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Probability Theory

Large deviations for invariant measures of general stochastic reaction–diffusion systems

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Abstract

In this paper we prove a large deviations principle for the invariant measures of a class of reaction–diffusion systems in bounded domains of $\mathbb{R}^d,\ d \geqslant 1$, perturbed by a noise of multiplicative type. We consider reaction terms which are not Lipschitz-continuous and diffusion coefficients in front of the noise which are not bounded and may be degenerate. ***To cite this article: S. Cerrai, M. Röckner, C. R. Acad. Sci. Paris, Ser. I 337 (2003).***

Résumé

Grandes déviations pour les mesures invariantes de systèmes généraux d'équations de réaction–diffusion stochastiques. Dans cet article on prouve un principe de grandes déviations pour les mesures invariantes de systèmes de réaction–diffusion stochastiques dans des domaines bornés de $\mathbb{R}^d,\ d \geqslant 1$, perturbés par un bruit multiplicatif. On considère des termes de réaction qui ne sont pas Lipschitz-continus et des coefficients de diffusion qui ne sont pas bornés et peuvent être dégénérés. ***Pour citer cet article : S. Cerrai, M. Röckner, C. R. Acad. Sci. Paris, Ser. I 337 (2003).***

Version française abrégée

On étudie le comportement asymptotique des systèmes de réaction–diffusion stochastiques dans des domaines bornés de $\mathbb{R}^d,\ d \geqslant 1$, perturbés par un bruit multiplicatif (cf. (1)).

Le terme de réaction est localement Lipschitzien et à croissance polynomiale. Le terme de diffusion est Lipschitz-continu et non borné. De plus, il peut s'annuler. Le bruit est blanc dans le temps et coloré dans l'espace. En dimension $d = 1$ il peut être pris blanc et en dimension $d > 1$ il doit être coloré, mais la covariance n'est jamais de classe Hilbert–Schmidt.

Dans [4] nous avons prouvé que pour chaque $\varepsilon > 0$ le système (1) a une solution u_ε^x dans l'espace des fonctions continues E et que pour chaque $x \in E$ et $a > 0$ la famille des probabilités $\{\mathcal{L}(u_\varepsilon^x(t))\}_{t \geqslant a}$ est tendue dans $(E, \mathcal{B}(E))$. Donc, il existe une suite $\{t_n\} \uparrow +\infty$ (qui peut dépendre de ε) telle que la suite des probabilités (voir (3)), converge

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faiblement vers une mesure ν_ε qui est invariante pour le système (1). Nous montrons ici que la famille des mesures invariantes $\{\nu_\varepsilon\}_{\varepsilon>0}$ obéit à un principe des grandes déviations dans l'espace E.

1. Introduction

We are here concerned with the study of the long-term behavior of the stochastic reaction–diffusion system

$$\begin{cases} \dfrac{\partial u_i}{\partial t}(t,\xi) = \mathcal{A}_i\, u_i(t,\xi) + f_i\big(\xi, u_1(t,\xi),\dots,u_r(t,\xi)\big) \\ \qquad\qquad + \varepsilon \sum_{j=1}^r g_{ij}\big(\xi, u_1(t,\xi),\dots,u_r(t,\xi)\big) Q_j \dfrac{\partial w_j}{\partial t}(t,\xi), \quad t \geqslant 0,\ \xi \in \overline{\mathcal{O}}, \\ u_i(0,\xi) = x_i(\xi),\ \xi \in \overline{\mathcal{O}}, \quad \mathcal{B}_i\, u_i(t,\xi) = 0, \quad t \geqslant 0,\ \xi \in \partial\mathcal{O},\ 1 \leqslant i \leqslant r. \end{cases} \tag{1}$$

Here $\mathcal{O}$ is a bounded open set of $\mathbb{R}^d$, with $d \geqslant 1$, having a C^∞ boundary. For each $i = 1,\dots,r$

$$\mathcal{A}_i(\xi, D) = \sum_{h,k=1}^d \frac{\partial}{\partial \xi_h}\left(a^i_{hk}(\xi)\frac{\partial}{\partial \xi_k}\right) - \alpha_i, \quad \xi \in \overline{\mathcal{O}}. \tag{2}$$

