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Existence, uniqueness and stability of backward stochastic differential equations with locally monotone coefficient

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Abstract We prove existence, uniqueness and stability of the solution for multidimensional backward stochastic differential equations (BSDE) with locally monotone coefficient. This is done with an almost quadratic growth coefficient and a square integrable terminal data. The coefficient could be neither locally Lipschitz in the variable y nor in the variable z. ***To cite this article: K. Bahlali et al., C. R. Acad. Sci. Paris, Ser. I 335 (2002) 757–762.***

Existence, unicité et stabilité des équations différentielles stochastiques rétrogrades à coefficient localement monotone

Résumé Nous prouvons l'existence, l'unicité et la stabilité des solutions d'équations différentielles stochastiques rétrogrades (EDSR), dont le coefficient vérifie une condition de type monotonie locale. Ces resultats sont obtenus avec un coefficient de croissance presque quadratique et une donnée terminale de carré intégrable. De plus le coefficient peut être ni localement Lipschitz en y ni en z. ***Pour citer cet article : K. Bahlali et al., C. R. Acad. Sci. Paris, Ser. I 335 (2002) 757–762.***

Version française abrégée

Soit $(W_t)_{0\leqslant t\leqslant T}$ un mouvement brownien r-dimensionnel, défini sur l'espace probabilisé $(\Omega, \mathcal{F}, P)$. On note $(\mathcal{F}_t)_{0\leqslant t\leqslant T}$ la filtration naturelle de (W_t), augmentée des ensembles P-négligeables. Soit ξ un vecteur aléatoire de $\mathbb{R}^d$ de carré intégrable et $\mathcal{F}_T$-mesurable. Soit f une fonction définie sur $\mathbb{R}_+ \times \Omega \times \mathbb{R}^d \times \mathbb{R}^{d\times r}$, à valeurs dans $\mathbb{R}^d$ et telle que pour tout $(y,z) \in \mathbb{R}^d \times \mathbb{R}^{d\times r}$, l'application $(t,\omega) \to f(t,\omega,y,z)$ est $\mathcal{F}_t$-progressivement mesurable. On considère l'EDSR suivante,

$$Y_t = \xi + \int_t^T f(s, Y_s, Z_s)\,\mathrm{d}s - \int_t^T Z_s\,\mathrm{d}W_s \quad (0 \leqslant t \leqslant T) \qquad (E^f)$$

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Compte tenu des applications des EDSR en mathématiques financières [13] et aux EDP non linéaires [20, 22], les problèmes d'existence et d'unicité des solutions d'EDSR sous des hypothèses assez générales ont été largement étudiées, voir par exemple [1–10,14–20,23,24] ainsi que les réferences qu'ils contiennent. Voir [11,12] pour une approche des EDSR en dimension infinie.

Cependant, en dehors de la dimension 1, du cas markovien [10] et de [1–3] toutes les études (d'existence et d'unicité des solutions) ont été faites sous des hypothèses globales sur le coefficient (quoique non nécessairement de type Lipschitz). Signalons que certaines methodes utilisées en dimension 1, basées sur des techniques de comparaison [9,14–16,18,19], ne marchent pas en dimension supérieure. En outre les techniques de localisation par des temps d'arrêt, utilisées dans les EDS progressives, ne fonctionnent pas pour les EDS rétrogrades. Récemment dans [1,2], l'existence, l'unicité ainsi que la stabilité des solutions ont été établies pour des EDSR multidimensionnlles avec simultannément des hypotèses locales sur le coefficient et une condition terminale uniquement de carré integrable. Cependant, dans [1,2] la croissance du coefficient est sous-linéaire.

Dans la présente Note, nous étendons les résultats de [1,2] dans plusieurs directions : d'une part, le coefficient est de croissance presque quadratique en sa variable y et sur-linéaire en sa variable z, et d'autre part, il vérifie une condition de type monotonie locale en la variable y. En outre, la condition verifiée par rapport à la variable z est plus faible que lipschitz locale. Les résultats principaux sont,

THÉROÈME 1 (Existence et unicité des solutions). – *Soit ξ une variable aléatoire à valeurs dans $\mathbb{R}^d$ de carré intégrable. Soit f un coefficient vérifiant les hypothèses* (H1)–(H4). *Alors l'équation* (E^f) *possède une solution unique.*

