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Analyse numérique/*Numerical Analysis*

Modeling and numerical treatment of boundary data in an eddy currents problem

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Abstract The aim of this paper is to analyze a finite element method to solve the eddy currents model in a bounded conductor domain. In particular we study a weak formulation in terms of the magnetic field. In order to impose suitable boundary conditions from a physical point of view, we introduce a Lagrange multiplier defined on the boundary and study the resulting mixed formulation by using classical techniques. ***To cite this article: A. Bermúdez et al., C. R. Acad. Sci. Paris, Ser. I 335 (2002) 633–638.***

Modélisation et traitement numérique des conditions aux limites dans un problème des courants de Foucault

Résumé L'objectif de cette Note est d'analyser une méthode d'éléments finis pour la résolution numérique du modèle des courants de Foucault. Nous étudions une formulation faible en termes de champ magnétique. Pour imposer des conditions aux limites réalistes d'un point de vue physique, nous introduisons un multiplicateur de Lagrange défini sur la frontière du domaine et nous étudions la formulation mixte correspondante, en utilisant des techniques classiques. ***Pour citer cet article : A. Bermúdez et al., C. R. Acad. Sci. Paris, Ser. I 335 (2002) 633–638.***

Version française abrégée

Les courants de Foucault sont généralement modélisés par les équations de Maxwell harmoniques en basse fréquence. Nous sommes nous intéressés à la résolution de ce problème dans un domaine conducteur borné Ω, traversé par un courant électrique alternatif de fréquence angulaire ω. Dans ce cas, le modèle se réduit à une seule équation (1) pour le champ complexe $\mathbf{H}$ associé au champ magnétique (voir [4] pour les détails), où μ est la perméabilité magnétique et σ est la conductivité électrique.

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Soit $\partial\Omega$ la frontière du domaine Ω, qui se décompose en deux morceaux Γ_{E} et Γ_{C}. Γ_{E} est la pointe de l'électrode où se produit l'arc électrique. Le reste de la frontière de l'électrode se décompose comme suit : $\Gamma_{\mathrm{C}} = \Gamma_{\mathrm{C}}^0 \cup \Gamma_{\mathrm{C}}^1 \cup \cdots \cup \Gamma_{\mathrm{C}}^{N_c}$, où Γ_{C}^n, $n = 1, \ldots, N_c$, sont les parties de la frontière connectées aux câbles qui apportent le courant électrique à l'électrode et Γ_{C}^0 la partie qui reste.

Les intensités qui traversent chaque câble, I_n, $n = 1, \ldots, N_c$, sont des données connues, tandis qu'aucun courant ne traverse Γ_{C}^0. Par conséquent, nous sommes amenés à imposer les conditions aux limites (2)–(6) ci-après.

Soient $\boldsymbol{\mathcal{X}} := \mathrm{H}(\mathbf{curl}, \Omega)$ et $a : \boldsymbol{\mathcal{X}} \times \boldsymbol{\mathcal{X}} \to \mathbb{C}$ la forme bilinéaire continue et elliptique définie par (7) ci-après. Soit $\mathcal{L} := \{\nu \in \mathrm{H}_{00}^{1/2}(\Gamma_{\mathrm{C}}) : \nu|_{\Gamma_{\mathrm{C}}^n} = \text{constante}, n = 1, \ldots, N_c\}$. Soit $\widetilde{I} \in \mathrm{L}^2(\Gamma_{\mathrm{C}})$ défini par (8), ci-après. Le problème mixte associé à la formulation variationnelle du problème (1)–(6) est le suivant :

PROBLÈME MP. – *Trouver* $\mathbf{H} \in \mathrm{H}(\mathbf{curl}, \Omega)$ *et* $\lambda \in \mathcal{L}$ *vérifiant*

$$a(\mathbf{H}, \overline{\mathbf{G}}) + \langle \mathbf{curl}\, \mathbf{G} \cdot \mathbf{n}, \lambda \rangle_{\Gamma_{\mathrm{C}}} = 0 \quad \forall \mathbf{G} \in \mathrm{H}(\mathbf{curl}, \Omega),$$

$$\langle \mathbf{curl}\, \mathbf{H} \cdot \mathbf{n}, \nu \rangle_{\Gamma_{\mathrm{C}}} = \int_{\Gamma_{\mathrm{C}}} \widetilde{I} \bar{\nu} \quad \forall \nu \in \mathcal{L}.$$

THÉORÈME 0.1. – *Il existe* $(\mathbf{H}, \lambda) \in \boldsymbol{\mathcal{X}} \times \mathcal{L}$ *solution unique du problème* MP.

