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Probability Theory

A uniqueness theorem for the solution of Backward Stochastic Differential Equations

Guangyan Jia [1]

School of Mathematics and System Sciences, Shandong University, Jinan, Shandong 250100, PR China

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Abstract

In this Note, we prove that if g is uniformly continuous in z, uniformly with respect to (ω, t) and independent of y, the solution to the backward stochastic differential equation (BSDE) with generator g, is unique. ***To cite this article: G. Jia, C. R. Acad. Sci. Paris, Ser. I 346 (2008).***

Résumé

Un théorême d'unicité de la solution d'une équation différentielle stochastique rétrograde. Dans cette Note, nous démontrons que pour une fonction g donnée, uniformément continue en z, uniformément en (ω, t) et indépendante de y l'équation différentielle stochastique, rétrograde de générateur g, admet une solution unique. ***Pour citer cet article : G. Jia, C. R. Acad. Sci. Paris, Ser. I 346 (2008).***

Version française abrégée

Dans cette Note, nous considérons dans $[0, T]$ l'EDSR suivante :

$$y_t = \xi + \int_t^T g(s, y_s, z_s)\,\mathrm{d}s - \int_t^T z_s\,\mathrm{d}W_s, \quad 0 \leqslant t \leqslant T, \tag{1}$$

où $g(t, \cdot)$ est uniformément continue et de plus satisfait les conditions :

H1 $g(\omega, t, \cdot)$ est uniformément continue par rapport à (ω, t) c'est-à-dire qu'il existe une fonction ϕ de $\mathbb{R}_+$ dans lui-même, continue, non décroissante, de croissance linéaire, sous additive, $\phi(0) = 0$ et telle que

E-mail address: jiagy@sdu.edu.cn.

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$$\left|g(\omega,t,z_1)-g(\omega,t,z_2)\right|\leqslant\phi\big(|z_1-z_2|\big),\quad P\text{-a.s., pour tout } t\in[0,T],\ z_1,z_2\in\mathbb{R}^d.$$

Nous notons A la constante de croissance linéaire, i.e., pour tout x :

$$0\leqslant\phi(x)\leqslant A(x+1)$$

pour tout $x\in\mathbb{R}_+$. De plus nous supposons que $g(t,0)_{t\in[0,T]}$ est bornée.

Sous ces hypothèses nous démontrons le résultat suivant :

Théorême 0.1. *Si g satisfait les hypothèses* H1 *et $\xi\in L^2(\Omega,\mathcal{F}_T,P)$. Alors la solution de l'équation* (1) *est unique.*

1. Introduction

One dimensional BSDEs are equations of the following type defined on $[0,T]$:

$$y_t=\xi+\int_t^T g(s,y_s,z_s)\,\mathrm{d}s-\int_t^T z_s\,\mathrm{d}W_s,\quad 0\leqslant t\leqslant T, \tag{2}$$

where W is a standard d-dimensional Brownian motion on a probability space $(\Omega,\mathcal{F},(\mathcal{F}_t)_{0\leqslant t\leqslant T},P)$ with $(\mathcal{F}_t)_{0\leqslant t\leqslant T}$ the filtration generated by W. The function $g:\Omega\times[0,T]\times\mathbb{R}\times\mathbb{R}^d\to\mathbb{R}$ is called generator of (2). Here T is the terminal time, and ξ is a $\mathbb{R}$-valued $\mathcal{F}_T$-adapted random variable; (g,T,ξ) are the parameters of (2). The solution $(y_t,z_t)_{t\in[0,T]}$ is a pair of $\mathcal{F}_t$-adapted and square integrable processes.

Nonlinear BSDEs were first introduced by Pardoux and Peng [7], who proved the existence and uniqueness of a solution under suitable assumptions on g and ξ, the most standard of which are the Lipschitz continuity of g with respect to (y,z) and the square integrability of ξ. An interesting and important question is to find weaker conditions rather than the Lipschitz one, under which the BSDE (2) still has a unique solution. As a matter of fact, there have been several works, such as Pardoux and Peng [8], Kobylanski [4] and Briand–Hu [1], etc. In this Note, we will give a new sufficient condition for the uniqueness of the solution to BSDEs.