THÉROÈME 2 (Stabilité des solutions). – *Soient f et ξ verifiant les hypothèses du Théorème* 1. *On suppose de plus que les conditions* (H5)–(H8) *sont satisfaites. Alors, pour tout $q < 2$ on a,*

$$\lim_{n\to+\infty} E\left(\sup_{0\leqslant t\leqslant T}\left|Y_t^n - Y_t\right|^q + \int_0^T \left|Z_s^n - Z_s\right|^q \mathrm{d}s\right) = 0.$$

L'idée consiste à combiner et développer les methodes utilisées précédemment dans [1–3,11,20]. Nous approximons f, suivant une famille appropriée de semi-normes, par une suite $(f_n)_{n>1}$ de fonctions lipchitziennes et nous utilisons, ensuite, une localisation pour identifier la limite comme solution de l'équation (E^f). Cependant, au lieu de travailler dans L^2-*fort* comme dans [1,2], nous montrons d'abord que la suite de solutions (Y^{f_n}, Z^{f_n}) converge dans L^2-*faible*, comme dans [11,20]. Nous identifions ensuite la limite comme solution de l'équation (E^f) en utilisant une localisation qui semble mieux adaptée aux EDSR que la localisation classique par des temps d'arrêt. L'identification de la limite, se fait par une technique différente de celles de [1–3,11,20]. En effet, on montre que les limites L^2-*faible* (Y, Z), sont au fait des limites *fortes* dans L^1. Ce qui est suffisant pour prouver que le terme à variation finie limite est de la forme $\int_t^T f(s, Y_s, Z_s)\,\mathrm{d}s$. Ceci est obtenu en appliquant la formule d'Itô à $(|Y^{f_n} - Y^{f_m}|^2 + \varepsilon)^\beta$ pour $0 < \beta < 1$ et non à $|Y^{f_n} - Y^{f_m}|^2$ comme cela se fait d'habitude. Ensuite, par passage à la limite successivement sur n, m et ε, nous prouvons l'existence des solutions pour un temps petit. Par continuation nous étendons le résultat à un temps fini quelconque. Ceci nous permet de traiter le cas où le coefficient est à croissance sur-linéaire en les deux variables y et z. L'unicité et la stabilité des solutions sont établies par des arguments similaires.

1. Introduction

Let $(W_t)_{0\leqslant t\leqslant T}$ be a r-dimensional Wiener process defined on a complete probability space $(\Omega, \mathcal{F}, P)$. Let $(\mathcal{F}_t)_{0\leqslant t\leqslant T}$ denote the natural filtration of (W_t) such that $\mathcal{F}_0$ contains all P-null sets of $\mathcal{F}$, and ξ be an $\mathcal{F}_T$-measurable d-dimensional square integrable random variable. Let f be an $\mathbb{R}^d$-valued process defined on $\mathbb{R}_+ \times \Omega \times \mathbb{R}^d \times \mathbb{R}^{d\times r}$ such that for all $(y,z) \in \mathbb{R}^d \times \mathbb{R}^{d\times r}$, the map $(t,\omega) \to f(t,\omega,y,z)$ is $\mathcal{F}_t$-progressively measurable. We consider the following BSDE,

$$Y_t = \xi + \int_t^T f(s, Y_s, Z_s)\,\mathrm{d}s - \int_t^T Z_s\,\mathrm{d}W_s, \quad 0 \leqslant t \leqslant T. \tag{E^f}$$

When the coefficient f is uniformly Lipschitz, the BSDE (E^f) has a unique solution which can be constructed by using Itô's representation theorem and the Picard approximation procedure, see for instance [21,13].

The applicability of backward stochastic differential equations both to finance theory [13] and to nonlinear partial differential equations [22] has motivated many efforts to establish existence and uniqueness of solutions under general hypotheses. Several works have attempted to relax the Lipschitz condition and the growth of the generator, see for instance [1–10,14–20,23,24] and the references therein. See [11,12] for infinite dimensional BSDE.

In the multidimensional case, only the BSDEs with global assumptions on the coefficient are considered in the previously cited papers. The difficulty in the improvement of the global (Lipschitz or monotone) condition is essentially due to the fact that the usual localization techniques, by means of stopping time, do not apply naturally to BSDEs. Notice also that the techniques used in the one dimensional case do not work in higher dimension. Recently in [1,2], the existence and uniqueness, as well as the stability of solutions have been established for multidimensional BSDEs with simultaneously a square integrable terminal data and a local assumptions (on the two variables y, z) on the coefficient. See also [3] for more developments. However in [1,2], the growth of the coefficient is at most linear.

