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C. R. Acad. Sci. Paris, Ser. I 347 (2009) 1201–1206

COMPTES RENDUS
MATHEMATIQUE

Probability Theory

One barrier reflected backward doubly stochastic differential equations with continuous generator

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Received 28 September 2008; accepted after revision 6 August 2009

Available online 16 September 2009

Presented by Marc Yor

Abstract

We prove the existence and uniqueness of solutions to Reflected Backward Doubly Stochastic Differential Equations (RBDSDEs) with one continuous barrier and uniformly Lipschitz coefficients. The existence of a maximal and a minimal solution for RBDSDEs with continuous generator is also established. ***To cite this article: K. Bahlali et al., C. R. Acad. Sci. Paris, Ser. I 347 (2009).***

Résumé

Équations différentielles doublement stochastiques rétrogrades réfléchies à une barrière. Nous établissons l'existence et l'unicité des solutions pour des équations différentielles doublement stochastiques rétrogrades réfléchies (EDDSRR) avec une barrière continue et des coefficients uniformement lipschitziens. Nous montrons également l'existence d'une solution maximale et d'une solution minimale pour des EDDSRR ayant un générateur continu. ***Pour citer cet article : K. Bahlali et al., C. R. Acad. Sci. Paris, Ser. I 347 (2009).***

Version française abrégée

Les résultats essentiels de cette note sont

Théorème 0.1. *Sous les conditions* (H1), (H2), (H3) *et* (H4), *l'Éq.* (1) *admet une solution unique.*

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[1] Partially supported by Marie Curie ITN, No. 213841-2.

[2] Partially supported by PHC Volubilis MA/142/06.

[3] Partially supported by PHC Tassili 07MDU705.

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doi:10.1016/j.crma.2009.08.001

Théorème 0.2 *(Comparaison). Soient* (ξ, f, g, S) *et* (ξ', f', g, S') *deux EDDSRR vérifiant les conditions* (H1), (H2), (H3) *et* (H4). *On suppose de plus que* :

(i) $\xi \leqslant \xi'$ *p.s.*
(ii) $f(t, y, z) \leqslant f'(t, y, z)\,\mathrm{d}P \times \mathrm{d}t$ *p.p.* $\forall (y, z) \in \mathbb{R} \times \mathbb{R}^d$.
(iii) $S_t \leqslant S'_t$, $0 \leqslant t \leqslant T$ *p.s.*

Soit (Y, Z, K) *une solution de EDDSRR* (ξ, f, g, S) *et* (Y', Z', K') *une solution de EDDSRR* (ξ', f', g, S'). *Alors*

$$Y_t \leqslant Y'_t, \quad 0 \leqslant t \leqslant T \quad p.s.$$

Théorème 0.3. *Sous les conditions* (H1), (H3), (H4) *et* (H5), *l'Éq.* (1) *admet une solution minimale et une solution maximale.*

1. Introduction

The backward doubly stochastic differential equations (BDSDE) were introduced by Pardoux and Peng in [5], where the existence and uniqueness of solutions are established under uniformly Lipschitz coefficients. In this Note, we study the case where the solution is forced to stay above a given stochastic process, called the obstacle. We obtain the real valued reflected backward doubly stochastic differential equation:

$$Y_t = \xi + \int_t^T f(s, Y_s, Z_s)\,\mathrm{d}s + \int_t^T g(s, Y_s, Z_s)\,\mathrm{d}B_s + K_T - K_t - \int_t^T Z_s\,\mathrm{d}W_s, \quad 0 \leqslant t \leqslant T \tag{1}$$

where the $\mathrm{d}W$ is a forward Itô integral and the $\mathrm{d}B$ is a backward Itô integral.

First, we establish the existence and uniqueness of solutions for the RBDSDE (1) in the case where the coefficients f and g are uniformly Lipschitz in the variables y and z.

Due to the fact that the solution should be adapted to a family $(\mathcal{F}_t)$ which is not a filtration, the usual techniques used in the classical reflected BSDEs (see e.g. [3]) does not work. Indeed, the section theorem cannot be easily used to derive that the solution stays above the obstacle for all time.