In fact, this problem came from a lecture given by Peng at a seminar of Shandong University in October 2005. In his lecture, Peng conjectured that if g is Hölder continuous in z and independent of y, then (2) has a unique solution. In this Note, we will prove this conjecture under a more general condition – uniform continuity – instead of Hölder continuity. In other words, g satisfies the following condition:

(H1) $g(\omega,t,\cdot)$ is uniformly continuous and uniformly with respect to (ω,t), i.e., there exists a function ϕ from $\mathbb{R}_+$ to itself, which is continuous, non-decreasing, subadditive and of linear growth, and $\phi(0)=0$ such that

$$\left|g(\omega,t,z_1)-g(\omega,t,z_2)\right|\leqslant\phi\big(|z_1-z_2|\big),\quad P\text{-a.s., for all } t\in[0,T],\ z_1,z_2\in\mathbb{R}^d.$$

Here we denote the constant of linear growth of ϕ by A, i.e.,

$$0\leqslant\phi(x)\leqslant A(x+1)$$

for all $x\in\mathbb{R}_+$ (see Crandall [3]). Moreover $(g(t,0))_{t\in[0,T]}$ is assumed to be bounded.

Remark 1. Clearly (H1) implies (H1′):

(H1′) $g(\omega,t,\cdot)$ is continuous, and of linear growth, i.e., there exists a positive real number B, such that

$$\left|g(\omega,t,z)\right|\leqslant B\big(|z|+1\big),\quad P\text{-a.s., for all } (t,z)\in[0,T]\times\mathbb{R}^d.$$

According to the result in [5], (H1′) guarantees the existence of a solution of (2).

This Note is organized as follows. In Section 2 we formulate the problem accurately and give some preliminary results. Finally, Section 3 is devoted to the proof of the main theorem.

2. Preliminaries

Let $(\Omega, \mathcal{F}, P)$ be a probability space and W be a d-dimensional standard Brownian motion on this space. Let $(\mathcal{F}_t)_{t\geqslant 0}$ be the filtration generated by this Brownian motion: $\mathcal{F}_t = \sigma\{W_s, s \in [0,t]\} \cup \mathcal{N}$, $\mathbb{F} = (\mathcal{F}_t)_{t\geqslant 0}$, where $\mathcal{N}$ is the set of all P-null subsets.

Let $T > 0$ be a fixed real number. In this Note, we always work in the space $(\Omega, \mathcal{F}_T, P)$. For a positive integer n and $z \in \mathbb{R}^n$, we denote by $|z|$ the Euclidean norm of z. We will denote by $\mathcal{H}_n^2 = \mathcal{H}_n^2(0, T; \mathbb{R}^n)$, the space of all $\mathbb{F}$-progressively measurable $\mathbb{R}^n$-valued processes such that $\mathbf{E}[\int_0^T |\psi_t|^2 \,dt] < \infty$, and by $\mathcal{S}^2 = \mathcal{S}^2(0, T; \mathbb{R})$ the elements in $\mathcal{H}_1^2$ with continuous paths such that $\mathbf{E}[\sup_{t\in[0,T]} |\psi_t|^2] < \infty$.

Now, let $\xi \in L^2(\Omega, \mathcal{F}_T, P)$ be a terminal value, $g : \Omega \times [0,T] \times \mathbb{R}^d \to \mathbb{R}$ be the generator, such that the process $g(\omega, t, z)_{t\in[0,T]} \in \mathcal{H}_1^2$ for any $z \in \mathbb{R}^d$. A solution of a BSDE is a pair of processes $(y_t, z_t)_{t\in[0,T]} \in \mathcal{S}^2 \times \mathcal{H}_d^2$ satisfying BSDE (2).