Nous considérons une famille de maillages tétraédriques régulière $\{\mathcal{T}_h\}$ de Ω où, comme d'habitude, h représente la taille du maillage. Le champ magnétique sera discrétisé en utilisant l'espace des éléments finis d'arête de Nédélec :

$$\boldsymbol{\mathcal{X}}_h := \left\{ \mathbf{G}_h \in \boldsymbol{\mathcal{X}} :\ \mathbf{G}_h(\mathbf{x}) = \mathbf{a} \times \mathbf{x} + \mathbf{b},\ \mathbf{a}, \mathbf{b} \in \mathbb{C}^3,\ \mathbf{x} \in K,\ \forall K \in \mathcal{T}_h \right\}.$$

Par ailleurs, le multiplicateur de Lagrange λ sera approché dans l'espace de dimension finie $\mathcal{L}_h := \{\lambda_h \in \mathcal{Q}_h^1(\Gamma_{\mathrm{C}}) : \lambda_h|_{\Gamma_{\mathrm{C}}^n} = \text{constant}, n = 1, \ldots, N_c\}$, où $\mathcal{Q}_h^1(\Gamma_{\mathrm{C}})$ est l'espace des fonctions continues linéaires par morceaux défini sur le maillage triangulaire induit par $\mathcal{T}_h$ sur la surface polyédrique Γ_{C}, qui s'annulent sur $\partial\Gamma_{\mathrm{C}}$. En utilisant ces espaces, on définit le problème discret suivant :

PROBLÈME DMP. – *Trouver* $\mathbf{H}_h \in \boldsymbol{\mathcal{X}}_h$ *et* $\lambda_h \in \mathcal{L}_h$ *vérifiant*

$$a(\mathbf{H}_h, \mathbf{G}_h) + \langle \mathbf{curl}\, \mathbf{G}_h \cdot \mathbf{n}, \lambda_h \rangle_{\Gamma_{\mathrm{C}}} = 0 \quad \forall \mathbf{G}_h \in \boldsymbol{\mathcal{X}}_h,$$

$$\langle \mathbf{curl}\, \mathbf{H}_h \cdot \mathbf{n}, \nu_h \rangle_{\Gamma_{\mathrm{C}}} = \int_{\Gamma_{\mathrm{C}}} \widetilde{I} \bar{\nu}_h \quad \forall \nu_h \in \mathcal{L}_h.$$

THEOREM 0.1. – *Le problème* DMP *admet une solution unique* $(\mathbf{H}_h, \lambda_h)$. *En plus, si la solution* $(\mathbf{H}, \lambda)$ *du problème* MP *vérifie* $\mathbf{H} \in \mathrm{H}^r(\mathbf{curl}, \Omega)$ *avec* $r > 1/2$, *alors on a*

$$\|\mathbf{H} - \mathbf{H}_h\|_{\mathrm{H}(\mathbf{curl}, \Omega)} \leqslant C \left(h^r \|\mathbf{H}\|_{\mathrm{H}^r(\mathbf{curl}, \Omega)} + \inf_{\nu_h \in \mathcal{L}_h} \|\lambda - \nu_h\|_{\mathcal{L}} \right).$$

1. Introduction

Numerical solution of the eddy currents model became an important research area in recent years because of many applications in electrical engineering (see, for instance, [5] and references therein). In particular, the present work is motivated by the need of a three-dimensional numerical simulation of metallurgical electrodes in an electric arc furnace (see, for instance, [3,7] for related works concerning axysimmetric models). A finite element method to solve the eddy currents problem in a bounded 3D-domain which contains not only the electrodes but the whole furnace is studied in [4]. However, the model presented in

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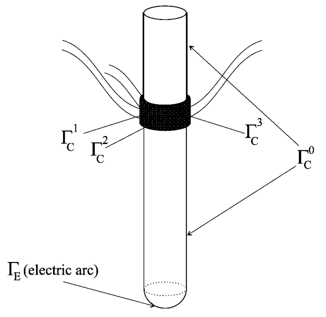

Figure 1. – Sketch of one single electrode.