We give here a method which allows us to overcome this difficulty. In the Lipschitz case, the idea consists to start from the basic RBDSDE with f and g independent from (y, z). We transform it to a RBDSDE with $f = g = 0$, for which we prove the existence and uniqueness of solutions by a penalization method. The section theorem is then used in this simple context ($f = g = 0$) to prove that the solution, of the RBDSDE with $f = g = 0$, stays above the obstacle at each time. The case where the coefficients f and g depend on (y, z) is then deduced by using a Picard type approximation.

Second, we consider the case where the coefficient f is continuous. We then approximate f by a monotone sequence of Lipschitz functions (f_n) and use a comparison theorem (which is established here for reflected BDSDEs) to derive the existence of a maximal and a minimal solution are then obtained by passing to the limit.

The Note is organized as follows. In Sections 2, we give some notations, assumptions and definitions. In Section 3, we present our main results. Section 4 is devoted to the (sketched) proofs.

2. Notations, definitions and assumptions

Let $(\Omega, \mathcal{F}, P)$ be a complete probability space and $T > 0$ be a fixed real number. Let $\{W_t,\ 0 \leqslant t \leqslant T\}$ and $\{B_t,\ 0 \leqslant t \leqslant T\}$ be two independent standard Brownian motions, defined on $(\Omega, \mathcal{F}, P)$, with values in $\mathbb{R}^d$ and $\mathbb{R}$ respectively. For $t \in [0, T]$, we define $\mathcal{F}_t := \mathcal{F}_t^W \vee \mathcal{F}_{t,T}^B$ and $\mathcal{G}_t := \mathcal{F}_t^W \vee \mathcal{F}_T^B$, where $\mathcal{F}_t^W := \sigma(W_s;\ 0 \leqslant s \leqslant t)$ and $\mathcal{F}_{t,T}^B := \sigma(B_s - B_t;\ t \leqslant s \leqslant T)$, completed with the P-null sets. It should be noted that $(\mathcal{F}_t)$ is not an increasing family of sub-σ-fields, and hence it is not a filtration. However, $(\mathcal{G}_t)$ is a filtration.

Let $M_T^2(0, T, \mathbb{R}^d)$ denote the set of d-dimensional, jointly measurable stochastic processes $\{\varphi_t;\ t \in [0, T]\}$, which satisfy:

(a) $E \int_0^T |\varphi_t|^2\,\mathrm{d}t < \infty$.
(b) φ_t is $\mathcal{F}_t$-measurable, for any $t \in [0, T]$.

We denote by $S^2_T([0,T],\mathbb{R})$, the set of continuous stochastic processes φ_t, such that:

(a′) $E\sup_{0\leqslant t\leqslant T}|\varphi_t|^2\,\mathrm{d}t<\infty$.
(b′) φ_t is $\mathcal{F}_t$-measurable, for any $t\in[0,T]$.

Definition 2.1. A solution of Eq. (1) is a $(\mathbb{R}\times\mathbb{R}^d\times\mathbb{R}_+)$-valued process $(Y_t,Z_t,K_t)_{0\leqslant t\leqslant T}$ which satisfies Eq. (1) and such that:

(i) $(Y,Z,K)\in S^2\times M^2\times L^2(\Omega)$.
(ii) $Y_t\geqslant S_t$.
(iii) (K_t) is continuous and increasing process with $K_0=0$ and $\int_0^T(Y_t-S_t)\,\mathrm{d}K_t=0$.

We consider the following assumptions:

(H1) $f:\Omega\times[0,T]\times\mathbb{R}\times\mathbb{R}^d\to\mathbb{R}$ and $g:\Omega\times[0,T]\times\mathbb{R}\times\mathbb{R}^d\to\mathbb{R}$ are measurable and satisfy, for every $(y,z)\in\mathbb{R}\times\mathbb{R}^d$, $f(.,y,z)\in M^2(0,T,\mathbb{R})$ and $g(.,y,z)\in M^2(0,T,\mathbb{R})$.

