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A Gutzwiller type formula for a reduced Hamiltonian within the framework of symmetry

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Abstract

For a classical Hamiltonian with a finite group of symmetries, we give semi-classical asymptotics in a neighbourhood of an energy E of a regularized spectral density of the quantum Hamiltonian restricted to symmetry subspaces of Peter–Weyl defined by irreducible characters of the group. If we suppose that the energy level Σ_E is compact, non-critical, and that its periodic orbits are non-degenerate, we get a Gutzwiller type formula for the reduced Hamiltonian, whose oscillating part involves the symmetry properties of closed trajectories of Σ_E. ***To cite this article: R. Cassanas, C. R. Acad. Sci. Paris, Ser. I 340 (2005).***

Résumé

Une formule de type Gutzwiller pour un hamiltonien réduit en présence de symétries. Pour un hamiltonien classique comportant des symétries issues d'un groupe fini, on donne le comportement semi-classique au voisinage d'une énergie E, d'une densité spectrale régularisée pour l'hamiltonien quantique restreint aux espaces de symétries de Peter–Weyl définis par les caractères irréductibles du groupe. Supposant que le niveau d'énergie Σ_E est compact sans point critique, et que ses orbites périodiques sont non dégénérées, on obtient pour l'opérateur restreint une formule du type Gutzwiller, dont la partie oscillante fait intervenir la symétrie des orbites périodiques de Σ_E. ***Pour citer cet article : R. Cassanas, C. R. Acad. Sci. Paris, Ser. I 340 (2005).***

Version française abrégée

Cette Note s'inscrit dans la lignée des articles de Helffer, Robert et El Houakmi [4,6]. On se donne un sous-groupe fini G du groupe orthogonal $O(d)$ et un hamiltonien lisse $H : \mathbb{R}^{2d} \to \mathbb{R}$ vérifiant :

$$\forall g \in G,\ \forall z \in \mathbb{R}^{2d}, \quad H\big(M(g)z\big) = H(z), \quad \text{où } M(g)(x,\xi) := (gx, g\xi). \tag{1}$$

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A cette action M de G sur l'espace des phases $\mathbb{R}^{2d}$, on associe une action $\widetilde{M}$ de G sur $L^2(\mathbb{R}^d)$ (voir (4)). Sous l'hypothèse (1), étant donné un caractère χ du groupe G, le quantifié de Weyl de H, noté $\widehat{H}$, commute avec le projecteur de Peter–Weyl P_χ construit comme moyenne barycentrique des $\widetilde{M}(g)$, et dont l'expression est donnée en (6). On note alors $\widehat{H}_\chi$ la restriction de $\widehat{H}$ à l'espace de symétrie $L^2_\chi(\mathbb{R}^d) := P_\chi(L^2(\mathbb{R}^d))$. Le hamiltonien réduit $\widehat{H}_\chi$ est alors autoadjoint sur $L^2_\chi(\mathbb{R}^d)$. Lorsque $E \in \mathbb{R}$ et $H^{-1}([E-\delta E, E+\delta E])$ est compact (où $\delta E > 0$), ψ étant une fonction lisse à support dans $]E-\delta E, E+\delta E[$, $\psi(\widehat{H}_\chi)$ est à trace. Si en outre $\hat{f}$ est lisse à support compact, et si le niveau d'énergie $\Sigma_E := \{H = E\}$ est sans point critique, on donne une asymptotique de la densité spectrale régularisée réduite $\mathcal{G}_\chi(h) := \mathrm{Tr}(\psi(\widehat{H}_\chi) f((E-\widehat{H}_\chi)/h))$ lorsque h tend vers zéro.

Dans sa thèse de doctorat, Zahra El Houakmi a étudié l'asymptotique de $\mathcal{G}_\chi(h)$ pour donner le comportement semi-classique de la fonction de comptage des valeurs propres de $\widehat{H}_\chi$ dans un intervalle de $\mathbb{R}$ donné (voir [3], ainsi que [4] dans le cas d'un groupe compact). Pour cela, elle a supposé que $\hat{f}$ était supportée près de zéro. Ici, on ne fait pas de restriction sur le support (compact) de $\hat{f}$. La quantité $\mathcal{G}_\chi(h)$ possède alors une asymptotique faisant intervenir des termes oscillants de la forme $\mathrm{e}^{\frac{\mathrm{i}}{\hbar}\alpha} \times h^k c_k(\hat{f})$.