Figure 1. – Dessin d'une seule électrode.

that paper is very complex and its numerical solution takes a lot of computer time. This is why it is useful to have simpler models to describe separate components of the whole system. In the present work, we study one such problem which consists in solving the eddy currents model in a domain including only one electrode (see Fig. 1).

One of the main difficulties to study the problem in a bounded domain is defining mathematically suitable and physically realistic boundary conditions. Essential and natural boundary conditions are considered in [1] and [4]. However, these boundary conditions are not directly related to the physical data in the case of an electrode. In the present paper, in order to deal with this question, we propose and analyze a finite element method to solve the eddy currents model in a bounded conductor domain by using the input current intensity as unique boundary data. We consider a formulation in terms of the magnetic field and impose the boundary conditions by means of Lagrange multipliers.

2. The eddy currents problem

Eddy currents are usually modeled by the low-frequency harmonic Maxwell equations. We are interested in solving the problem in a bounded conductor domain Ω crossed by an alternating electric current of angular frequency ω. In this case, the model reduces to one single equation for the complex field $\mathbf{H}$ associated with the magnetic field (see [4] for details):

$$\mathrm{i}\omega\mu\mathbf{H} + \mathbf{curl}\left(\frac{1}{\sigma}\,\mathbf{curl}\,\mathbf{H}\right) = \mathbf{0}, \tag{1}$$

where μ is the magnetic permeability and σ is the electric conductivity.

Let $\partial\Omega$ be the boundary of the domain Ω which splits into two pieces Γ_{E} and Γ_{C}. Γ_{E} is the tip of the electrode where the electric arc arises. In its turn, the rest of the electrode boundary splits as follows, $\Gamma_{\mathrm{C}} = \Gamma_{\mathrm{C}}^0 \cup \Gamma_{\mathrm{C}}^1 \cup \cdots \cup \Gamma_{\mathrm{C}}^{N_c}$, where Γ_{C}^n, $n = 1, \ldots, N_c$, are the parts of the boundary connected to the wires supplying electric current to the electrode, and Γ_{C}^0 the remaining (see Fig. 1). We suppose that Γ_{E} is nonempty and connected, and that $\Gamma_{\mathrm{C}}^n \cap \Gamma_{\mathrm{E}} = \emptyset$, $n = 1, \ldots, N_c$.

The input current intensities through each wire, I_n, $n = 1, \ldots, N_c$, are known data, whereas no current goes through Γ_{C}^0. Hence we are led to impose the following boundary conditions:

$$\int_{\Gamma_{\mathrm{C}}^n} \mathbf{curl}\,\mathbf{H}\cdot\mathbf{n} = I_n \quad \text{on } \Gamma_{\mathrm{C}}^n,\ n = 1, \ldots, N_c, \tag{2}$$

$$\mathbf{curl}\,\mathbf{H}\times\mathbf{n} = \mathbf{0} \quad \text{on } \Gamma_{\mathrm{C}}^n,\ n = 1, \ldots, N_c, \tag{3}$$

$$\mathbf{curl}\,\mathbf{H}\cdot\mathbf{n} = 0 \quad \text{on } \Gamma_{\mathrm{C}}^0, \tag{4}$$

$$\mu\mathbf{H}\cdot\mathbf{n} = 0 \quad \text{on } \Gamma_{\mathrm{C}}. \tag{5}$$

On the other hand, the natural boundary condition corresponding to free current exit on the electrode tip yields

$$\mathbf{curl}\,\mathbf{H} \times \mathbf{n} = \mathbf{0} \quad \text{on } \Gamma_{\mathrm{E}}. \tag{6}$$

It can be shown that conditions (3) and (5) turn out to be natural boundary conditions of the weak formulation of the problem, when (2) and (4) are imposed by Lagrange multiplier techniques.