In this Note, we extend the result of [1,2] in many directions. First, the coefficient is 'almost' quadratic in the variable y and super-linear in z. Second the coefficient satisfies a monotonicity condition only locally. Third, the coefficient could be neither locally Lipschitz in the variable y nor in the variable z. Moreover, the terminal data is assumed to be square integrable only. The method we use here develops those used in [1,2]: we approximate f by a sequence $(f_n)_{n>1}$ of Lipschitz functions via a suitable family of semi-norms. Then we use an appropriate localization to identify the limit as a solution of Eq. (E^f). The new idea, here, mainly consists to establishing that the sequence of solutions (Y^{f_n}, Z^{f_n}) converges weakly in L^2 and strongly in L^1. This last property is obtained by applying Itô's formula to $(|Y^{f_n} - Y^{f_m}|^2 + \varepsilon)^\beta$ for some $0 < \beta < 1$ and $\varepsilon > 0$, instead of $|Y^{f_n} - Y^{f_m}|^2$ as is usually done. This allows us to treat multidimentional BSDE with a coefficient which has a super-linear growth coefficient in both variables y and z. We first prove the existence and uniqueness of a solution for a small time duration and then we use the continuation procedure to extend the result to an arbitrarily prescribed time duration. The stability of the solution is established by similar arguments.

To illustrate our result, let us consider the following example:

Example. – Let $0 < \varepsilon < 1$ and

$$f_1(t,\omega,y,z) = g(t,\omega,y)\left[|z|\sqrt{-\log|z|}\mathbf{1}_{\{|z|<\varepsilon\}} + h(z)\mathbf{1}_{\{\varepsilon\leqslant|z|<1+\varepsilon\}} + |z|\sqrt{\log|z|}\mathbf{1}_{\{|z|>1+\varepsilon\}}\right],$$

where g is a bounded function which is continuous in y such that $g(t,\omega,0) = 0$ and $\langle y - y', g(t,y) - g(t,y')\rangle \leqslant 0$. h is a Lipschitz and positive function which is choosed such that f_1 is continuous.

Let $f_2(t,\omega,y,z)$ be a continuous function in (y,z) such that:

(i) There exist $M > 0$, $\gamma < \frac{1}{2}$ and $\eta \in \mathbb{L}^1([0,T]\times\Omega)$ such that $\langle y, f_2(t,\omega,y,z)\rangle \leqslant \eta + M|y|^2 + \gamma|z|^2$.

(ii) There exist $M > 0$, $1 < \alpha < 2$ and $\overline{\eta} \in \mathbb{L}^{\alpha'}([0,T]\times\Omega)$; $|f_2(t,\omega,y,0)| \leqslant \overline{\eta} + M|y|^\alpha$.

(iii) There exists a constant $C > 0$:

$$\begin{aligned}&\left\langle y - y', f_2(t,y,z) - f_2(t,y',z')\right\rangle \\ &\quad\leqslant C|y-y'|^2\left[1 + \left|\log|y-y'|\right|\right] + C|y-y'||z-z'|\left[1 + \sqrt{\left|\log|z-z'|\right|}\right].\end{aligned}$$

Our work shows that Eq. $(E^{f_1+f_2})$ has a unique solution. It should be noted that to our knowledge this example is not covered by any previous result.

The paper is organized as follows. In Section 2, we state the assumptions and define some notations. Section 3 is devoted to the statement of our results, as well as to several lemmas which are the main steps of the proofs.

2. Definitions, assumptions and notations

We denote by $\mathcal{E}$ the set of $\mathbb{R}^d \times \mathbb{R}^{d\times r}$-valued processes (Y, Z) defined on $[0, T] \times \Omega$ which are $\mathcal{F}_t$-adapted and such that: $\|(Y, Z)\|^2 = E(\sup_{0\leqslant t\leqslant T} |Y_t|^2 + \int_0^T |Z_s|^2\,\mathrm{d}s) < +\infty$. The couple $(\mathcal{E}, \|\cdot\|)$ is then a Banach space.

DEFINITION 1. – A solution of Eq. (E^f) is a couple (Y, Z) which belongs to the space $(\mathcal{E}, \|\cdot\|)$ and satisfies (E^f).

