Available online at www.sciencedirect.com
ScienceDirect
C. R. Acad. Sci. Paris, Ser. I 346 (2008) 1169–1174
COMPTES RENDUS MATHEMATIQUE
http://france.elsevier.com/direct/CRASS1/

Dynamical Systems

A generic incompressible flow is topological mixing

Mário Bessa [1]

ESTGOH – IPC, Rua General Santos Costa, 3400-124, Oliveira do Hospital and CMUP, Rua do Campo Alegre, 687, 4169-007 Porto, Portugal

Received 16 April 2008; accepted after revision 16 July 2008

Available online 15 October 2008

Presented by Étienne Ghys

Abstract

In this Note we prove that there exists a residual subset of the set of divergence-free vector fields defined on a compact, connected Riemannian manifold M, such that any vector field in this residual satisfies the following property: Given any two nonempty open subsets U and V of M, there exists $\tau \in \mathbb{R}$ such that $X^t(U) \cap V \neq \emptyset$ for any $t \geqslant \tau$. ***To cite this article: M. Bessa, C. R. Acad. Sci. Paris, Ser. I 346 (2008).***

Résumé

Un flot générique incompressible est mélangeant. Dans cette Note nous montrons qu'il existe une partie résiduelle $\mathcal{R}$ dans l'ensemble des champs vectoriels qui préservent l'élément de volume pour laquelle tout $X \in \mathcal{R}$ est topologiquement mélangeant. ***Pour citer cet article : M. Bessa, C. R. Acad. Sci. Paris, Ser. I 346 (2008).***

Version française abrégée

Soit M une variété riemannienne, connexe, compacte. Notons μ la mesure de Lebesgue sur M. Soit $\mathfrak{X}^1_\mu(M)$ l'ensemble des champs vectoriels qui préservent l'élément de volume muni de la topologie C^1. Rappelons qu'une partie de $\mathfrak{X}^1_\mu(M)$ est dite *résiduelle* si elle contient une intersection dénombrable d'ouverts denses.

Le champ vectoriel X est *topologiquement mélangeant*, si, pour tout couple ouvert U, $V \subset M$, il existe $\tau > 0$ tel que $X^\tau(U) \cap V \neq \emptyset$ pour tout $t \geqslant \tau$.

Dans cette Note nous montrons qu'il existe une partie résiduelle $\mathcal{R} \subset \mathfrak{X}^1_\mu(M)$ pour laquelle pour tout $X \in \mathcal{R}$ est topologiquement mélangeant.

Ce résultat généralise un résultat établi par Abdenur, Avila et Bochi [1, Theorem A] pour la classe des champs vectoriels qui préservent l'élément de volume. En fait, dans [1, Theorem B] il est démontré qu'une classe homocline (non triviale) générique d'un champ vectoriel est topologiquement mélangeant. Ici, grâce à une version d'un résultat fondamental de Bonatti et Crovisier [5], nous savons que les classes homoclines génériques de $X \in \mathfrak{X}^1_\mu(M)$ sont égales à M. Ainsi, nous pouvons obtenir la propriété de mélange sur toute la variété M. Notre démonstration utilise

E-mail address: bessa@fc.up.pt.
[1] Partially supported by FCT-FSE, SFRH/BPD/20890/2004.

1631-073X/$ – see front matter
doi:10.1016/j.crma.2008.07.012

des démonstrations de [1] et un théorème de perturbations de champs vectoriels qui préservent l'élément de volume de Arbieto et Matheus [2].

Noter que dans [1, Theorem B], la topologie utilisée est la topologie C^r ($r \geqslant 1$). Toutefois, ici nous nous sommes limités à la topologie de classe C^1, même si nos perturbations appartiennent à la classe C^r, malheureusement les versions C^r ($r \geqslant 2$) des résultats de Bonatti et Crovisier et de Arbieto et Matheus mentionnés ci-dessus ne sont pas connus.

Nous terminons en soulignant que les versions temps discret des résultats de [1] et de ceux établis dans cette Note sont encore des problèmes ouverts.

1. Introduction, basic definitions and statement of the results

Let M be a n-dimensional compact, connected, boundaryless C^∞ Riemannian manifold and let μ be the measure induced by a volume-form defined in M. We call μ the Lebesgue measure. For any $r \geqslant 1$ the class of C^r *divergence-free* (or *zero divergence*) vector fields defined on M will be denote by $\mathfrak{X}^r_\mu(M)$.