3. Analysis of the weak formulation of the problem

Let us assume that Ω has a Lipschitz-continuous connected boundary. We use standard notation for Sobolev spaces and norms. We also use the space

$$\mathrm{H}^r(\mathbf{curl}, \Omega) := \left\{ \mathbf{G} \in \mathrm{H}^r(\Omega)^3 : \mathbf{curl}\,\mathbf{G} \in \mathrm{H}^r(\Omega)^3 \right\},$$

for each positive real number r. This is a Hilbert space endowed with the norm

$$\|\mathbf{G}\|_{\mathrm{H}^r(\mathbf{curl},\Omega)} := \left[\|\mathbf{G}\|^2_{\mathrm{H}^r(\Omega)^3} + \|\,\mathbf{curl}\,\mathbf{G}\|^2_{\mathrm{H}^r(\Omega)^3} \right]^{1/2}.$$

Let $\mathrm{H}_{00}^{-1/2}(\Gamma_{\mathrm{C}})$ be the dual space of $\mathrm{H}_{00}^{1/2}(\Gamma_{\mathrm{C}})$ which, in its turn, is the space of functions defined on Γ_{C} that extended by 0 to $\partial\Omega \setminus \Gamma_{\mathrm{C}}$ belong to $\mathrm{H}^{1/2}(\partial\Omega)$. We denote by $\langle \cdot, \cdot \rangle_{\Gamma_{\mathrm{C}}}$ the duality pairing between $\mathrm{H}_{00}^{-1/2}(\Gamma_{\mathrm{C}})$ and $\mathrm{H}_{00}^{1/2}(\Gamma_{\mathrm{C}})$.

We assume that $\mu, \sigma \in \mathrm{L}^{\infty}(\Omega)$ and that there exist constants $\underline{\mu}$ and $\underline{\sigma}$ such that

$$\mu(\mathbf{x}) \geqslant \underline{\mu} > 0, \quad \sigma(\mathbf{x}) \geqslant \underline{\sigma} > 0, \quad \text{a.e. in } \Omega.$$

To obtain a weak formulation of the boundary value problem (1)–(6) in terms of the magnetic field, we notice that the boundary conditions (5), (6) imply that the tangential component of $\frac{1}{\sigma}\,\mathbf{curl}\,\mathbf{H}$ on the boundary of Ω is a gradient. In particular, we obtain by formal calculus that $\frac{1}{\sigma}\,\mathbf{curl}\,\mathbf{H} \times \mathbf{n} = \nabla\phi \times \mathbf{n}$ on $\partial\Omega$ for some scalar function ϕ with $\phi = 0$ on Γ_{E}. Then, multiplying the equation (1) by a test function $\mathbf{G}$ such that $\mathbf{curl}\,\mathbf{G} \cdot \mathbf{n} = 0$ on Γ_{C} and using a Green's formula, we obtain

$$\mathrm{i}\omega \int_{\Omega} \mu \mathbf{H} \cdot \overline{\mathbf{G}} + \int_{\Omega} \frac{1}{\sigma}\,\mathbf{curl}\,\mathbf{H} \cdot \mathbf{curl}\,\overline{\mathbf{G}} = 0.$$

Let $\boldsymbol{\mathcal{X}} := \mathrm{H}(\mathbf{curl}, \Omega)$ and $a : \boldsymbol{\mathcal{X}} \times \boldsymbol{\mathcal{X}} \to \mathbb{C}$ be the sesquilinear continuous and elliptic form defined by

$$a(\mathbf{H}, \mathbf{G}) := \mathrm{i}\omega \int_{\Omega} \mu \mathbf{H} \cdot \overline{\mathbf{G}} + \int_{\Omega} \frac{1}{\sigma}\,\mathbf{curl}\,\mathbf{H} \cdot \mathbf{curl}\,\overline{\mathbf{G}}. \tag{7}$$

Let us define

$$\widetilde{I} := \begin{cases} \dfrac{I_n}{\mathrm{meas}(\Gamma_{\mathrm{C}}^n)} & \text{on } \Gamma_{\mathrm{C}}^n,\ n = 1, \ldots, N_c, \\ 0 & \text{on } \Gamma_{\mathrm{C}}^0. \end{cases} \tag{8}$$