(H2) There exist constants $C>0$ and $0<\alpha<1$, such that for every $(t,\omega)\in\Omega\times[0,T]$ and $(y,z)\in\mathbb{R}\times\mathbb{R}^d$,

$$\begin{cases}|f(t,y,z)-f(t,y',z')|\leqslant C(|y-y'|+|z-z'|),\\ |g(t,y,z)-g(t,y',z')|^2\leqslant C|y-y'|^2+\alpha|z-z'|^2.\end{cases}$$

(H3) ξ be a square integrable random variable which is $\mathcal{F}_T$-measurable.

(H4) The obstacle $\{S_t,\ 0\leqslant t\leqslant T\}$, is a continuous $\mathcal{F}_t$-progressively measurable real-valued process satisfying $E(\sup_{0\leqslant t\leqslant T}(S_t)^2)<\infty$. We shall always assume that $S_T\leqslant\xi$ a.s.

(H5) (i) For a.e. (t,w), the map $(y,z)\mapsto f(t,y,z)$ is continuous.
(ii) There exist constants $\kappa>0$, $L>0$ and $\alpha\in\,]0,1[$, such that for every $(t,\omega)\in\Omega\times[0,T]$ and $(y,z)\in\mathbb{R}\times\mathbb{R}^d$,

$$\begin{cases}|f(t,y,z)|\leqslant\kappa(1+|y|+|z|),\\ |g(t,y,z)-g(t,y',z')|^2\leqslant L|y-y'|^2+\alpha|z-z'|^2.\end{cases}$$

3. The main results

Lemma 3.1. *For $i=1,2$, let (η^i) be square integrable and $\mathcal{G}_T$-measurable random variables. Let $h^i:[0,T]\times\Omega\times\mathbb{R}\to\mathbb{R}$ be such that: for every $\mathcal{G}_t$-adapted process satisfying $E(\sup_{t\leqslant T}{Y_t}^2)<\infty$, the process $h^i(.,Y.)$ is $\mathcal{G}_t$-adapted and satisfies $E\int_0^T(h^i(s,Y_s))^2\,\mathrm{d}s<\infty$.*

Let (Y^i,Z^i) be a solution of the following BSDE:

$$\begin{cases}Y^i_t=\eta^i+\int_t^T h^i(s,Y^i_s)\,\mathrm{d}s-\int_t^T Z^i_s\,\mathrm{d}W_s,\\ E(\sup_{t\leqslant T}|Y^i_t|^2+\int_0^T|Z^i_s|^2\,\mathrm{d}s)<\infty.\end{cases}$$

Assume that

(i) *h^1 is a uniformly Lipschitz in the variable y.*
(ii) *$\eta^1\leqslant\eta^2$ a.s.*
(iii) *$h^1(t,Y^2_t)\leqslant h^2(t,Y^2_t)\,\mathrm{d}P\times\mathrm{d}t$ a.e.*

Then, $Y^1_t\leqslant Y^2_t$, $0\leqslant t\leqslant T$, a.s.

We first consider the following basic RBDSDE

$$\begin{cases} Y_t = \xi + \int_t^T f(s)\,\mathrm{d}s + K_T - K_t + \int_t^T g(s)\,\mathrm{d}B_s - \int_t^T Z_s\,\mathrm{d}W_s, \\ Y_t \geqslant S_t, \\ \int_t^T (Y_s - S_s)\,\mathrm{d}K_s = 0. \end{cases} \tag{2}$$

Proposition 3.1. *Under assumptions* (H1), (H3) *and* (H4), *the basic RBDSDE* (2) *has a unique solution.*

Theorem 3.1. *Assume that* (H1), (H2), (H3) *and* (H4) *hold. Then, the RBDSDE* (1) *has a unique solution.*

Theorem 3.2. *Let* (ξ, f, g, S) *and* (ξ', f', g, S') *be two RBDSDEs. Each one satisfying all the previous assumptions* (H1), (H2), (H3) *and* (H4). *Assume moreover that*:

(i) $\xi \leqslant \xi'$ *a.s.*
(ii) $f(t,y,z) \leqslant f'(t,y,z)\,\mathrm{d}P \times \mathrm{d}t$ *a.e.* $\forall (y,z) \in \mathbb{R} \times \mathbb{R}^d$.
(iii) $S_t \leqslant S'_t$, $0 \leqslant t \leqslant T$ *a.s.*