On retrouve ainsi les résultats d'El Houakmi lorsque $\hat{f}$ est supportée près de zéro (cf. Theorem 2.1). On fait par ailleurs une hypothèse de non-dégénérescence sur certaines orbites périodiques du système classique du niveau d'énergie Σ_E qui ont une période dans $[-|G|T, |G|T]$, où $T > 0$ est tel que $\mathrm{Supp}\,\hat{f} \subset [-T, T]$, de telle sorte que ces orbites soient en nombre fini. On obtient alors une formule de type Gutzwiller pour $\widehat{H}_\chi$, qui fait intervenir la symétrie des orbites précédentes et de nombreuses quantités caractéristiques du système classique associé à H (voir Theorem 2.2).

Si l'on suppose de plus que $g = \mathrm{Id}_{\mathbb{R}^d}$ est l'unique élément de G ayant un point fixe dans Σ_E, alors l'espace réduit Σ_E/G hérite d'une structure de variété lisse, le système dynamique classique restreint à Σ_E passe au quotient, et on obtient une formule de Gutzwiller dans Σ_E/G (cf. Corollary 2.3).

La méthode employée diffère de celle des articles précédemment cités : on évite la méthode BKW, les opérateurs de Fourier intégraux et les problèmes de caustiques en utilisant un théorème de propagation des états cohérents issu des travaux de M. Combescure et D. Robert sur ce sujet. Comme dans l'article [1], on aboutit à un problème de phase stationnaire, à g fixé dans G, avec phase complexe et variété compacte de points critiques donnée par :

$$\mathcal{C}_{E,g} = \{(t,z) \in]-T, T[\times \mathbb{R}^{2d}\colon\ z \in \Sigma_E,\ M(g)\Phi_t(z) = z\}. \tag{2}$$

Pour $(t,z) \in \mathcal{C}_{E,g}$, on a, pour $k \in \mathbb{N}$, $\Phi_{kt}(z) = M(g^{-k})z$. Le groupe G étant fini, l'orbite de z est périodique et $|G|t$ en est une période, ce qui explique en partie les quantités intervenant au Théorème 2.2.

1. Introduction

This Note is closely related to many papers [1,2,5,7], and above all, to the articles of Helffer, Robert, El Houakmi [6,4] and El Houakmi's Ph.D. Thesis [3]. It follows the same landscape: let $H : \mathbb{R}^{2d} \to \mathbb{R}$ be a smooth Hamiltonian and G a finite subgroup of the orthogonal group $O(d)$. If $g \in G$, we set:

$$M(g) := \begin{pmatrix} g & 0 \\ 0 & g \end{pmatrix} \in O(2d) \cap \mathrm{Sp}(d).$$

M is the action of G on the phase space $\mathbb{R}^{2d}$. We assume that G is a symmetry for H, i.e.:

$$\forall g \in G,\ \forall z \in \mathbb{R}^{2d}, \quad H(M(g)z) = H(z). \tag{3}$$

As usual, we make suitable assumptions (see [1]) to have nice properties for the Weyl quantization of H, which is defined as follows: for $u \in \mathcal{S}(\mathbb{R}^d)$,

$$\mathrm{Op}_h^w(H)u(x) = (2\pi h)^{-d} \int\limits_{\mathbb{R}^d}\int\limits_{\mathbb{R}^d} \mathrm{e}^{\frac{\mathrm{i}}{h}(x-y)\xi} H\left(\frac{x+y}{2}, \xi\right) u(y)\,\mathrm{d}y\,\mathrm{d}\xi.$$

In particular, $\mathrm{Op}_h^w(H)$ is essentially selfadjoint on $\mathcal{S}(\mathbb{R}^d)$ and we denote by $D(\widehat{H})$, $\widehat{H}$ its selfadjoint extension.

