Claude Comte

The Fundamental Role of the Reference Frame Revisited

Séminaire de Philosophie et Mathématiques, 1994, fascicule 2
« Vers une théorie quantique non standard », , p. 35-44
<http://www.numdam.org/item?id=SPHM_1994___2_A2_0>

THE FUNDAMENTAL ROLE OF THE REFERENCE FRAME REVISITED

Claude COMTE

REHSEIS, University Paris VII, 2 Place Jussieu 75251 PARIS cedex 05 FRANCE

I - The reference frame question is of central importance in any physical theory. Indeed, it is closely related to the question of the conditions of the applicability of physical laws and principles. It seems that this problem does not get the attention it deserves. Dear reader, could you help me find a book on quantum mechanics which does more than simply keep quiet about it ?

As a matter of fact, the formulation of physical laws makes sense only inasmuch as these apply to as many laboratory situations as possible, or what amounts to the same, to the largest set of equivalent observer's points of view with respect to physical phenomena. The corresponding space-time transformations connecting different equivalent situations or observer's points of view **relative to one another** are the invariance transformations of physical theory, or in other words the fundamental symmetries of space-time.

The notion of invariance is inherent in physical theory : all physicists assume that the physical world is understandable. This requires that physical theory be true anywhere and anytime in the universe. As only **reproducible phenomena** are accessible to scientific investigation, the first task of any kind of physical theory is to select the whole set of situations in which physical phenomena can be reproduced. Only then is it possible to describe reproducible phenomena by **invariant laws.**

Giordano Bruno and Galileo remarked for the first time that the laws of mechanics (the laws of free fall) are the same on an uniformly moving ship as on solid land. Although they did not lay it down as a principle, their statement must be considered as the earliest expression of the **relativity principle.** Moreover, Giordano Bruno introduced the very useful concept of an **isolated physical system**, and pointed out the impossibility of detecting the motion of such a system (the ship, for instance) by internal experiments. This last statement is equivalent to the postulate that **physical phenomena are reproducible in any isolated physical system.**

What is meant by **reproducibility**? "Like causes always produce like effects". This well known statement is a summary of strict Laplacian determinism. But quantum processes are at variance with this doctrine, and even in classical physics, processes obeying Laplacian determinism are an exception, sensitivity to small variations of the initial conditions being a rule. In celestial mechanics, for instance, sensitivity comes into play as soon as the number of interacting bodies exceeds two. Let us introduce the term **"multideterminism" to** characterize the following kind of behaviour : once the initial state is fixed with all possible accuracy, the final state cannot be predicted

any more : this state may be one among a quite large set of possible final states, which by repetition of the experiment prove to occur with different probabilities ; conversely the initial state cannot be "retrodicted" from knowledge of the final state : it is similarly one among a set of possible initial states.

The following physical hypothesis is supported by a large accumulation of experimental facts. If an isolated system evolves from an initial state i to one amongaset F of possible final states, then any state f belonging to F may be reached starting from the same set I of possible initial states, and any initial state i' belonging to I leads to one among the states of F and no more. In other words, **prediction and "retrodiction" work between sets of states I and F. Laplacian determinism corresponds to the special case where the sets I and F are reduced to one sole element.**

From the one-to-one correspondance between sets of initial and final states, it is easy to prove the existence of **conservation laws** (2) directly, without recourse to Hamiltonian formalism or to any additional postulates. Our knowledge of multideterministic evolutions is restricted, on one hand to the conservation laws which limit the range of accessible states (spectrum), and on the other hand to the probabilities of these states. **What is meant by reproducibility in a multideterministic process?** By repeating the same experiment a number of times, starting from the same initial conditions fixed with the same accuracy, we shall verify, in our laboratory assumed to be in the optimal situation of an **isolated system**, that **we reproduce the same spectrum of states always with the same probabilities.**

The absence of reproducibility in the preceding sense will be looked upon by physicists as the mark of some not yet identified causes, the influence of which being different in each performance of the experiment. Physicists have always succeeded in restoring reproducibility, by enlarging the frontiers of the system in order to embrace the supplementary causes, and after setting them under experimental control.