Let (Y, Z, K) [*resp.* (Y', Z', K')] *be a solution to the RBDSDE* (ξ, f, g, S) [*resp.* (ξ', f', g, S')]. *Then,* $Y_t \leqslant Y'_t$, $0 \leqslant t \leqslant T$ *a.s.*

Theorem 3.3. *Under the assumptions* (H1), (H3), (H4) *and* (H5), *the RBDSDE* (1) *has a solution* (Y, Z, K) *which is a minimal one, in the sense that, if* (Y^*, Z^*) *is any other solution, then* $Y \leqslant Y^*$, P*-a.s.*

4. Proofs

Proof of Lemma 3.1. It follows by applying Itô's formula to $|(Y_t^1 - Y_t^2)^+|^2$. □

Proof of Proposition 3.1. We first prove the existence of solutions. We define

$$Y_t^n := \xi + \int_t^T f(s)\,\mathrm{d}s + n\int_t^T \left(S_s - Y_s^n\right)^+ \mathrm{d}s + \int_t^T g(s)\,\mathrm{d}B_s - \int_t^T Z_s^n\,\mathrm{d}W_s$$

and put

$$\begin{cases} \bar{\xi} := \xi + \int_0^T f(s)\,\mathrm{d}s + \int_0^T g(s)\,\mathrm{d}B_s, \\ \bar{S}_t := S_t + \int_0^t f(s)\,\mathrm{d}s + \int_0^t g(s)\,\mathrm{d}B_s, \\ \bar{Y}_t^n := Y_t^n + \int_0^t f(s)\,\mathrm{d}s + \int_0^t g(s)\,\mathrm{d}B_s. \end{cases}$$

We then have

$$\bar{Y}_t^n = \bar{\xi} + n\int_t^T \left(\bar{S}_s - \bar{Y}_s^n\right)^+ \mathrm{d}s - \int_t^T Z_s^n\,\mathrm{d}W_s. \tag{$*$}$$

Let $\Lambda_t := E^{\mathcal{G}_t}[\bar{\xi} \vee \sup_{s\leqslant T} \bar{S}_s]$.

By Lemma 3.1, we have

$$\bar{Y}_t^0 = E^{\mathcal{G}_t}[\bar{\xi}] \leqslant \bar{Y}_t^n \leqslant \bar{Y}_t^{n+1} \leqslant \Lambda_t = E^{\mathcal{G}_t}\Big[\bar{\xi} \vee \sup_{s\leqslant T} \bar{S}_s\Big].$$

Using Itô's formula and passing to the expectations, we show that

$$E\int_0^T \left|Z_s^n\right|^2 \mathrm{d}s \leqslant 2E\Big|\sup_{s\leqslant T}(\bar{S}_s - \bar{\xi})^+\Big|^2 + 2E\int_0^T |\gamma_s|^2\,\mathrm{d}s.$$

Returning to Eq. $(*)$, we get

$$E\left(n\int_0^T\big(\overline{S}_s-\overline{Y}_s^n\big)^+\,\mathrm{d}s\right)^2\leqslant 4E\left|\sup_{s\leqslant T}(\overline{S}_s-\overline{\xi})^+\right|^2.$$

Hence, there exists a nondecreasing and right continuous process K satisfying $E(K_T^2)<\infty$ such that along a subsequence (which still denoted n) we have for all $\varphi\in\mathbb{L}^2(\Omega;\mathcal{C}([0,T]))$,

$$\lim_n E\int_0^T\varphi_s n\big(S_s-Y_s^n\big)^+\,\mathrm{d}s=E\int_0^T\varphi_s\,\mathrm{d}K_s. \tag{3}$$

Set $\overline{Y}_t:=\sup_n\overline{Y}_t^n$ and $Y_t:=\overline{Y}_t-\int_0^t f(s)\,\mathrm{d}s-\int_0^t g(s)\,\mathrm{d}B_s:=\sup_n Y_t^n$.

Let $\widetilde{Y}_t^n:=\overline{S}_T+n\int_t^T(\overline{S}_s-\widetilde{Y}_s^n)\,\mathrm{d}s-\int_t^T\widetilde{Z}_s^n\,\mathrm{d}W_s$.