G acts on the quantum space $L^2(\mathbb{R}^d)$ by $\widetilde{M}$ defined for $g \in G$ by:

$$\widetilde{M}(g)(f)(x) = f(g^{-1}x), \quad \forall f \in L^2(\mathbb{R}^d),\ \forall x \in \mathbb{R}^d. \tag{4}$$

Under assumption (3), we have: $\forall g \in G, \quad [\widehat{H}, \widetilde{M}(g)] = 0.$ (5)

Let $\widehat{G}$ be the set of algebraic characters of irreducible complex representations of G. If $\chi \in \widehat{G}$, we introduce:

$$P_\chi := \frac{d_\chi}{|G|} \sum_{g \in G} \overline{\chi(g)}\, \widetilde{M}(g), \tag{6}$$

the associated orthogonal projector, where d_χ is the degree of the representation ($d_\chi = \chi(\mathrm{Id}_{\mathbb{R}^d})$, see [9]). We call $L^2_\chi(\mathbb{R}^d) := P_\chi(L^2(\mathbb{R}^d))$ the *symmetry subspace associated to* χ. We apply the Peter–Weyl theorem (see [10]) to claim that:

$$L^2(\mathbb{R}^d) = \bigoplus_{\chi \in \widehat{G}}^{\perp} L^2_\chi(\mathbb{R}^d).$$

Thanks to (5), we have $[\widehat{H}, P_\chi] = 0$. If we denote by $\widehat{H}_\chi$ the restriction of $\widehat{H}$ to $D(\widehat{H}_\chi) := D(\widehat{H}) \cap L^2_\chi(\mathbb{R}^d)$, then $(D(\widehat{H}_\chi), \widehat{H}_\chi)$ is a selfadjoint operator on the Hilbert space $L^2_\chi(\mathbb{R}^d)$, which we call the *reduced* (*quantum*) *Hamiltonian*.

In her Ph.D. Thesis, El Houakmi gave the semi-classical behaviour as $h \to 0^+$ of $N_\chi(I) := \mathrm{Tr}(\mathbb{1}_I(\widehat{H}_\chi))$, the eigenvalues counting function of $\widehat{H}_\chi$ in a given interval I of $\mathbb{R}$ (see [4] in the case of a compact group G). As usual, the idea is to regularize: suppose that $E \in \mathbb{R}$ is such that the energy level $\Sigma_E := \{H = E\}$ is compact and without critical point for H. Then one can study the *reduced regularized spectral density*:

$$\mathcal{G}_\chi(h) := \mathrm{Tr}\left(\psi(\widehat{H}_\chi) f\left(\frac{E - \widehat{H}_\chi}{h}\right)\right),$$

where ψ is smooth, compactly supported in a neighbourhood $]E - \delta E, E + \delta E[$ of E such that $H^{-1}([E - \delta E, E + \delta E])$ is compact ($\psi(\widehat{H}_\chi)$ is an energy cut-off which is trace class), f is smooth and $\hat{f}$ (the Fourier transform of f) is compactly supported.

In previous publications on the asymptotic of $N_\chi(I)$, it was enough to suppose that $\hat{f}$ is supported near zero. When $\hat{f}$ is with compact support without any other restriction, oscillating terms in h may appear, giving rise to a Gutzwiller type formula which is the main goal of this Note. The method employed is close to [1]: unlike articles previously quoted, which made use of an approximation of the propagator $\mathrm{e}^{-\mathrm{i}\frac{t}{h}\widehat{H}}$ by some FIO following the BKW method, we will use here the work of Combescure and Robert on the propagation of coherent states.

2. Asymptotics

2.1. Weyl term

We denote by $\Phi_t(z)$ the classical flow of H at time t and initial condition $z \in \mathbb{R}^{2d}$.[1] We introduce the following set for $g \in G$:

[1] $\Phi_t(z)$ is the solution of the Hamiltonian system $\dot{z_t} = J\nabla H(z_t)$ with initial condition z, where $J := \begin{pmatrix} 0 & I_d \\ -I_d & 0 \end{pmatrix}$.

$$\mathcal{L}_{E,g} := \{t \in \mathbb{R}\colon\ \exists z \in \Sigma_E\colon\ M(g)\Phi_t(z) = z\}.$$

We slightly precise the result of El Houakmi by the following theorem:

Theorem 2.1. *We have* $\mathcal{G}_\chi(h) = \frac{d_\chi}{|G|}\sum_{g\in G}\overline{\chi(g)}I_{g,E}(h)$. *For* g *in* G *let us denote by*

$$\nu_g := \dim\ker(g - \mathrm{Id}_{\mathbb{R}^d}),\qquad F_g := \ker\big(M(g) - \mathrm{Id}_{\mathbb{R}^{2d}}\big)\quad \textit{and}\quad \widetilde{F}_g := \ker(g - \mathrm{Id}_{\mathbb{R}^d}).$$

Then, under previous assumptions, we have: if $\mathrm{Supp}\,\hat{f} \cap \mathcal{L}_{E,g} = \emptyset$, *then* $I_{g,E}(h) = \mathrm{O}(h^{+\infty})$.

If $\mathrm{Supp}\,\hat{f} \cap \mathcal{L}_{E,g} = \{0\}$ *then we have the following expansion modulo* $\mathrm{O}(h^{+\infty})$:

$$I_{g,\lambda}(h) \asymp h^{1-\nu_g}\sum_{k\geqslant 0} c_k(\hat{f}, g)h^k,\quad \text{as } h \to 0^+ \tag{7}$$

uniformly in λ *in a small neighborhood of* E, *where* $c_k(\hat{f}, g)$ *are distributions in* $\hat{f}$ *with support in* $\{0\}$, *and, if* $d(\Sigma_\lambda \cap F_g)$ *denote the Euclidian measure on* $\Sigma_\lambda \cap F_g$, *we have*:

$$c_0(\hat{f}, g) = \psi(\lambda)\hat{f}(0)\frac{(2\pi)^{-\nu_g}}{\det((\mathrm{Id}_{\mathbb{R}^d} - g)|_{\tilde{F}_g^{\perp}})}\int_{\Sigma_\lambda\cap F_g}\frac{d(\Sigma_\lambda \cap F_g)(z)}{|\nabla H(z)|}. \tag{8}$$

2.2. Oscillating terms

If $g \in G$ and γ is a periodic orbit of Σ_E globally stable by $M(g)$, we set:

$$\mathcal{L}_{g,\gamma} := \{t \in \mathrm{Supp}\,\hat{f}\colon\ \exists z \in \gamma\colon\ M(g)\Phi_t(z) = z\}.$$

If $t_0 \in \mathcal{L}_{g,\gamma}$, $z \in \gamma$, then P_{γ,g,t_0} denotes the Poincaré map of γ between z and $M(g^{-1})z$ at time t_0, restricted to Σ_E. The characteristic polynomial of P_{γ,g,t_0} does not depend on $z \in \gamma$.

Theorem 2.2. *Suppose* $\Psi = 1$ *in a neighbourhood of* E. *Under previous assumptions, suppose that* $0 \notin \mathrm{Supp}\,\hat{f} \subset\ ]-T, T[$, *where* $T > 0$. *Make the following hypothesis of non-degeneracy: if* $\gamma \subset \Sigma_E$, *is such that* $\exists g \in G$ *and* $\exists t_0 \in \mathcal{L}_{g,\gamma}$, $t_0 \neq 0$, *then* 1 *is not an eigenvalue of* $M(g)dP_{\gamma,g,t_0}$. *Then the set of such* γ*'s is finite and the following expansion holds true modulo* $\mathrm{O}(h^{+\infty})$, *as* $h \to 0^+$:

$$\mathcal{G}_\chi(h) \asymp \frac{d_\chi}{|G|}\sum_{\substack{\gamma \text{ periodic}\\ \text{orbit of } \Sigma_E}}\ \sum_{\substack{g\in G \text{ s.t.}\\ M(g)\gamma=\gamma}}\overline{\chi(g)}\sum_{\substack{t_0\in\mathcal{L}_{g,\gamma}\\ t_0\neq 0}} \mathrm{e}^{\frac{\mathrm{i}}{h}S_\gamma(t_0)}\sum_{k\geqslant 0} d_k^{\gamma,g,t_0}(\hat{f})h^k.$$