We consider the following assumptions:

(H1) f is continuous in (y, z) for almost all (t, ω),

(H2) There exist $M > 0$, $\gamma < \frac{1}{2}$ and $\eta \in \mathbb{L}^1([0, T] \times \Omega)$ such that,

$$\bigl\langle y, f(t, \omega, y, z)\bigr\rangle \leqslant \eta + M|y|^2 + \gamma|z|^2 \quad P\text{-a.s., a.e. } t \in [0, T].$$

(H3) There exist $M_1 > 0$, $0 \leqslant \alpha < 2$, $\alpha' > 1$ and $\overline{\eta} \in \mathbb{L}^{\alpha'}([0, T] \times \Omega)$ such that:

$$\bigl|f(t, \omega, y, z)\bigr| \leqslant \overline{\eta} + M_1\bigl(|y|^\alpha + |z|^\alpha\bigr).$$

(H4) There exists a real valued sequence $(A_N)_{N>1}$ and constants $M > 1$, $r > 1$ such that:

  (i) $\forall N > 1$, $1 < A_N \leqslant N^r$.

  (ii) $\lim_{N\to\infty} A_N = \infty$.

  (iii) For every $N \in \mathbb{N}$, $\forall y, y'z, z'$ such that $|y|, |y'|, |z|, |z'| \leqslant N$, we have

$$\begin{aligned}&\bigl\langle y - y', f(t, y, z) - f(t, y', z')\bigr\rangle \\ &\quad\leqslant M|y - y'|^2 \log A_N + M|y - y'||z - z'|\sqrt{\log A_N} + M A_N^{-1}.\end{aligned}$$

For a given f, the solutions of Eq. (E^f) will be denoted by (Y^f, Z^f). When the assumption (H3) are satisfied, we can define a family of semi-norms $(\rho_n(f))_{n\in N}$ by,

$$\rho_n(f) = E\int_0^T \sup_{|y|,\ |z|\leqslant n} \bigl|f(s, y, z)\bigr|\,\mathrm{d}s.$$

3. The main results

THEOREM 2. – *Let ξ be a square integrable random variable which is $\mathcal{F}_1$-measurable. Assume that* (H1)–(H4) *are satisfied. Then Eq. (E^f) has a unique solution.*

In the following, we give a stability result for the solution with respect to the data (f, ξ). Roughly speaking, if f_n converges to f in the metric defined by the family of semi-norms (ρ_N) and ξ_n converges to ξ in $\mathrm{L}^2(\Omega)$ then (Y^n, Z^n) converges to (Y, Z) in $\mathrm{L}^q(\Omega \times [0, T])$ for all $q < 2$. Let (f_n) be a sequence of processes which are $\mathcal{F}_t$-progressively measurable for each n. Let (ξ_n) be a sequence of random variables which are $\mathcal{F}_T$-measurable for each n and such that $E(|\xi_n|^2) < \infty$. We will assume that for each n, the BSDE (E^{f_n,ξ_n}) corresponding to the data (f_n, ξ_n) has a (not necessarily unique) solution. Each solution of Eq. (E^{f_n,ξ_n}) will be denoted by (Y^n, Z^n). We suppose also that the following assumptions (H5)–(H7) are fulfilled:

(H5) For every N, $\rho_N(f_n - f) \to 0$ as $n \to \infty$.

(H6) $E(|\xi_n - \xi|^2) \to 0$ as $n \to \infty$.

(H7) There exist $M > 0$, $\gamma < \frac{1}{2}$ and $\eta \in \mathbb{L}^1([0,T]\times\Omega)$ such that,

$$\sup_n \langle y, f_n(t,\omega,y,z)\rangle \leqslant \eta + M|y|^2 + \gamma|z|^2 \quad P\text{-a.s., a.e. } t\in[0,T].$$

(H8) There exist $M_1 > 0$, $1 < \alpha < 2$, $\alpha' > 1$ and $\overline{\eta} \in \mathbb{L}^{\alpha'}([0,T]\times\Omega)$ such that:

$$\sup_n \left| f_n(t,\omega,y,z)\right| \leqslant \overline{\eta} + M_1\left(|y|^\alpha + |z|^\alpha\right).$$

THEOREM 3. – *Let f and ξ be as in Theorem* 2. *Assume that* (H5)–(H7) *and* (H8) *are satisfied. Then, for all $q < 2$ we have*

$$\lim_{n\to+\infty}\left(E\sup_{0\leqslant t\leqslant T}\left|Y_t^n - Y_t\right|^q + E\int_0^T \left|Z_s^n - Z_s\right|^q \,\mathrm{d}s\right) = 0.$$

To prove Theorem 2, we need some lemmas. The following can be proved by truncation and regularization.