Since $\overline{S}_T\leqslant\overline{\xi}$, then Lemma 3.1 shows that, for every $t\in[0,T]$, $\overline{Y}_t^n\geqslant\widetilde{Y}_t^n$ a.s.

Let σ be a $\mathcal{G}_t$-stopping time, and put $\tau:=\sigma\wedge T$. The sequence of processes $(\widetilde{Y}^n)$ satisfies then the equality $\widetilde{Y}_\tau^n=E^{\mathcal{G}_\tau}[\overline{S}_Te^{-n(T-\tau)}+n\int_\tau^T\overline{S}_se^{-n(s-\tau)}\,\mathrm{d}s]$ and therefore converges to $\overline{S}_\tau$ a.s. This implies that $Y_\tau\geqslant S_\tau$ a.s. It follows from the section theorem ([2], p. 220) that for every $t\in[0,T]$, $Y_t\geqslant S_t$ a.s.

Let $N\in\mathbb{N}^*$ and $n,m\geqslant N$. Using Itô's formula and the Burkholder–Davis–Gundy inequality, one can show that there exists a positive constant C such that

$$\limsup_{n,m}\left(E\Big(\sup_{t\leqslant T}\big(Y_t^n-Y_t^m\big)^2\Big)+E\int_0^T\big|Z_s^n-Z_s^m\big|^2\,\mathrm{d}s\right)\leqslant 2CE\int_0^T\big(S_s-Y_s^N\big)\,\mathrm{d}K_s.$$

Letting N tend to ∞ we obtain, since $Y\geqslant S$ a.s. and K is increasing,

$$\limsup_{n,m}\left(E\Big(\sup_{t\leqslant T}\big(Y_t^n-Y_t^m\big)^2\Big)+E\int_0^T\big|Z_s^n-Z_s^m\big|^2\,\mathrm{d}s\right)\leqslant 2CE\int_0^T(S_s-Y_s)\,\mathrm{d}K_s\leqslant 0.$$

This shows that the sequence (Y^n,Z^n) converges suitably to a process (Y,Z). And, it is not difficult to prove that (Y,Z,K) satisfies the RBDSDE (2) and (Y,K) is continuous. Since $Y\geqslant S$, we deduce that, $\int_0^T(Y_t-S_t)\,\mathrm{d}K_t\geqslant 0$. On the other hand, we have for each n, $\int_0^T(Y_t^n-S_t)\,\mathrm{d}K_t^n\leqslant 0$. Hence, $\int_0^T(Y_t-S_t)\,\mathrm{d}K_t=0$.

Uniqueness. Let $(\Delta Y,\Delta K,\Delta Z)$ be the difference between two arbitrary solutions. Since $\int_t^T(\Delta Y_s-\Delta S_s)\times d(\Delta K_s)\leqslant 0$, the uniqueness follows. □

Proof of Theorem 3.1. Existence. Define the sequence $(Y_t^n,Z_t^n,K_t^n)_{0\leqslant t\leqslant T}$ by $Y_t^0:=S_t$, $Z_t^0:=0$ and for every $t\in[0,T]$ and every $n\in\mathbb{N}^*$,

$$\begin{cases} Y_t^{n+1}:=\xi+\int_t^T f(s,Y_s^n,Z_s^n)\,\mathrm{d}s+\int_t^T g(s,Y_s^n,Z_s^n)\,\mathrm{d}B_s+\int_t^T\mathrm{d}K_s^{n+1}-\int_t^T Z_s^{n+1}\,\mathrm{d}W_s,\\ Y_t^{n+1}\geqslant S_t\quad\text{a.s.},\\ \int_t^T(Y_s^{n+1}-S_s)\,\mathrm{d}K_s^{n+1}=0.\end{cases}$$

Such a sequence (Y^n,Z^n,K^n) exists by Proposition 3.1.

Put $\overline{Y}^{n+1}:=Y^{n+1}-Y^n$.