Integrating any C^1 vector field X we obtain its associated *flow* X^t, which is a 1-parameter group of C^1 volume-preserving diffeomorphisms. For this reason we call X^t an *incompressible* (or *volume-preserving*) flow. The infinitesimal generator of the flow X^t is the vector field X say, $\frac{\mathrm{d}X^t}{\mathrm{d}t}|_{t=s}(p) = X(X^s(p))$.

We say that a subset $\mathcal{R} \subset \mathfrak{X}^1_\mu(M)$ is a *residual* subset, or a *generic* subset, if it contains a countable intersection of dense and open sets with respect to the Whitney C^1-topology.

We say that $x \in M$ is a *singularity* of the vector field X if $X(x) = 0$.

A vector field X is said to be *transitive* if its flow has a dense orbit in M or, equivalently, given any nonempty open sets $U, V \subseteq M$, there exists $\tau > 0$ such that $X^\tau(U) \cap V \neq \emptyset$. Now we consider a more rigid definition. We say that a vector field X is *topologically mixing* if given any nonempty open sets $U, V \subseteq M$, there exists $\tau > 0$ such that, for all $t \geqslant \tau$ we have $X^\tau(U) \cap V \neq \emptyset$.

We say that a closed orbit γ of period $P = P_{X,\gamma}$ is *hyperbolic* if the spectrum of the time-P derivative of the Poincaré transversal map does not intersect $\mathbb{S}^1$. A closed orbit γ of period $P = P_{X,\gamma}$ is *elliptic* if the spectrum of the time-P derivative of the Poincaré transversal map lies in $\mathbb{S}^1 \setminus \mathbb{R}$. It is well-known that given any hyperbolic closed orbit γ and $x \in \gamma$, x has smooth *stable* and *unstable manifolds* (see e.g. [7]) which are defined respectively by:

$$W^{s/u}(x) = \Big\{ y \in M \; d\big(X^t(x), X^t(y)\big) \underset{t \to +\infty/-\infty}{\longrightarrow} 0 \Big\},$$

where $d(\cdot,\cdot)$ is the distance inherit by the Riemannian structure on M. The saturated of $W^s(x)$ by the flow X^t gives us the *stable manifold of the closed orbit* which we denote by $W^s(\gamma)$. In the same way we define $W^u(\gamma)$. The *index* of γ is the dimension of the unstable manifold.

Now, given any hyperbolic closed orbit γ of a flow X^t, we define its *homoclinic class* by $H_{X,\gamma} = \overline{W^s(\gamma) \pitchfork W^u(\gamma)}$, where $\bar{A}$ denotes the closure of the set A and $\pitchfork$ the transversal intersection of manifolds. We denote by $\mathrm{Per}(X)$ the set of all periodic points of X, and by $\mathrm{Per}_n(X)$ the subset of periodic points with period less than n.

In this Note we prove the following result:

Theorem 1.1. *There exists a C^1-residual subset $\mathcal{R} \subset \mathfrak{X}^1_\mu(M)$ such that, if $X \in \mathcal{R}$ then X is a topological mixing vector field.*

This result generalizes a result given in Abdenur, Avila and Bochi's Theorem A [1] for the divergence-free class. Actually, in [1, Theorem B] it is proved that a nontrivial homoclinic class of a generic vector field is topologically mixing. Here, and due to an important result of Bonatti and Crovisier (see Theorem 2.1 and Section 4) we know that generically homoclinic classes of $X \in \mathfrak{X}^1_\mu(M)$ are global, i.e. equal to M. Thus, we were able to obtain the topologically mixing statement in all M.

Notice that, in [1] (Theorem B), the topology involved is the C^r ($r \geqslant 1$) topology. However, here we are constrained to the C^1 topology class since, even if our perturbation lemma (Lemma 2.2) was done in the C^r class, unfortunately the C^r ($r \geqslant 2$) version of Theorem 2.1 aforementioned is not available.

We finish the introduction noting that the discrete-time version of the results in [1] and those in this paper are still open.

2. Proof of Theorem 1.1

In this section we present the proof of Theorem 1.1. A key ingredient that we are going to use is the remarkable Bonatti–Crovisier C^1-connecting lemma for pseudo-orbits, namely Théorème 1.3 of [5].