The constants α_i are positive, the coefficients a^i_{hk} are in $C^\infty(\overline{\mathcal{O}})$ and the matrices $a^i(\xi) := [a^i_{hk}(\xi)]_{hk}$ are non-negative and symmetric for any $\xi \in \overline{\mathcal{O}}$ and fulfill a uniform ellipticity condition, that is $\inf_{\xi\in\overline{\mathcal{O}}}\langle a^i(\xi)h, h\rangle \geqslant \lambda_i\,|h|^2,\ h \in \mathbb{R}^d$, for some positive constant λ_i. Finally, the operators $\mathcal{B}_i$ act on $\partial\mathcal{O}$ and are assumed either of Dirichlet or of co-normal type.

The mapping $f := (f_1,\dots,f_r) : \overline{\mathcal{O}} \times \mathbb{R}^r \to \mathbb{R}^r$ is only locally Lipschitz-continuous and has polynomial growth. The mapping $g := [g_{ij}] : \overline{\mathcal{O}} \times \mathbb{R}^r \to \mathcal{L}(\mathbb{R}^r)$ is Lipschitz-continuous, without any global boundedness and non-degeneracy assumptions.

The linear operators Q_j are bounded on $L^2(\mathcal{O})$ and may be taken to be equal to the identity operator in case of space dimension $d = 1$. The noisy perturbations $\partial w_j/\partial t$ are independent cylindrical Wiener processes, defined on the same stochastic basis $(\Omega, \mathcal{F}, \mathcal{F}_t, \mathbb{P})$.

In [3] (see also [2]) it has been proved that for any $\varepsilon > 0$ system (1) admits a unique global solution u^x_ε in the space E of continuous functions on $\overline{\mathcal{O}}$ and for each initial datum $x \in E$ and $a > 0$ the family of probability measures $\{\mathcal{L}(u^x_\varepsilon(t)\}_{t\geqslant a}$ is tight in $(E, \mathcal{B}(E))$. In particular, due to the Krylov–Bogoliubov theorem it has been shown that there exists a sequence $\{t_n\} \uparrow +\infty$ (possibly depending on ε) such that the sequence of probability measures defined by

$$\nu_{\varepsilon,n}(\Gamma) := \frac{1}{t_n} \in t_0^{t_n} \mathbb{P}\big(u^0_\varepsilon(s) \in \Gamma\big)\, \mathrm{d}s, \quad \Gamma \in \mathcal{B}(E), \tag{3}$$

converges weakly to some measure ν_ε, which is invariant for system (1).

In the earlier paper [4] we have proved that the process $\{u^x_\varepsilon\}_{\varepsilon>0}$ is governed by a large deviation principle in $C([0,T]; E)$, for any $T > 0$. Our aim here is to prove that the family of invariant measures $\{\nu_\varepsilon\}_{\varepsilon>0}$ defined as the weak limits of the sequences of measures as in (3) obeys a large deviation principle in E.

2. Assumptions

In what follows we shall denote by H the Hilbert space $L^2(\mathcal{O}; \mathbb{R}^r)$, endowed with the scalar product $\langle\cdot,\cdot\rangle_H$ and the corresponding norm $|\cdot|_H$. Moreover we shall denote by A the realization in H of the differential operator $\mathcal{A} = (\mathcal{A}_1,\dots,\mathcal{A}_r)$ defined in (2), endowed with the boundary conditions $\mathcal{B} = (\mathcal{B}_1,\dots,\mathcal{B}_r)$, where for each $i = 1,\dots,r$

$$\mathcal{B}_i u = u, \quad \text{or} \quad \mathcal{B}_i u = \langle a^i \nu, \nabla u\rangle \tag{4}$$

(here ν is the normal vector at $\partial\mathcal{O}$).