The only admissible causes are **observable facts**, in Mach's sense of the term. (7). Sound physical theory dispenses with made-to-order hypotheses relying on fictitious causes with no other indication of their existence than the sole fact which they are introduced to explain ; the phenomenon which is to be explained must be brought into relationship with other observable facts.

Confronted with the task of finding the system of reference in which Galileo's principle of inertia and the other laws of mechanics were to hold, Newton (8) came to postulate the existence of an absolute space and an absolute time. "Absolute Time flows equably without regard to any thing external" and "Absolute Space, on its own nature, without regard to any thing external, remains always similar and immoveable". Newton's famous vessel experiment, where centrifugal forces drive the liquid to the walls, was intended to prove that the cause of centrifugal forces is Absolute Space and nothing else, and that these forces are not to be regarded as resulting from motion relative to other masses, such as the fixed stars, but as resulting from absolute rotation in empty space.

The logical bases of Newtonian concepts were for the first time thoroughly analyzed by Mach in his critical account of mechanics (7), which had a profound influence on Einstein. He rejected **empty** space as a cause, because it is not an observable fact. For as we have no other indication of its existence than centrifugal forces, we are supporting the hypothesis of absolute space only by the fact which it was introduced to explain. He expressed for the first time the idea that the totality of masses in the universe must be regarded as the real cause of centrifugal forces, and of inertial forces in general. He emphasized that "the only lesson to be drawn from Newton's rotating vessel experiment, is that the relative motion of water with respect to the walls does not cause perceptible centrifugal forces, the seat of these forces being in the relative motion with respect to the masses of the Earth and other heavenly bodies". What would happen if the walls of the vessel grew larger and larger, until they embraced the totality of the mass of the universe ? There would be no more centrifugal force in the vessel. For there would be no factual cause of them any more.

Only relative motions, contrary to Newton's views, can generate centrifugal forces. One is led to demand that the laws of physics involve only relative motions of bodies. Along this road, Einstein (3) came to the statement that of all imaginable reference systems, "in any kind of motion relative to one another, there is none which we may look as privileged a priori", and to a considerable extension of the principle of relativity : "The laws of physics must be of such nature that they apply to systems of reference in any kind of motion". The mathematical translation consists in the requirement that "the general laws of nature are to be expressed by equations which hold good for all systems of coordinates, that is, are covariant with respect to any substitutions whatever (generally covariant)".

In fact, the laws of physics are deduced in an arbitrary sytem of coordinates, from their well-behaved special relativistic expressions, which hold true in local Galilean frames. Einstein's conception raises two questions - is **linear** covariance a necessary condition ? - Do local inertial frames not play some privileged role at least for the task of establishing the form of physical laws ?

Now, Kretschmann (5) pointed out that the principle of general covariance lacks physical sense, because any theory can be expressed in covariant form (as is the case with Maxwell's equations in arbitrary coordinates), and this principle is therefore at most of heuristic value (4).

In order to overcome this last difficulty, it is possible to propose a modern definition of what constitutes a reference frame which does not depart in spirit from the standpoint of

Giordano Bruno and Galileo on the question. **Isolated systems are undeniably privileged, indeed, as they constitute the only ones in which physical phenomena are reproducible, and therefore the only ones in which physical laws can be established at all.** Any sound physical theory must start by taking account of them. To recognize this point does not contradict General Relativity ; it may on the other hand serve to strengthen its foundations.