Theorem 2.1 *(Bonatti–Crovisier). There exists a residual C^1-subset $\mathcal{R}_1$ of the volume-preserving C^1 diffeomorphisms defined on M such that if $f \in \mathcal{R}_1$, then f is a transitive diffeomorphism. Moreover, f has a unique homoclinic class.*

The definition of transitivity and of homoclinic class for the discrete-time case are obvious. As another results of this type, like e.g. Hayashi's connecting lemma (see [10]), Theorem 2.1 is also true for the class of divergence-free vector fields (see Section 4). The other ingredient is the following lemma whose proof we postpone to Section 3:

Lemma 2.2. *There exists a residual $\mathcal{R}_2 \subset \mathfrak{X}^1_\mu(M)$ such that if $X \in \mathcal{R}_2$, then given any closed orbits γ_1 and γ_2 with $\gamma_1 \neq \gamma_2$ we have*

$$\frac{P_{X,\gamma_1}}{P_{X,\gamma_2}} \in \mathbb{R} \setminus \mathbb{Q}.$$

Now we are ready to prove our main result:

Proof of Theorem 1.1. Let $\mathcal{R}_1$ be the residual subset given by Theorem 2.1 and $\mathcal{R}_2$ the residual subset given by Lemma 2.2. Define $\mathcal{R} = \mathcal{R}_1 \cap \mathcal{R}_2$. The proof from now on follows *ipsis verbis* [1] (proof of Theorem B) nevertheless, for the sake of completeness, we present it here.

We want to prove that given $X \in \mathcal{R}$ and any two nonempty open sets U and V in M, there exists some $\tau \in \mathbb{R}$ such that $X^t(U) \cap V \neq \emptyset$ for any $t \geqslant \tau$.

Since $X \in \mathcal{R}_1$ there exists a unique homoclinic class M, so we take two distinct closed orbits γ_1 and γ_2 of period P_1 and P_2 respectively, with the same index[2] and satisfying $\gamma_1 \cap U \neq \emptyset$ and $\gamma_2 \cap V \neq \emptyset$. Since $X \in \mathcal{R}_2$ we have $P_1/P_2 \in \mathbb{R} \setminus \mathbb{Q}$.

Let $x \in \gamma_1 \cap U$, $y \in \gamma_2 \cap V$ and $z \in W^u(x) \cap W^s(y)$. There exists $\tau_1 > 0$ such that

$$\left\{X^{-(\tau_1+kP_1)}(z)\right\}_{k\in\mathbb{N}} \subset W^u(x) \quad \text{and} \quad X^{-(\tau_1+kP_1)}(z) \underset{k\to+\infty}{\longrightarrow} x.$$

Hence, for some $t_1 > 0$ we have $X^{-(t_1+kP_1)}(z) \in U$ for all $k \in \mathbb{N}$. In the same way there exist $t_2 > 0$ and a sufficiently small $\epsilon > 0$ such that $X^{\tau_2+\ell P_2+s}(z) \in V$ for all $\ell \in \mathbb{N}$ and $|s| < \epsilon$.

It is easy to see that the set $\{kP_1 + \ell P_2 + s\colon\ k, \ell \in \mathbb{N} \text{ and } |s| < \epsilon\}$ contains no interval of the form $[T, +\infty)$ for some $T > 0$.

Define $\tau = t_1 + t_2 + T$. Therefore, for any $t \geqslant \tau$, there exist $k, \ell \in \mathbb{N}$ and a small $|s| < \epsilon$ such that $t = t_1 + t_2 + kP_1 + \ell P_2 + s$. Now, since $z \in X^{t_1+kP_1}(U)$ and $X^{t_2+\ell P_2+s}(z) \in V$ for all $k, \ell \in \mathbb{N}$ we get that $X^{t_2+\ell P_2+s}(z) \in X^t(U) \cap V$ and X is topologically mixing. □

3. Proof of Lemma 2.2

To prove Theorem 1.1 in the divergence-free setting it will be necessary to perform the perturbations in this context, so the following result, due to Arbieto and Matheus (see [2] Theorem 3.1), is crucial:

Lemma 3.1 *(C^2-Pasting Lemma). Given $\epsilon > 0$ there exists $\delta > 0$ such that if $X \in \mathfrak{X}^2_\mu(M)$, $K \subset M$ is a compact set and $Y \in \mathfrak{X}^2_\mu(M)$ is δ-C^1-close to X in a small neighborhood $U \supset K$, then there exist $Z \in \mathfrak{X}^2_\mu(M)$, V and W with $K \subset V \subset U \subset W$ such that $Z|_V = Y$, $Z|_{\mathrm{int}(W^c)} = X$ and Z is ϵ-C^1-close to X.*

[2] We use the fact that they have the same index, together with the fact that the close hyperbolic orbits of some index are dense in the homoclinic class.