The operator A generates an analytic semigroup e^{tA} in each $L^p(\mathcal{O};\mathbb{R}^r)$, with $1 \leqslant p \leqslant \infty$, which is self-adjoint on H. Moreover, e^{tA} is compact on $L^p(\mathcal{O};\mathbb{R}^r)$, for all $1 \leqslant p \leqslant \infty$ and $t > 0$, and the spectrum $\{-\alpha_n\}$ is independent of p. Our first hypothesis concerns the eigenvalues of A.

Hypothesis 2.1. *The complete orthonormal system of H which diagonalizes A is equi-bounded in the sup-norm.*

Next, we assume that $Q := (Q_1, \dots, Q_r) : H \to H$ is a bounded linear operator which satisfies the following conditions.

Hypothesis 2.2. *Q is non-negative and diagonal with respect to the complete orthonormal basis which diagonalizes A, with eigenvalues $\{\lambda_n\}$. Moreover, if $d \geqslant 2$ we have*

$$\text{there exists} \quad \begin{cases} \varrho < \infty & \text{if } d = 2 \\ \varrho < \dfrac{2d}{d-2} & \text{if } d > 2 \end{cases} \quad \text{such that} \quad \|Q\|_\varrho := \left(\sum_{k=1}^{\infty} \lambda_n^\varrho\right)^{1/\varrho} < \infty. \tag{5}$$

In Hypotheses 2.3 and 2.4 below we give conditions on the coefficients f and g.

Hypothesis 2.3. *The mapping $g : \overline{\mathcal{O}} \times \mathbb{R}^r \to \mathcal{L}(\mathbb{R}^r)$ is continuous. Moreover the mapping $g(\xi, \cdot) : \mathbb{R}^r \to \mathcal{L}(\mathbb{R}^r)$ is Lipschitz-continuous, uniformly with respect to $\xi \in \overline{\mathcal{O}}$, that is*

$$\sup_{\xi \in \overline{\mathcal{O}}} \sup_{\substack{\sigma, \rho \in \mathbb{R}^r \\ \sigma \neq \rho}} \frac{\|g(\xi, \sigma) - g(\xi, \rho)\|_{\mathcal{L}(\mathbb{R}^r)}}{|\sigma - \rho|} < \infty.$$

In what follows for any $x, y : \overline{\mathcal{O}} \to \mathbb{R}^r$ we set $(G(x)y)(\xi) := g(\xi, x(\xi))y(\xi),\ \xi \in \overline{\mathcal{O}}$.
Next, setting $f := (f_1, \dots, f_r)$, for any $x : \overline{\mathcal{O}} \to \mathbb{R}^r$ we define $F(x)(\xi) := f(x(\xi)),\ \xi \in \overline{\mathcal{O}}$.

Hypothesis 2.4. (i) *The mapping $F : E \to E$ is locally Lipschitz-continuous and there exists $m \geqslant 1$ such that*

$$\left|F(x)\right|_E \leqslant c\left(1 + |x|_E^m\right), \quad x \in E. \tag{6}$$

Moreover, $F(0) = 0$.
(ii) *For any $x, h \in E$*

$$\left\langle F(x+h) - F(x), \delta_h \right\rangle_E \leqslant 0,$$

for some $\delta_h \in \partial |h|_E := \{h^\star \in E^\star\colon\ |h^\star|_{E^\star} = 1,\ \langle h, h^\star\rangle_E = |h|_E\}$.
(iii) *There exist $a > 0$ and $c \geqslant 0$ such that for each $x, h \in E$*

$$\left\langle F(x+h) - F(x), \delta_h \right\rangle_E \leqslant -a|h|_E^m + c\left(1 + |x|_E^m\right), \tag{7}$$

for some $\delta_h \in \partial |h|_E$.

The next conditions assure the compactness of level sets for the quasi-potential.