Applying Itô's formula to $|Y|^2e^{\beta t}$ and using the fact that $\int_t^T e^{\beta s}\overline{Y}_s^{n+1}(\mathrm{d}K_s^{n+1}-\mathrm{d}K_s^n)\leqslant 0$, we show that:

$$E\int_t^T\big(\overline{C}\big|\overline{Y}_s^{n+1}\big|^2+\big|\overline{Z}_s^{n+1}\big|^2\big)e^{\beta s}\,\mathrm{d}s\leqslant\left(\frac{1+\alpha}{2}\right)^n E\int_t^T\big(\overline{C}\big|\overline{Y}_s^1\big|^2+\big|\overline{Z}_s^1\big|^2\big)e^{\beta s}\,\mathrm{d}s.$$

Since $\frac{1+\alpha}{2}<1$, the sequence (Y^n,Z^n) converges in $M^2\times M^2$. We easily deduce that (Y^n) convergence in S^2. The **uniqueness** can be proved by using Theorem 3.2 which is proved below. □

Proof of Theorem 3.2. Applying Itô's formula to $|(Y_t - Y'_t)^+|^2$, then passing to the expectations and using the fact that f is Lipschitz we obtain, since $Y_t > S'_t \geqslant S_t$ on the set $\{Y_s > Y'_s\}$,

$$
\begin{aligned}
&E\big|(Y_t - Y'_t)^+\big|^2 + E\int_t^T 1_{\{Y_s > Y'_s\}}\big|Z_s - Z'_s\big|^2\,\mathrm{d}s \\
&\quad\leqslant \left(3C + \frac{1}{\varepsilon}C^2\right)E\int_t^T \big|Y_s - Y'_s\big|^2 1_{\{Y_s > Y'_s\}}\,\mathrm{d}s + (\varepsilon + \alpha)E\int_t^T \big|Z_s - Z'_s\big|^2 1_{\{Y_s > Y'_s\}}\,\mathrm{d}s.
\end{aligned}
$$

Now, choose $\varepsilon = \frac{1-\alpha}{2}$, $\overline{C} = 3C + \frac{1}{\varepsilon}C^2$ and use Gronwall's lemma to show that $Y_t \leqslant Y'_t$, $\forall t$ a.s. □

Proof of Theorem 3.3. The sequence $f_n(t,x) := \inf_{y\in\mathbb{Q}}\{f(t,y) + n|x-y|\}$ for every $n > K$, f_n is uniformly n-Lipschitz, with linear growth and (f_n) converges suitably to f (see e.g. [1]).

We get from Theorem 3.1, that for every $n \in \mathbb{N}^*$, there exists a unique solution $\{(Y^n_t, Z^n_t, K^n_t),\ 0 \leqslant t \leqslant T\}$ for the following RBDSDE

$$
\begin{cases}
Y^n_t = \xi + \int_t^T f_n(s, Y^n_s, Z^n_s)\,\mathrm{d}s + K^n_T - K^n_t + \int_t^T g(s, Y^n_s, Z^n_s)\,\mathrm{d}B_s - \int_t^T Z^n_s\,\mathrm{d}W_s, \quad 0 \leqslant t \leqslant T, \\
Y^n_t \geqslant S_t, \\
\int_0^T (Y^n_s - S_s)\,\mathrm{d}K^n_s = 0.
\end{cases}
\tag{4.1}
$$

Using the properties of f_n, we prove that the sequence (Y^n, Z^n, K^n) converges to a process (Y, Z, K) which is a minimal solution to the RBDSDE (1). Approximating f by sup-convolution, i.e. by the sequence $f_n(t,x) := \sup_{y\in\mathbb{Q}}\{f(y) - n|x-y|\}$, one can prove that the RBDSDE (1) has a maximal solution. □

Remark. In contrast to the classical BSDEs [4], when the barrier S is constant, the reflection process K is not necessary absolutely continuous with respect to the Lebesgue measure. Indeed, if we take $S = 0$, $\xi = 0$, $f = 0$ and $g = 1$, one can show that $Z = 0$ and $K_t = (\sup_{0\leqslant s\leqslant T} B_s) - (\sup_{t\leqslant s\leqslant T} B_s)$.

Acknowledgement

The authors are sincerely grateful to the referee for many useful suggestions which have lead to an improvement of the Note.

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