Proof of Lemma 2.2. First we consider the case when the dimension of M is greater than three. By Robinson's conservative version of the Kupka–Smale Theorem [9] we obtain, in particular, that the subset of $\mathfrak{X}^1_\mu(M)$ defined by,

$$\mathcal{A}_n = \{X \in \mathfrak{X}^1_\mu(M)\colon \text{ Sing}(X) \text{ and } \text{Per}_n(X) \text{ are hyperbolic}\},$$

is open and dense in $\mathfrak{X}^1_\mu(M)$. We also define the *open* set,

$$\mathcal{B}_n = \left\{X \in \mathcal{A}_n\colon \text{ if } \gamma_1, \gamma_2 \in \text{Per}_n(X) \text{ and } \gamma_1 \neq \gamma_2, \text{ then } \frac{P_{X,\gamma_1}}{P_{X,\gamma_2}} \notin \{r_i\}_{i=1}^n\right\},$$

where $\{r_i\}_{i=1}^\infty$ are the positive rational numbers.

To obtain the *density* we must prove that given $\epsilon > 0$ and a vector field $X_1 \in \mathfrak{X}^1_\mu(M)$, for all $n \in \mathbb{N}$ there exists $Z \in \mathcal{B}_n$, ϵ-C^1-close to X_1. By Zuppa's Theorem [11] the vector field X_1 can be $\epsilon/2$-C^1-approximated by a C^2 vector field $X_2 \in \mathcal{A}_n$. By definition X_2^t has a finite number of hyperbolic closed orbits of period less than n, which we denote by $\{\gamma_i\}_{i=1}^m$.

Given $s_i > 0$, for $i = 1, \ldots, m$, the vector fields defined by

$$Y_i(\cdot) = (1 + s_i)^{-1} X_2(\cdot),$$

are divergence-free and, for each $i = 1, \ldots, m$, if s_i is close to zero, then Y_i is close to X_2.

Now, for $i = 1, \ldots, m$, we take tubular compact neighborhoods $K_i \supset \gamma_i$ and we choose K_i sufficiently thin in order that some open neighborhoods $\mathcal{W}_i \supset K_i$ should be pairwise disjoint.

Now we are going to define recursively a vector field $Z_m \in \mathfrak{X}^2_\mu(M)$ $\epsilon/2$-C^1-close to X_2 such that $Z_m|_{\gamma_i} = Y_i|_{\gamma_i}$.

Define $Y_0 = X_2$. Now for each $i = 1, \ldots, m$, by Lemma 3.1 there exists $\delta > 0$ such that if Y_i is δ-C^1-close to Y_{i-1} in a small neighborhood $U_i \supset K_i$, then there exist $Z_i \in \mathfrak{X}^2_\mu(M)$, V_i and W_i with $K_i \subset V_i \subset U_i \subset W_i$ such that:

1) $Z_i|_{V_i} = Y_i$;
2) $Z_i|_{\text{int}(W_i^c)} = Y_{i-1}$;
3) Z_i is $\frac{\epsilon}{2m}$-C^1-close to Y_{i-1}.

At the end we obtain the claimed vector field Z_m which satisfies:

4) Z_m converges to X_2, in the C^1-topology, as s_i converges to 0;
5) $P_{Z_m,\gamma_i} = (1 + s_i) P_{X_2,\gamma_i}$ for $i = 1, \ldots, m$.

Therefore, we consider adequate small $s_1, s_2, \ldots, s_m$ such that

6) $Z_m \in \mathcal{A}_n$;
7) $\frac{P_{Z_m,\gamma_i}}{P_{Z_m,\gamma_j}} \notin \{r_i\}_{i=1}^n$ for $i \neq j$.

Finally, if such a vector field Z_m has another closed orbit then, by 5) its period is greater or equal than n, thus $Z_m \in \mathcal{B}_n$. Define the residual subset claimed by the lemma by $\mathcal{R}_2 = \bigcap_{n \in \mathbb{N}} \mathcal{B}_n$.

If M is tridimensional we define:

$$\mathcal{A}_n = \{X \in \mathfrak{X}^1_\mu(M)\colon \text{ Sing}(X) \text{ and } \text{Per}_n(X) \text{ are hyperbolic or elliptic}\},$$

which is open and dense, and we do exactly the same proof as before. □

4. Theorem 2.1 for incompressible flows – guidelines for a proof

Along this section we assume some familiarity with Bonatti and Crovisier's proof [5] and also with the nomenclature used there.