Hypothesis 2.5. *Either $G(0) = 0$ or there exists a continuous increasing function $c(t)$ such that for any $t \geqslant 0$*

$$\left|Q\left[G(0)\right]^\star e^{t[A+F'(0)]^\star} h\right|_H \geqslant c(t)\left|Q e^{tA} h\right|_H, \quad h \in H. \tag{8}$$

In the case (8) *is verified, the following conditions hold.*

(i) *If* $\{-\alpha_n\}$ *and* $\{\lambda_n\}$ *are respectively the eigenvalues of* A *and* Q, *then*

$$\frac{1}{c}\alpha_n^{-\delta} \leqslant \lambda_n \leqslant c\alpha_n^{-\delta}, \tag{9}$$

for some $c > 0$ *and some* δ *such that*

$$\delta \geqslant 0, \quad \text{if } d = 1, \qquad \delta > \frac{d-2}{4}, \quad \text{if } d \geqslant 2. \tag{10}$$

(ii) *The mappings* f *and* g *are of class* C^∞ *on* $\overline{\mathcal{O}} \times \mathbb{R}^r$.

(iii) *If* δ *is the constant in* (9), *then for any* $\gamma \leqslant \delta$ *and* $u, v \in H^{2\gamma,2}(\mathcal{O};\mathbb{R}^r)$ *we have*

$$\mathcal{B}_\gamma u_{|\partial O} = \mathcal{B}_\gamma v_{|\partial O} = 0 \quad\Longrightarrow\quad \mathcal{B}_\gamma F(u)_{|\partial O} = \mathcal{B}_\gamma \big(G(u)v\big)_{|\partial O} = 0, \tag{11}$$

where the boundary conditions $\mathcal{B}_\gamma$ *are defined by*

$$\mathcal{B}_{i,\gamma} := \big\{\mathcal{B}_i, \mathcal{B}_i\mathcal{A}_i, \ldots, \mathcal{B}_i\mathcal{A}_i^k\big\}, \quad \text{if } \gamma \in (k+m_i, k+1+m_i],\ k \in \mathbb{N}\cup\{0\},$$

and $\mathcal{B}_{i,\gamma} := \emptyset$, *if* $\gamma \in [0, m_i]$, *with* $m_i := \operatorname{ord}\mathcal{B}_i$. *Moreover, if* $u, v, w \in H^{2\delta,2}(\mathcal{O};\mathbb{R}^r)$ *we have*

$$\mathcal{B}_\delta u_{|\partial O} = \mathcal{B}_\delta v_{|\partial O} = \mathcal{B}_\delta w_{|\partial O} = 0 \quad\Longrightarrow\quad \mathcal{B}_\delta\big(F'(u)v\big)_{|\partial O} = \mathcal{B}_\delta\big(\big[G'(u)v\big]w\big)_{|\partial O} = 0. \tag{12}$$

3. The skeleton equation

With the notations introduced in the previous section, system (1) can be written more concisely as

$$\mathrm{d}u(t) = \big[Au(t) + F\big(u(t)\big)\big]\,\mathrm{d}t + G\big(u(t)\big)Q\,\mathrm{d}w(t), \quad u(0) = x. \tag{13}$$

Now, for any $-\infty \leqslant t_1 < t_2 \leqslant +\infty$, $x \in E$ and $\varphi \in L^2(t_1, t_2; H)$ we denote by $z_{t_1}^x(\varphi)$ any solution belonging to $C([t_1, t_2]; E)$ of the deterministic problem

$$z'(t) = Az(t) + F\big(z(t)\big) + G\big(z(t)\big)Q\varphi(t), \quad z(t_1) = x. \tag{14}$$

Theorem 3.1. *Under Hypotheses* 2.1–2.4, *for any* $r \geqslant 0$ *there exists a constant* $c_r > 0$ *such that for any* $T \in \mathbb{R}$ *and* $x \in E$

$$\sup_{|\varphi|_{L^2(T,\infty;H)} \leqslant r} \big|z_T^x(\varphi)\big|_{C([T,\infty);E)} \leqslant c_r\big(1 + |x|_E\big). \tag{15}$$

Moreover, there exists $\theta_\star \in (0,1)$ *and* $c_r \in (0, +\infty)$ *such that for any* $t > T$ *and* $x \in E$

$$\sup_{|\varphi|_{L^2(T,\infty;H)} \leqslant r} \big|z_T^x(\varphi)(t)\big|_{C^{\theta_\star}(\overline{\mathcal{O}};\mathbb{R}^r)} \leqslant c_r\big(1 + |x|_E^m\big)\big(1 + (t-T)^{-\theta_\star/2}\big). \tag{16}$$

Finally, if we take $x = 0$ *we have* $\lim_{|\varphi|_{L^2(T,\infty;H)} \to 0} |z_T^0(\varphi)|_{C([T,\infty);E)} = 0$.