We now introduce a useful lemma which plays an important role in this Note. First we define

$$\underline{f}_n(t,z) \triangleq \inf_{u\in\mathbb{Q}^d} \{ f(t,u) + n|z-u| \} \quad \text{and} \quad \bar{f}_n(t,z) \triangleq \sup_{u\in\mathbb{Q}^d} \{ f(t,u) - n|z-u| \},$$

where f satisfies (H1) and $n \in \mathbb{N}$. Also we define $C = \max\{A, B\}$. Then one has:

Lemma 2. *Let f satisfy* (H1) *and $\bar{f}_n, \underline{f}_n$ be defined as above. Then for $n > C$:*

(i) $-C(|z|+1) \leqslant \underline{f}_n(t,z) \leqslant f(t,z) \leqslant \bar{f}_n(t,z) \leqslant C(|z|+1)$ *P-a.s. for any* $(t,z) \in [0,T] \times \mathbb{R}^d$;

(ii) $\underline{f}_\cdot(t,z)$ *is non-decreasing and* $\bar{f}_\cdot(t,z)$ *is non-increasing for any* $(t,z) \in [0,T] \times \mathbb{R}^d$;

(iii) $|\bar{f}_n(t,z_1) - \bar{f}_n(t,z_2)| \leqslant n|z_1 - z_2|$ *and* $|\underline{f}_n(t,z_1) - \underline{f}_n(t,z_2)| \leqslant n|z_1 - z_2|$ *P-a.s. for any* $t \in [0,T]$, $z_1, z_2 \in \mathbb{R}^d$;

(iv) *If* $z^n \to z$ *as* $n \to \infty$, *then* $\underline{f}_n(t,z^n) \to f(t,z)$ *and* $\bar{f}_n(t,z^n) \to f(t,z)$ *P-a.s. as* $n \to \infty$;

(v) $0 \leqslant f(t,z) - \underline{f}_n(t,z) \leqslant \phi(\frac{2C}{n-C})$ *and* $0 \leqslant \bar{f}_n(t,z) - f(t,z) \leqslant \phi(\frac{2C}{n-C})$ *P-a.s. for any* $(t,z) \in [0,T] \times \mathbb{R}^d$.

Proof. It is not hard to check (i)–(iv) (see [5]).

We now prove (v). It follows from (H1) that, for given $(t,z) \in [0,T] \times \mathbb{R}^d$, one has:

$$f(t,u) \geqslant f(t,z) - \phi(|z-u|) \geqslant f(t,z) - A(|z-u|+1) \geqslant f(t,z) - C(|z-u|+1), \quad \text{for any } u \in \mathbb{R}^d. \tag{3}$$

Given $n > C$, we define

$$\Lambda_n \triangleq \{ u \in \mathbb{Q}^d\colon\ n|z-u| \geqslant C(|z-u|+2) \}.$$

Clearly, Λ_n is not empty and $\mathbb{Q}^d = \Lambda_n \cup \Lambda_n^c$ where $\Lambda_n^c = \{u \in \mathbb{Q}^d\colon\ n|z-u| < C(|z-u|+2)\}$ is the complementary set of Λ_n (which is not empty too). For any $u \in \Lambda_n$, it follows from (3) that

$$f(t,u) + n|z-u| \geqslant f(t,u) + C(|z-u|+2) \geqslant f(t,z) + C.$$

Then by (i) of this lemma, one has:

$$f(t,u) + n|z-u| > f(t,z) + \frac{C}{2} > f(t,z) \geqslant \inf_{v\in\Lambda_n\cup\Lambda_n^c} \{ f(t,v) + n|z-v| \}, \quad \text{for any } u \in \Lambda_n.$$

Therefore,

$$\begin{aligned}
\underline{f}_n(t,z) &= \inf_{u\in\Lambda_n\cup\Lambda_n^c} \{ f(t,u) + n|z-u| \} = \inf_{u\in\Lambda_n^c} \{ f(t,u) + n|z-u| \} \\
&= \inf\{ f(t,u) + n|z-u|\colon\ u \in \mathbb{Q}^d \text{ and } n|z-u| < C(|z-u|+2) \} \quad \text{by the definition of } \Lambda_n^c \\
&\geqslant \inf\{ f(t,u)\colon\ u \in \mathbb{Q}^d \text{ and } n|z-u| < C(|z-u|+2) \} \\
&\geqslant \inf\left\{ f(t,z) - \phi(|z-u|)\colon\ u \in \mathbb{Q}^d \text{ and } |z-u| \leqslant \frac{2C}{n-C} \right\} \quad \text{by the first inequality of (3)} \\
&= f(t,z) - \phi\left(\frac{2C}{n-C}\right).
\end{aligned}$$

By analogy, we can prove the second part of (vi). The proof is complete. □

Remark 3. If f satisfies (H1), then $0 \leqslant \bar{f}_n(t,z) - \underline{f}_n(t,z) \leqslant 2\phi(\frac{2C}{n-C})$ P-a.s. for any $(t,z) \in [0,T] \times \mathbb{R}^d$ and $n > C$.