4.1. Existence of perturbation flowboxes of large length

Let $\mathcal{C}_\varphi \subset \mathbb{R}^n$ be an $(n-1)$-dimensional tiled cube (see [5, Fig. 1]) and $\varphi = \varphi_x : U \to \mathbb{R}^n$ be a volume-preserving chart based at $x \in M$ (see [6, Lemma 2]). We consider the tiled cube (of a fixed chart based at x) $\mathcal{C} := \varphi^{-1}(\mathcal{C}_\varphi) \subset \Sigma$ where Σ is an $(n-1)$-dimensional submanifold transversal to the flow direction. Given $N \geqslant 1$ we say that $\{x_i\}_{i=0}^n$ *preserves the tiling* in the flowbox[3] $\mathcal{F}_X(\mathcal{C}, N) := \bigcup_{t\in[0,N]} X^t(\mathcal{C})$ if:

- $x_0, x_n \notin \mathcal{F}_X(\mathcal{C}, N)$;
- If $x_i \in \mathcal{T}_i \subset \mathcal{C}$ (for $i = 0, \ldots, n-1$ and some tile $\mathcal{T}_i$), then $X^{-1}(x_{i+1}) \in \mathcal{T}_i$;
- If $x_i \in X^j(\mathcal{C})$ (for $j = 1, \ldots, N-1$), then $x_{i+1} = X^1(x_i)$.

Fix $X \in \mathfrak{X}^1_\mu(M)$. We say that a tiled cube $\mathcal{C}$ is an *ϵ-perturbation flowbox of length $N \geqslant 1$* if any sequence $\{x_i\}_{i=0}^n$ preserving the tiling in the flowbox $\mathcal{F}_X(\mathcal{C}, N)$, there exists Y ϵ-C^1-close to X such that $Y = X$ outside $\mathcal{F}_X(\mathcal{C}, N)$ and exists $\{y_i\}_{i=0}^n$ (with $y_0 = x_0$ and $y_n = x_n$) such that, if $y_i \in \mathcal{F}(\mathcal{C}, N-1)$, then $y_{i+1} = Y^1(y_i)$.

Theorem 4.1 *(Connecting Lemma for pseudo-orbits). Given $X \in \mathfrak{X}^1_\mu(M)$ and $\epsilon > 0$, there exists $N \in \mathbb{N}$ such that for all $x \in M$, if any tiled cube $\mathcal{C}$ (of a fixed chart based at x) is a flowbox of length N, then $\mathcal{C}$ is an ϵ-perturbation flowbox of length N.*

The proof of the previous theorem follows straightforward from the arguments in [5]. However, since we are in the conservative flow setting, care must be taken to perturb the orbit segments to cancel the jumps of the pseudo-orbits and remain inside $\mathfrak{X}^1_\mu(M)$. For that we use a beautiful argument given by Pugh and Robinson [8, Section 8(c)]. In broad terms, we start by making a small perturbation in the original orbit in order to perform the desired action. Then, the way this C^2-trajectory γ is realized by an incompressible flow C^1-close to the original one is by constructing a volume-form adequate to γ. This is done in two steps; first, and since the goal is to obtain a closed transversal $(n-1)$-form λ along γ, we consider a local $(n-2)$-form η so that λ can be defined as $d\eta$. Second, by using the Whitney extension theorem, we make η global.

4.2. Existence of topological towers for flows

Consider a (transversal) section Σ, a map $R : \Sigma \to \Sigma$ and a ceiling function $h : \Sigma \to [c, +\infty]$, with $c > 0$. The flow $\mathcal{S}^s : \Sigma \times \mathbb{R} \to \Sigma \times \mathbb{R}$ defined by $(x, r) \mapsto (R^n(x), r + s - \sum_{i=0}^{n-1} h(R^i(x)))$ where $n \in \mathbb{Z}$ is uniquely defined by $\sum_{i=0}^{n-1} h(R^i(x)) \leqslant r + s < \sum_{i=0}^{n} h(R^i(x))$ is called a *special flow*. Roughly speaking $\mathcal{S}^s$ moves a given point (x, r) to $(x, r + s)$ at velocity one until hits the graph of h, after that the point returns to the transversal section Σ (base). It is well known that any aperiodic[4] flow is equivalent to some special flow (see [3] and the references therein). In [3, Section 3.6.1] we use this fact to build a Kakutani castle with very high towers in order to avoid overlapping of the local perturbations and, moreover, to have enough *time* to perform lots of small perturbations. One of the main steps to prove Bonatti–Crovisier's results is based in an analogue argument involving the so-called topological towers (see [5, Section 3]). In fact, we would like to prove that there is a finite family of perturbation flowboxes of length N with disjoint supports and such that every orbit in M enter inside some flowbox in finite time. Notice that the bottom of each tower is an open set in the section. Clearly, closed orbits of period $\leqslant N$ and singularities goes against the existence of topological towers. Nevertheless, the notion can be adapted to pass this difficulty. In conclusion the proof splits into two cases. In the first one we obtain a family of perturbation flowboxes far from closed orbits of small period and also singularities. For that we just have to borrow the arguments along Sections 3 and 4 of [5]. Finally, in the second case, we have to show how to deal with periodic hyperbolic orbits. This is done exactly as in [5, Proposition 4.2] with the obvious modifications. The only novelty is the existence of hyperbolic singularities, however, they are treated like a hyperbolic fixed points was in [5, Proposition 4.2].