The next theorem shows that under some stronger conditions on F, Q and G it is possible to give estimates of $|z^x(\varphi)(t)|_E$ which are uniform with respect to the initial datum $x \in E$.

Theorem 3.2. *Assume that there exists* $\gamma \in [0,1]$ *such that* $\sup_{\xi \in \overline{\mathcal{O}}} |g(\xi,\sigma)|_{\mathcal{L}(\mathbb{R}^r)} \leqslant c(1 + |\sigma|^\gamma)$, $\sigma \in \mathbb{R}^r$, *and* $m > 1 + (2+d)\gamma[1 - d(\varrho - 2)/2\varrho]^{-1}$, *where* ϱ *and* m *are the constants introduced respectively in* (5) *and* (7). *Then, under Hypotheses* 2.1–2.4, *for any* $r \geqslant 0$ *there exists* $c_r > 0$ *such that*

$$\sup_{x \in E} \sup_{|\varphi|_{L^2(0,\infty;H)} \leqslant r} \big|z^x(\varphi)(t)\big|_E \leqslant c_r\big(1 + (t \wedge 1)^{-1/(m-1)}\big), \quad t > 0. \tag{17}$$

4. Compactness of the level sets of the quasi-potential

For any $t > 0$ and $z \in C([0,t];E)$ we define $I_t(z) := \frac{1}{2}\inf\{|\varphi|^2_{L^2(0,t;H)};\ z = z(\varphi)\}$, where $z(\varphi)$ is the solution of the skeleton equation (14) in the interval $[0,t]$, corresponding to the control φ. Analogously, for any $z \in C((-\infty,0];E)$ we define $I_{-\infty}(z)$.

Next, for any $x \in E$ we define the *quasi-potential*

$$V(x) := \inf\big\{I_t(z);\ t > 0,\ z \in C\big([0,t];E\big),\ \text{with } z(0) = 0 \text{ and } z(t) = x\big\}.$$

First of all notice that the functional V has a unique minimum at $x = 0$; namely $V(x) = 0$ iff $x = 0$.

Theorem 4.1. *Under Hypotheses* 2.1–2.5*, for any $r \geqslant 0$ the level set $K(r) := \{x \in E\colon\ V(x) \leqslant r\}$ is compact in E.*

The two key results which allows us to prove Theorem 4.1 are stated in the following proposition.

Proposition 4.2. *Assume Hypotheses* 2.1–2.5*. Then*

(i) *for any $r \geqslant 0$ the set $K_{-\infty}(r)$ is compact in $C((-\infty,0];E)$;*
(ii) *if condition* (8) *holds, for any $x \in E$*

$$V(x) = \min\Big\{I_{-\infty}(z);\ z \in C\big((-\infty,0];E\big),\ z(0) = x,\ \lim_{t\to-\infty}\big|z(t)\big|_E = 0\Big\}. \tag{18}$$

The proof of (18) is quite delicate, as we are dealing with a colored noise, when $d > 1$, and the multiplication term G can vanish. Thus we have to use some arguments of locally exact controllability. Namely we have to prove that there exists $T_0 > 0$ such that system (14) is locally exactly controllable, with state space $V := D((-A)^{\delta+1/2})$ and control space $U := L^2(0,T;H)$, for any $T \leqslant T_0$. To this purpose, the crucial step is showing that the solution of Eq. (14) verifies some further regularity property in $D((-A)^{\delta+1/2})$.

5. Lower and upper bounds

Theorem 5.1. *For any $\delta, \gamma > 0$ and $\bar{x} \in E$ there exists $\varepsilon_0 > 0$ such that*

$$\nu_\varepsilon\big(\big\{x \in E\colon\ |x - \bar{x}|_E < \delta\big\}\big) \geqslant \exp\left(-\frac{V(\bar{x}) + \gamma}{\varepsilon^2}\right), \qquad \varepsilon \leqslant \varepsilon_0.$$

Unlike in [7], here the skeleton equation (14) is not null controllable, and then the proof of Theorem 5.1 requires this crucial lemma.