3. Main theorem

To begin with, we introduce two sequences of BSDE as follows:

$$\underline{y}_t^n = \xi + \int_t^T \underline{g}_n(s, \underline{z}_s^n)\,\mathrm{d}s - \int_t^T \underline{z}_s^n\,\mathrm{d}W_s \tag{4}$$

and

$$\bar{y}_t^n = \xi + \int_t^T \bar{g}_n(s, \bar{z}_s^n)\,\mathrm{d}s - \int_t^T \bar{z}_s^n\,\mathrm{d}W_s. \tag{5}$$

Clearly, for any given $n > C$, both (4) and (5) have unique adapted solutions, for which we denote them by $(\underline{y}_t^n, \underline{z}_t^n)_{t\in[0,T]}$ and $(\bar{y}_t^n, \bar{z}_t^n)_{t\in[0,T]}$ respectively. Moreover we denote the maximal solution and the minimal one of (2) respectively by $(\bar{y}_t, \bar{z}_t)_{t\in[0,T]}$ and $(\underline{y}_t, \underline{z}_t)_{t\in[0,T]}$, and any given solution of (2) by $(y_t, z_t)_{t\in[0,T]}$. We now have the following lemma.

Lemma 4. *Let g satisfy* (H1) *and $\xi \in L^2(\Omega, \mathcal{F}_T, P)$. Then one has,*

(i) $\bar{y}_t^n \geqslant \bar{y}_t^{n+1} \geqslant \bar{y}_t \geqslant y_t \geqslant \underline{y}_t \geqslant \underline{y}_t^{n+1} \geqslant \underline{y}_t^n$, *$P$-a.s. for $t \in [0,T]$ and $n > C$. Moreover,* $\mathbf{E}[|\bar{y}_t^n - \bar{y}_t|^2] + \mathbf{E}[\int_0^T |\bar{z}_t^n - \bar{z}_t|^2\,\mathrm{d}t] \to 0$ *and* $\mathbf{E}[|\underline{y}_t^n - \underline{y}_t|^2] + \mathbf{E}[\int_0^T |\underline{z}_t^n - \underline{z}_t|^2\,\mathrm{d}t] \to 0$ *as $n \to \infty$;*
(ii) *In addition, there exists some positive constant M_0 depending only on C, T and ξ, such that* $E[|\bar{y}_t^n|^2] \leqslant M_0$, $E[\int_0^T |\bar{z}_t^n|^2\,\mathrm{d}t] \leqslant M_0$; *and* $\mathbf{E}[|\underline{y}_t^n|^2] \leqslant M_0$, $E[\int_0^T |\underline{z}_t^n|^2\,\mathrm{d}t] \leqslant M_0$ *for any $n > C$;*
(iii) *For any $n > C$,* $\mathbf{E}[|\bar{y}_t^n - \underline{y}_t^n|] \leqslant 2\phi(\frac{2C}{n-C})T$.

Proof. The proofs of (i) and (ii) can be found in [5]. We now prove (iii). Here we always assume $n > C$. By (4) and (5),

$$\bar{y}_t^n - \underline{y}_t^n = \int_t^T \left(\bar{g}_n(s, \bar{z}_s^n) - \underline{g}_n(s, \underline{z}_s^n)\right)\mathrm{d}s - \int_t^T (\bar{z}_s^n - \underline{z}_s^n)\,\mathrm{d}W_s, \quad t \in [0,T]. \tag{6}$$