[3] Notice that we consider *injective* flowboxes, that is, any $y \in \mathcal{F}_X(\mathcal{C}, N)$ can be written in a unique way as $y = X^t(x)$ where $x \in \mathcal{C}$ and $t \in [0, N]$.

[4] A flow X^t is *aperiodic* if the union of $\mathrm{Per}(X)$ with the set of singularities has zero Lebesgue measure.

4.3. End of the proof of Theorem 2.1 for incompressible flows

Like in [5] we denote by $x \dashv y$ every time that it is possible to connect x to y by an ϵ-pseudo-orbit with arbitrary small $\epsilon > 0$. If $\dim(M) \geqslant 4$, then, following the same arguments in [5] page 79, we are able to obtain Theorem 4.2 below and thus conclude the proof of Theorem 2.1 for our setting.

Theorem 4.2. *Given $X \in \mathfrak{X}^1_\mu(M)$ and $\epsilon > 0$ we suppose that any closed orbit is hyperbolic.*[5] *Then for all pair (x, y) of points in M such that $x \dashv y$ there exist Y ϵ-C^1-close to X and $t > 0$ such that $Y^t(x) = y$.*

If $\dim(M) = 3$ we have to take in account that the elliptic points are stable, thus generically the closed orbits are hyperbolic or elliptic (see [9]). In this case, once again we borrow the arguments along [5, Section 6.2], and we use the C^1-perturbation results available, namely a version of Franks' lemma for incompressible flows (see [4, Lemma 3.2]) and also Hayashi's connecting lemma for incompressible flows (see [10]). Finally, we obtain the conservative three-dimensional version of Theorem 2.1 and we are done.

Acknowledgements

We would like to thank Jorge Rocha for the careful reading of the preliminary version. We would like also to thank the referee for useful remarks.

References

[1] F. Abdenur, A. Avila, J. Bochi, Robust transitivity and topological mixing for C^1-flows, Proc. Amer. Math. Soc. 132 (3) (2003) 699–705.
[2] A. Arbieto, C. Matheus, A pasting lemma and some applications for conservative systems, Ergodic Theory Dynam. Systems 27 (6) (2007) 1399–1417.
[3] M. Bessa, The Lyapunov exponents of generic zero divergence three-dimensional vector fields, Ergodic Theory Dynam. Systems 27 (6) (2007) 1445–1472.
[4] M. Bessa and J. Rocha, On C^1-robust transitivity of volume-preserving flows, J. Differential Equations (2008), in press.
[5] C. Bonatti, S. Crovisier, Récurrence et généricité, Invent. Math. 158 (1) (2004) 33–104.
[6] J. Moser, On the volume elements on a manifold, Trans. Amer. Math. Soc. 120 (1965) 286–294.
[7] J. Palis, W. de Melo, Geometric Theory of Dynamical Systems, Springer Verlag, 1982.
[8] C. Pugh, C. Robinson, The C^1 closing lemma, including Hamiltonians, Ergodic Theory Dynam. Systems 3 (1983) 261–313.
[9] C. Robinson, Generic properties of conservative systems, Amer. J. Math. 92 (1970) 562–603.
[10] L. Wen, Z. Xia, C^1 connecting lemmas, Trans. Amer. Math. Soc. 352 (2000) 5213–5230.
[11] C. Zuppa, Regularisation C^∞ des champs vectoriels qui préservent l'elément de volume, Bol. Soc. Bras. Mat. 10 (2) (1979) 51–56.

[5] This property is generic if $\dim(M) \geqslant 4$, see [9].