Lemma 5.2. *For any $\bar{x} \in E$, with $V(\bar{x}) < \infty$, and for any $\delta, \gamma, R > 0$ there exist $T_0 > 0$ and $\varphi_0 \in L^2(0,T_0;H)$ such that*

$$\frac{1}{2}|\varphi_0|^2_{L^2(0,T_0;H)} \leqslant V(\bar{x}) + \frac{\gamma}{2}, \qquad \sup_{|x|_E \leqslant R}\big|z_0^x(\varphi_0)(T_0) - \bar{x}\big|_E \leqslant \frac{\delta}{2}.$$

Concerning the upper bounds we have to distinguish the case of bounded and the case of unbounded G. Actually, whereas for bounded G it is possible to use exponential tail estimates for the solution of system (1) proved in [4], for unbounded G this is no more possible and then uniform estimates (17) are required.

Theorem 5.3. *Assume that Hypotheses* 2.1–2.5 *hold. Moreover, assume that*

(i) *either $g : \overline{\mathcal{O}} \times \mathbb{R}^r \to \mathbb{R}^r$ is bounded,*

(ii) *or*

$$\sup_{\xi\in\overline{\mathcal{O}}}\big|g(\xi,\sigma)\big|_{\mathcal{L}(\mathbb{R}^r)}\leqslant c\big(1+|\sigma|^\gamma\big),\quad \sigma\in\mathbb{R}^r, \tag{19}$$

where $m>1+(2+d)\gamma[1-d(\varrho-2)/2\varrho]^{-1}$, *and* ϱ *and* m *are the constants introduced respectively in* (5) *and* (7).

Then for any $s,\delta,\gamma>0$ *there exists* $\varepsilon_0>0$ *such that*

$$\nu_\varepsilon\big(\big\{x\in E;\ \mathrm{dist}_E\big(x,K(s)\big)\geqslant\delta\big\}\big)<\exp\left(-\frac{s-\gamma}{\varepsilon^2}\right),\quad \varepsilon\leqslant\varepsilon_0.$$

We first prove the following preliminary result.

Lemma 5.4. *Under Hypotheses* 2.1–2.5, *for any* $\delta,s>0$ *there exist* $\lambda>0$ *and* $\overline{T}>0$ *such that*

$$\big\{z(t);\ z\in K_{\Sigma_\lambda,t}(s)\big\}\subseteq\left\{x\in E;\ \mathrm{dist}_E\big(x,K(s)\big)<\frac{\delta}{2}\right\},\quad t\geqslant\overline{T},$$

where $\Sigma_\lambda:=\{x\in E;\ |x|_E\leqslant\lambda\}$.

Once we have proved this lemma, we have

Lemma 5.5. *Assume Hypotheses* 2.1–2.4. *Then*

(i) *if* G *is bounded, for any* $\rho,s,\delta>0$ *there exists* $\bar n\in\mathbb{N}$ *such that*

$$\beta_{\bar n}:=\inf\big\{I_{\bar n}(z);\ z\in C\big([0,\bar n];E\big);\ \big|z(0)\big|_E\leqslant\rho,\ \big|z(j)\big|_E\geqslant\lambda,\ j=1,\dots,\bar n\big\}>s,$$

where λ *is the constant introduced in Lemma* 5.4 *corresponding to* s *and* δ;

(ii) *if* G *fulfills* (19), *then for any* $s,\delta>0$ *there exists* $\bar n\in\mathbb{N}$ *such that*

$$\beta_{\bar n}:=\inf\big\{I_{\bar n}(z);\ z\in C\big([0,\bar n];E\big);\ \big|z(j)\big|_E\geqslant\lambda,\ j=1,\dots,\bar n\big\}>s.$$

The lemma above allows us to conclude the proof of Theorem 5.3 not too differently from [7] (see also [6], [5] and [1] for background references).

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