Consider the following closed linear manifold of $\boldsymbol{\mathcal{X}}$,

$$\boldsymbol{\mathcal{W}}(\widetilde{I}) := \left\{ \mathbf{G} \in \boldsymbol{\mathcal{X}} : \langle \mathbf{curl}\,\mathbf{G} \cdot \mathbf{n}, \nu \rangle_{\Gamma_{\mathrm{C}}} = \left\langle \widetilde{I}, \nu \right\rangle_{\Gamma_{\mathrm{C}}} \ \forall \nu \in \mathrm{H}_{00}^{1/2}(\Gamma_{\mathrm{C}}),\ \nu|_{\Gamma_{\mathrm{C}}^n} = \text{constant},\ n = 1, \ldots, N_c \right\},$$

and its associated subspace,

$$\mathcal{W}(0)=\left\{\mathbf{G}\in\mathcal{X}:\langle\mathbf{curl}\,\mathbf{G}\cdot\mathbf{n},\nu\rangle_{\Gamma_{\mathrm{C}}}=0\ \forall\nu\in\mathrm{H}^{1/2}_{00}(\Gamma_{\mathrm{C}}),\ \nu|_{\Gamma^n_{\mathrm{C}}}=\text{constant},\ n=1,\dots,N_c\right\}.$$

Now we introduce the following

PROBLEM P. – *To find* $\mathbf{H}\in\mathcal{W}(\widetilde{I})$ *satisfying*

$$a(\mathbf{H},\mathbf{G})=0\quad\forall\mathbf{G}\in\mathcal{W}(0).\tag{9}$$

THEOREM 3.1. – *Problem* P *has a unique solution.*

Proof. – The result is an immediate consequence of the facts that a is elliptic in $\mathcal{W}(0)$ and $\mathcal{W}(\widetilde{I})$ is a non-empty closed convex manifold of $\mathrm{H}(\mathbf{curl},\Omega)$. □

To avoid dealing with functions that satisfy the constraints associated with $\mathcal{W}(\widetilde{I})$ and $\mathcal{W}(0)$ we consider a mixed formulation of the problem. It consists in handling the boundary conditions (2) and (4) in a weak sense by introducing a Lagrange multiplier defined on Γ_{C}.

Let

$$\mathcal{L}:=\left\{\nu\in\mathrm{H}^{1/2}_{00}(\Gamma_{\mathrm{C}}):\nu|_{\Gamma^n_{\mathrm{C}}}=\text{constant},\ n=1,\dots,N_c\right\},$$

endowed with the norm $\|\nu\|_{\mathcal{L}}:=\inf\{\|f\|_{\mathrm{H}^1(\Omega)}:f\in\mathrm{H}^1(\Omega),\ f|_{\Gamma_{\mathrm{C}}}=\nu$ and $f|_{\Gamma_{\mathrm{E}}}=0\big\}$, which is equivalent to that of $\mathrm{H}^{1/2}_{00}(\Gamma_{\mathrm{C}})$. Let b be the sesquilinear form defined in $\mathcal{X}\times\mathcal{L}$ by

$$b(\mathbf{G},\lambda):=\langle\mathbf{curl}\,\mathbf{G}\cdot\mathbf{n},\lambda\rangle_{\Gamma_{\mathrm{C}}}.$$

The mixed problem associated with problem P is the following:

PROBLEM MP. – *To find* $\mathbf{H}\in\mathrm{H}(\mathbf{curl},\Omega)$ *and* $\lambda\in\mathcal{L}$ *satisfying*

$$a(\mathbf{H},\mathbf{G})+b(\overline{\mathbf{G}},\lambda)=0\quad\forall\mathbf{G}\in\mathcal{X},\tag{10}$$

$$b(\mathbf{H},\nu)=\int_{\Gamma_{\mathrm{C}}}\widetilde{I}\,\bar{\nu}\quad\forall\nu\in\mathcal{L}.\tag{11}$$

THEOREM 3.2. – *There exists a unique* $(\mathbf{H},\lambda)\in\mathcal{X}\times\mathcal{L}$ *solution of problem* MP.

Proof. – The proof is based on the classical Babuška–Brezzi theory. In particular we prove the inf–sup condition for the bilinear form b by using results concerning vector potentials in $\mathbb{R}^3$ (see [2]). □

4. Numerical solution

We consider a family of regular tetrahedral meshes $\{\mathcal{T}_h\}$ of Ω where, as usual, h denotes the corresponding mesh-size.