Terms $d_k^{\gamma,g,t_0}(\hat{f})$ *are distributions in* $\hat{f}$ *with support in* $\{t_0\}$, $S_\gamma(t_0) := \int_0^{t_0} p_s\dot{q}_s\,\mathrm{d}s$, $((q_s, p_s) := \Phi_s(z))$, *and*

$$d_0^{\gamma,g,t_0}(\hat{f}) = \frac{T_\gamma^*\,\mathrm{e}^{\mathrm{i}\frac{\pi}{2}\sigma_\gamma(g,t_0)}}{2\pi|\det(M(g)dP_{\gamma,g,t_0} - \mathrm{Id})|^{1/2}}\hat{f}(t_0)$$

where T_γ^* *is the primitive period of* γ *and* $\sigma_\gamma(g, t_0) \in \mathbb{Z}$ *is a Maslov index of* γ.

If one omits the hypothesis of non-degeneracy, we still get a more general asymptotic, but we did not calculate first terms, which depend on the connected components of the sets (for $g \in G$):

$$C_{E,g} := \{(t, z) \in\ ]-T, T[\ \times \Sigma_E\colon\ M(g)\Phi_t(z) = z\}. \tag{9}$$

Corollary 2.3. *Under previous assumptions, suppose that $g = \mathrm{Id}_{\mathbb{R}^d}$ is the only element of G to have a fix point on Σ_E. Then the reduced space Σ_E/G is a smooth manifold, the dynamics of H on Σ_E drop down to the quotient. Moreover, if π denotes the projection on the quotient and $\bar{\gamma}$ is a periodic orbit in Σ_E/G, if $\pi(\gamma) = \bar{\gamma}$, then, there is only one g_γ in G such that, $\forall z \in \gamma$, $M(g_\gamma)\Phi_{T^*_{\bar{\gamma}}}(z) = z$. If $\pi(\gamma_1) = \pi(\gamma_2)$ then g_{γ_1} and g_{γ_2} are conjugated in G, and we denote by $\chi(g_{\bar{\gamma}})$ the quantity $\chi(g_{\gamma_1}) = \chi(g_{\gamma_2})$. We then have*:

$$\mathcal{G}_\chi(h) = d_\chi \sum_{\substack{\bar{\gamma}\ \text{periodic}\\ \text{orbit of } \Sigma_E/G}} \sum_{\substack{n\in\mathbb{Z}^*\ \text{s.t.}\\ nT^*_{\bar{\gamma}}\in\operatorname{Supp}\hat{f}}} \hat{f}(nT^*_{\bar{\gamma}})\overline{\chi(g^n_{\bar{\gamma}})}\mathrm{e}^{\frac{\mathrm{i}}{h}nS_{\bar{\gamma}}} \frac{T^*_{\bar{\gamma}}\mathrm{e}^{\mathrm{i}\frac{\pi}{2}\sigma_{\bar{\gamma},n}}}{2\pi|\det((dP_{\bar{\gamma}})^n - \mathrm{Id})|^{1/2}} + \mathrm{O}(h) \quad \text{as } h \to 0^+,$$

where $S_{\bar{\gamma}} := \int_0^{T^*_{\bar{\gamma}}} p_s\dot{q}_s\,\mathrm{d}s$, $P_{\bar{\gamma}}$ is the Poincaré map of $\bar{\gamma}$ in Σ_E/G, and $\sigma_{\bar{\gamma},n}$ is a Maslov index of $\bar{\gamma}$.

3. Sketch of the proof

Classically, we have $\mathcal{G}_\chi(h) = \mathrm{Tr}(\psi(\widehat{H})f((E-\widehat{H})/h)P_\chi)$, so that we decompose

$$\mathcal{G}_\chi(h) = \frac{d_\chi}{|G|}\sum_{g\in G}\overline{\chi(g)}I_{g,E}(h), \quad \text{where } I_{g,E}(h) = \mathrm{Tr}\left(\psi(\widehat{H})f\left(\frac{E-\widehat{H}}{h}\right)\widetilde{M}(g)\right).$$

