q$, provided that the quadratic module $Q = \Sigma A^2 + q_0 \Sigma A^2 + \cdots + q_n \Sigma A^2$ possesses the moment property.

In its turn, the moment property of the cone $Q$ can be restated as a density of $Q$ in the set of all elements of $A$ which are non-negative on $P(Q)$, in the strongest locally convex topology carried by $A$.


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