Note that

$$\bar{g}_n(s, \bar{z}_s^n) - \underline{g}_n(s, \underline{z}_s^n) = \underline{g}_n(s, \bar{z}_s^n) - \underline{g}_n(s, \underline{z}_s^n) + \bar{g}_n(s, \bar{z}_s^n) - \underline{g}_n(s, \bar{z}_s^n) = \underline{g}_n(s, \bar{z}_s^n) - \underline{g}_n(s, \underline{z}_s^n) + \hat{g}_t^n,$$

where $\hat{g}_t^n := \bar{g}_n(s, \bar{z}_s^n) - \underline{g}_n(s, \bar{z}_s^n)$. It follows from (v) of Lemma 2 that $0 \leqslant \hat{g}_t^n \leqslant 2\phi(\frac{2C}{n-C})$ P-a.s. for $t \in [0,T]$.

We set $\hat{y}_t^n \triangleq \bar{y}_t^n - \underline{y}_t^n$, $\hat{z}_t^n \triangleq \bar{z}_t^n - \underline{z}_t^n$, and denote by $\bar{z}_t^{n,i}$, $\underline{z}_t^{n,i}$ the components of $\bar{z}_t^n$ and $\underline{z}_t^n$ respectively. Define $z_t^{n,0} \triangleq \bar{z}_t^n$ and $z_t^{n,i} \triangleq (\underline{z}_t^{n,1}, \ldots, \underline{z}_t^{n,i}, \bar{z}_t^{n,i+1}, \ldots, \bar{z}_t^{n,d})$ and

$$b_t^{n,i} \triangleq \mathbf{1}_{\{\bar{z}_t^{n,i} \neq \underline{z}_t^{n,i}\}} \frac{\underline{g}_n(t, z_t^{n,i-1}) - \underline{g}_n(t, z_t^{n,i})}{\bar{z}_t^{n,i} - \underline{z}_t^{n,i}},$$

for $1 \leqslant i \leqslant d$ where $\mathbf{1}$ is the indicator function. Eq. (6) can rewritten as

$$\hat{y}_t^n = \int_t^T (b_s^n \hat{z}_s^n + \hat{g}_s^n)\,\mathrm{d}s - \int_t^T \hat{z}_s^n\,\mathrm{d}W_s,$$

for $t \in [0,T]$ where $b_s^n := (b_s^{n,1}, \ldots, b_s^{n,d})$ $(i = 1, \ldots, d)$.

We now set $q_t^n := \exp[\int_0^t b_s^n \, \mathrm{d}W_s - \frac{1}{2}\int_0^t |b_s^n|^2 \, \mathrm{d}s]$. Since $\underline{g}_n$ satisfies a Lipschitz condition, $|b_s^n| \leqslant n$ for any given n. Applying Itô formula to $q_t^n \hat{y}_t^n$ on $[t,T]$ and then taking conditional expectation yields:

$$\hat{y}_t^n = (q_t^n)^{-1}\mathbf{E}\left[\int_t^T q_s^n \hat{g}_s^n \, \mathrm{d}s \,|\, \mathcal{F}_t\right] = \mathbf{E}\left[\int_t^T \exp\left(\int_t^s b_r^n \, \mathrm{d}W_r - \frac{1}{2}\int_t^s |b_r^n|^2 \, \mathrm{d}r\right)\hat{g}_s^n \, \mathrm{d}s \,|\, \mathcal{F}_t\right].$$

It follows from the property of exponential martingale that, for $s \geqslant t$,

$$\mathbf{E}\left[\exp\left(\int_t^s b_r^n \, \mathrm{d}W_r - \frac{1}{2}\int_t^s |b_r^n|^2 \, \mathrm{d}r\right)\right] = 1.$$