The magnetic field is a function of $\mathrm{H}(\mathbf{curl},\Omega)$ and it will be discretized by using Nédélec edge finite elements (see [8]). Then, fields in $\mathcal{X}$ will be approximated in the finite dimensional space

$$\mathcal{X}_h:=\left\{\mathbf{G}_h\in\mathcal{X}:\mathbf{G}_h|_K\in\mathcal{N}(K)\ \forall K\in\mathcal{T}_h\right\},$$

where

$$\mathcal{N}(K):=\left\{\mathbf{G}_h\in\mathcal{P}_1(K)^3:\ \mathbf{G}_h(\mathbf{x})=\mathbf{a}\times\mathbf{x}+\mathbf{b},\ \mathbf{a},\mathbf{b}\in\mathbb{C}^3,\ \mathbf{x}\in K\right\}.$$

On the other hand, the Lagrange multiplier will be approximated in the finite dimensional space

$$\mathcal{L}_h:=\left\{\lambda_h\in\mathcal{Q}^1_h(\Gamma_{\mathrm{C}}):\lambda_h|_{\Gamma^n_{\mathrm{C}}}=\text{constant},\ n=1,\dots,N_c\right\},$$

where $\mathcal{Q}_h^1(\Gamma_C)$ is the space of piecewise linear continuous functions, defined on the triangular mesh $\mathcal{T}_h^{\Gamma_C}$ induced by $\mathcal{T}_h$ on the polyhedral surface Γ_C, which vanish on $\partial\Gamma_C$; namely,

$$\mathcal{Q}_h^1(\Gamma_C) := \left\{ q_h \in \mathrm{H}_0^1(\Gamma_C) : q_h|_T \in \mathcal{P}_1(T)\ \forall T \in \mathcal{T}_h^{\Gamma_C} \right\}. \tag{12}$$

By using these spaces, we define the following discrete problem:

PROBLEM DMP. – *To find* $\mathbf{H}_h \in \boldsymbol{\mathcal{X}}_h$ *and* $\lambda_h \in \mathcal{L}_h$ *satisfying*

$$a(\mathbf{H}_h, \mathbf{G}_h) + b(\overline{\mathbf{G}}_h, \lambda_h) = 0 \quad \forall \mathbf{G}_h \in \boldsymbol{\mathcal{X}}_h,$$
$$b(\mathbf{H}_h, \nu_h) = \int_{\Gamma_C} \widetilde{I}\,\bar{\nu}_h \quad \forall \nu_h \in \mathcal{L}_h.$$

THEOREM 4.1. – *Problem* DMP *attains a unique solution* $(\mathbf{H}_h, \lambda_h)$. *Furthermore, if the solution* $(\mathbf{H}, \lambda)$ *of problem* MP *satisfies* $\mathbf{H} \in \mathrm{H}^r(\mathbf{curl}, \Omega)$ *with* $r > 1/2$, *then*

$$\|\mathbf{H} - \mathbf{H}_h\|_{\mathrm{H}(\mathbf{curl},\Omega)} \leqslant C\left(h^r \|\mathbf{H}\|_{\mathrm{H}^r(\mathbf{curl},\Omega)} + \inf_{\nu_h \in \mathcal{L}_h} \|\lambda - \nu_h\|_{\mathcal{L}} \right). \tag{13}$$

Proof. – We first prove that the problem is well posed. Then we obtain the estimate by following similar arguments to those developed in the proof of Theorem 1.1 in Chapter II of [6]. For that, we have to prove that the finite-dimensional space analogue to $\boldsymbol{\mathcal{W}}(\widetilde{I})$ is nonempty. □

Remark 1. – This theorem provides error estimates for the approximation of the magnetic field **H** and its **curl**, which in most applications are the variables of interest. To obtain error estimates for the Lagrange multiplier, a discrete inf–sup condition would be needed. However, the constant corresponding to this discrete inf–sup condition does not seem to remain bounded away from zero when h goes to 0. Actually, by means of numerical experiments, we have observed that this constant converges to zero with a linear dependence on h. Alternatively, if $\mathbf{curl}\,\mathbf{H} \cdot \mathbf{n} \in \mathrm{L}^2(\Gamma_C)$ is assumed, then the Lagrange multiplier can be chosen in $\mathrm{L}^2(\Gamma_C)$. In this case, it can be discretized by piecewise constant functions and we can prove a uniform discrete inf–sup condition. We are currently studying the corresponding continuous and discrete problems under this assumption, which will be the subject of a forthcoming paper.

We have implemented the method described above in a MATLAB program to test the performance and convergence properties of the method.

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