By using the Fourier inversion formula, we make appear the h-quantum unitary group $U_h(t) := \mathrm{e}^{-\mathrm{i}\frac{t}{h}\widehat{H}}$ to obtain:

$$I_{g,E}(h) = \frac{1}{2\pi}\int_{\mathbb{R}} \mathrm{e}^{\mathrm{i}tE/h}\hat{f}(t)\,\mathrm{Tr}\big(\psi(\widehat{H})U_h(t)\widetilde{M}(g)\big)\,\mathrm{d}t. \tag{10}$$

Then, we use a trace formula with coherent states, for A trace class operator on $L^2(\mathbb{R}^d)$:

$$\mathrm{Tr}(A) = (2\pi h)^{-d}\int_{\mathbb{R}^{2d}_\alpha} \langle A\varphi_\alpha;\varphi_\alpha\rangle_{L^2(\mathbb{R}^d)}\,\mathrm{d}\alpha. \tag{11}$$

Here, φ_α is the coherent state centered at $\alpha := (q,p)$ defined for $x \in \mathbb{R}^d$ by:[2]

$$\varphi_\alpha(x) = \frac{1}{(h\pi)^{d/4}}\exp\left(\mathrm{i}\frac{p}{h}\left(x-\frac{q}{2}\right)\right)\exp\left(-\frac{|x-q|^2}{2h}\right). \tag{12}$$

Besides, coherent states behave well with symmetries: $\widetilde{M}(g)\varphi_\alpha = \varphi_{M(g)\alpha}$. (13)

Eventually, we have:

$$I_g(h) = \frac{h^{-d}}{(2\pi)^{d+1}}\int_{\mathbb{R}}\int_{\mathbb{R}^{2d}_\alpha} \mathrm{e}^{\mathrm{i}\frac{tE}{h}}\hat{f}(t)m_h(\alpha,t,g)\,\mathrm{d}\alpha\,\mathrm{d}t, \tag{14}$$

where

$$m_h(\alpha,t,g) := \big\langle U_h(t)\varphi_\alpha;\psi(\widehat{H})\varphi_{M(g)^{-1}\alpha}\big\rangle_{L^2(\mathbb{R}^d)}. \tag{15}$$

The term $\psi(\widehat{H})\varphi_{M(g)^{-1}\alpha}$ can be easily developped by functional calculus and action of a pseudo-differential operator on a coherent state. For the term $U_h(t)\varphi_\alpha$ we use the result of Combescure and Robert on the propagation of

[2] See [1] for more details.

coherent states (see for example [1] or [8]). After several calculations, we are lead to a stationnary phase problem with complex phase and a compact smooth manifold of critical points. We have to find the asymptotic of quantities of the following form:

$$K(h) := \int_{\mathbb{R}_t} \int_{\mathbb{R}^{2d}_z} \chi_0(z) \hat{f}(t) \big(\partial^\gamma(\psi \circ H)\big)(g^{-1}z) c(t,z) \exp\left(\frac{\mathrm{i}}{h}\varphi_{E,g}(t,z)\right) \mathrm{d}t\, \mathrm{d}z \tag{16}$$

where c is smooth, χ_0 is smooth, compactly supported in $\mathbb{R}^{2d}$, and if $t \in \mathbb{R}$, $z \in \mathbb{R}^{2d}$, $g \in G$, then:

$$\varphi_{E,g}(t,z) := S(t,z) + Et - \frac{1}{2}(gq_t - q)(gp_t + p) + \frac{\mathrm{i}}{4}\big\langle (I - W_t)\big(z_t - M(g^{-1})z\big), z_t - M(g^{-1})z\big\rangle, \tag{17}$$

where $z_t = (q_t, p_t)$ is the solution with $z_0 = z$ of $\dot{z_t} = J\nabla H(z_t)$, $S(t,z) = \int_0^t \dot{q}_t p_t - H(z_t)\,\mathrm{d}t$ is the classical action, $\Im\varphi_{E,g} \geqslant 0$ and $|||W_t||| < 1$. The critical set of the phase (17) defined by $\{\nabla_{(t,z)}\varphi_{E,g} = 0\} \cap \{\Im\varphi_{E,g} = 0\}$ is given by (9). As the group G is finite, one can see that if $(t_0, z) \in \mathcal{C}_{E,g}$, then z has a periodic orbit with period $|G|t_0$. This gives an idea of the origin of quantities appearing in Theorem 2.2.

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