Therefore,

$$\begin{aligned}
\mathbf{E}[\hat{y}_t^n] &= \mathbf{E}\left[\mathbf{E}\left[\int_t^T \exp\left(\int_t^s b_r^n \, \mathrm{d}W_r - \frac{1}{2}\int_t^s |b_r^n|^2 \, \mathrm{d}r\right)\hat{g}_s^n \, \mathrm{d}s \,|\, \mathcal{F}_t\right]\right] \\
&= \mathbf{E}\left[\int_t^T \exp\left(\int_t^s b_r^n \, \mathrm{d}W_r - \frac{1}{2}\int_t^s |b_r^n|^2 \, \mathrm{d}r\right)\hat{g}_s^n \, \mathrm{d}s\right] \\
&\leqslant 2\phi\left(\frac{2C}{n-C}\right)\mathbf{E}\left[\int_t^T \exp\left(\int_t^s b_r^n \, \mathrm{d}W_r - \frac{1}{2}\int_t^s |b_r^n|^2 \, \mathrm{d}r\right)\mathrm{d}s\right] \leqslant 2\phi\left(\frac{2C}{n-C}\right)T.
\end{aligned}$$

The proof is complete. □

The following result is our main theorem:

Theorem 5. *Let g satisfy* (H1) *and $\xi \in L^2(\Omega, \mathcal{F}_T, P)$. Then the solution of* (2) *is unique.*

Proof. From Lemma 4(iii), it follows that $\mathbf{E}[|\bar{y}_t^n - \underline{y}_t^n|] \to 0$ as $n \to \infty$ for $t \in [0,T]$. Therefore

$$\mathbf{E}\big[|\bar{y}_t - \underline{y}_t|\big] \leqslant \mathbf{E}\big[|\bar{y}_t - \bar{y}_t^n|\big] + \mathbf{E}\big[|\bar{y}_t^n - \underline{y}_t^n|\big] + \mathbf{E}\big[|\underline{y}_t^n - \underline{y}_t|\big] \to 0,$$

as $n \to \infty$ for $t \in [0,T]$. The proof is complete. □

Remark 6. In the case when g depends on y and is uniformly continuous condition in y, the uniqueness of solution does not hold in general. For example, let us consider the following equation:

$$y_t = \int_t^1 \sqrt{|y_s|} \, \mathrm{d}s - \int_t^1 z_s \, \mathrm{d}W_s \quad \text{for } t \in [0,1].$$

Clearly, $g(y) = \sqrt{|y|}$ is uniformly continuous. It is not hard to check that for each $c \in [0,1]$,

$$(y_t, z_t)_{t\in[0,1]} = \left(\left[\max\left(0, \frac{c-t}{2}\right)\right]^2, 0\right)_{t\in[0,1]},$$

is a solution of the above BSDE.

Certainly, if g is Lipschitz continuous with respect to y or satisfies some kind of monotonic condition just like used in [6], the result in Theorem 6 also holds true, this point is not difficult to be found in the proofs of Theorem 6 and Lemma 4.

Remark 7. It is worth noting that there is an important difference between the BSDE satisfying standard condition and the BSDE discussed in this note: although we still have the associated comparison theorem for this kind of BSDEs, the associated strict comparison theorem – see [2, (ii) of Proposition 2.1] – (which says, if $\xi_1 \geqslant \xi_2$ P-a.s. and $P(\xi_1 > \xi_2) > 0$, then $y_0^{\xi_1} > y_0^{\xi_2}$ where $(y_t^{\xi_i}, z_t^{\xi_i})_{t\in[0,T]}$ denotes the solution of (g, T, ξ_i), $i = 1, 2$) does not hold in general.

For example, let us consider a BSDE as follows:

$$y_t^X = X + \int_t^T \frac{3}{2}|z_s^X|^{2/3} - \int_t^T z_s^X \,\mathrm{d}W_s,$$

where W is a one-dimensional Brownian motion, $g = \frac{3}{2}|z|^{2/3}$. It is not hard to check that for each constant $c \in \mathbb{R}$,

$$(y_t, z_t)_{t\in[0,T]} = \left(c - \frac{1}{4}W_t^4, -W_t^3\right)_{t\in[0,T]}$$

is the solution of $(g, T, c - \frac{1}{4}W_T^4)$, hence $y_0^{c-\frac{1}{4}W_T^4} = y_0^c = c$. But $c \geqslant c - \frac{1}{4}W_T^4$ P-a.s. and $P(c > c - \frac{1}{4}W_T^4) > 0$. In economics, this means that there exist infinitely many opportunities of arbitrage.

More detailed discussions about this phenomenon and the corresponding PDE problem will appear in another paper.

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