LEMMA 4. – *Let f be a function which satisfies* (H1)–(H3). *Then there exists a sequence of functions (f_n) such that*

(a) *For each n, f_n is bounded and globally Lipschitz in (y,z) t a.e. and P-a.s.*

(b) *There exists $M' > 0$, such that*

$$\sup_n \left| f_n(t,\omega,y,z)\right| \leqslant \overline{\eta} + M' + M\left(|y|^\alpha + |z|^\alpha\right) \quad P\text{-a.s., a.e. } t\in[0,T].$$

(c) *There exists $M' > 0$, such that*

$$\sup_n \langle y, f_n(t,\omega,y,z)\rangle \leqslant \eta + M' + M|y|^2 + \gamma|z|^2.$$

(d) *For every N, $\rho_N(f_n - f)\to 0$ as $n\to\infty$.*

Using standard arguments in BSDEs one can prove the following estimates.

LEMMA 5. – *Let f and ξ be as in Theorem* 1. *Let (f_n) be the sequence of functions associated to f by Lemma* 1 *and denotes by (Y^{f_n}, Z^{f_n}) the solution of Eq. (E^{f_n}). Then, there exits a universal constant l such that*

(a) $E\int_0^T \mathrm{e}^{2Ms}|Z_s^{f_n}|^2\,\mathrm{d}s \leqslant \frac{\mathrm{e}^{2MT}}{(1-2\gamma)}\left[E|\xi|^2 + 2E\int_0^T(\eta + M')\,\mathrm{d}s\right] = K_1.$

(b) $E\sup_{0\leqslant t\leqslant T}(\mathrm{e}^{2Mt}|Y_t^{f_n}|^2) \leqslant \frac{l}{(1-2\gamma)}\mathrm{e}^{2MT}\left[E|\xi|^2 + 2E\int_0^T(\eta+M')\,\mathrm{d}s\right] = K_2.$

(c) $E\int_0^T \mathrm{e}^{2Ms}|f_n(s,Y_s^{f_n},Z_s^{f_n})|^{\overline{\alpha}}\,\mathrm{d}s \leqslant 4^{(\overline{\alpha}-1)}\left[E\int_0^T \mathrm{e}^{2Ms}((\overline{\eta}+M')^{\overline{\alpha}} + 4)\,\mathrm{d}s + M_1^{\overline{\alpha}}(K_1 + TK_2)\right] = K_3.$

(d) $E\int_0^T \mathrm{e}^{2Ms}|f(s,Y_s^{f_n},Z_s^{f_n})|^{\overline{\alpha}}\,\mathrm{d}s \leqslant K_3$, *where* $\overline{\alpha} = \min(\alpha', \frac{2}{\alpha})$.

Applying Itô's formula to the quantity $[|Y_t^{f_n} - Y_t^{f_m}|^2 + (A_N\log A_N)^{-1}]^{\beta/2}$ and using a localization argument we establish

LEMMA 6. – *For every $R > 2$, $\beta\in\,]1,\min(3-\alpha,2)[$, $\delta' < (\beta-1)\min\left(\frac{1}{4M^2}, \frac{3-\alpha-\beta}{2rM^2\beta}\right)$ and $\varepsilon > 0$, there exists $N_0 > R$ such that for all $N > N_0$:*

$$\limsup_{n,m\to+\infty} E\sup_{(T'-\delta')^+\leqslant t\leqslant T'}\left|Y_t^{f_n} - Y_t^{f_m}\right|^\beta + E\int_{(T'-\delta')^+}^{T'}\frac{|Z_s^{f_n} - Z_s^{f_m}|^2}{(|Y_s^{f_n} - Y_s^{f_m}|^2 + \nu_R)^{(2-\beta)/2}}\,\mathrm{d}s$$
$$\leqslant \varepsilon + \ell\,\mathrm{e}^{C_N\delta'}\limsup_{n,m\to+\infty} E\left|Y_{T'}^{f_n} - Y_{T'}^{f_m}\right|^\beta,$$

where $\nu_R = \sup((A_N\log A_N)^{-1}, N\geqslant R)$, $C_N = \frac{2M^2\beta}{(\beta-1)}\log A_N$ and ℓ is a universal positive constant.

Taking successively $T' = T$, $T' = (T-\delta')^+$, $T' = (T-2\delta')^+,\ldots,$ in Lemma 6, we establish the existence of a solution. The uniqueness as well as the stability (Theorem 3) of the solutions can be established by similar estimates.

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