A freely moving spaceship provides a modern construction of a reference frame, which replaces Bruno's and Galilei's ship. As it is driven by no external forces of an electromagnetic or nuclear nature, our isolated spaceship is said to be in inertial motion. Gravity is not interpreted as an external force, but as a deviation of its inertial motion from uniform rectilinear motion, due to the distribution of the surrounding heavinly masses. This is made possible by the remarkable property of the gravitation field of imparting the same acceleration to all bodies. So far our spaceship corresponds to what is meant by a local Galilean frame in General Relativity. As it is well known Galilean frames can only be constructed in small regions, and their construction requires stopping the rotation motion of the spaceship with respect to the heavinly masses. Provided this last condition is fulfilled, **any small region at rest inside our spaceship can itself be considered as an isolated system,** because matter put inside that region is not subject to any centrifugal force.

A local Galilean frame is ordinarily defined as a system in which Galileo's principle of inertia holds true. Although in practice it amounts to the same, the definition of the local inertial frame proposed here, which refers to isolated systems and reproducibility instead of Galileo's principle, constitutes a theoretical improvement :

1 / Indeed, we stress the notion of invariance of physical laws (or symmetries of space-time) as a consequence of the reproducibility of physical phenomena. **The principle of inertia of Galileo follows from the symmetries of space-time**, as shown by Levy-Leblond (6) :Inside the walls of our spaceship we have at our disposal an ideal laboratory : if we translate or rotate our experimental set up within the limited inner space, let it be in a state of inertial motion, or repeat the experiment at another time, nothing will change in the process and results of our experiment. **There is no experiment to contradict the idea that our system is isolated.** The spatial location and the state of inertial motion of our spaceship with respect to the masses of the universe cannot be detected by any internal experiment, as it was the case for Bruno's and Galileo's ship.

2 / By amplifying the central rôle of invariance transformations (or inertial transformations, which is the same) i.e **homogeneity and isotropy of space-time, relativity principle,** a drastic **reduction of the set of fundamental principles can be achieved.** Invariance is a severely constraining condition, which allows little freedom to the form of general physical laws.

This will be illustrated in the case of classical (in the sense of non-quantum) mechanics. A universal deductive scheme which generalizes the methodology developed in my previous work (1, 2) allows to establish very simply and in a fully analogous way the laws of statics (equilibrium theory), dynamics and kinematics. As a result, the general form of the corresponding laws is completely determined up to the value of some universal constants, which turn out to be the curvature R of space in the case of statics, and the upper limit C of the velocity of energy propagation in the case of dynamics and kinematics. The constants R and C must be measured by experiments. The general expressions of the laws correspond to Einstein's dynamics in non -Euclidean space, including those of Newton's mechanics as the limiting case $R \to \infty$, and $C \to \infty$ (9).

From the standpoint of the philosophy of science, these considerations tend to credit the following thesis : **the sole fact that some rational discourse on natural phenomena is possible strongly limits the very form of this discourse.**

For the task of discovering general physical laws, like the mass-energy equivalence for instance, it turns out to be absolutely unnecessary to have recourse to any kind of fancy model of mechanical, electromagnetic or other type, concerning the structure of matter and interactions. Conversely, Relativity theory tells us nothing about the structure of matter itself. Indeed, **Relativity amounts to nothing more than drawing all the consequences of the notion of invariance.**

3 / The key idea of a program of further investigations will be the generalization of our considerations on equilibrium theory to general dynamical processes where particles are created and annihilated at different points in space-time. We expect as much curvature parameters R as there are local independent inertial transformations : one time translation , three spatial translations, three spatial rotations, and three velocities. A variation of these ten curvature parameters when the situation of the local Galilean frame changes is in no way excluded. The sole external material cause with which such a variation can be connected, if it happens, is the relative situation of the local Galilean frame with respect to the surrounding masses in the universe. So the influence of the distant masses - in other terms of the gravitation field - may be described by a set of ten curvature parameters : this approach will provide an alternative formulation in which inertia and gravitation amalgamate into a concept of higher order in such a way that inertial motion is determined only by the distribution of the distant masses.

II - A universal deductive scheme

1) Statics

Let us consider the equilibrium of a massless lever to which perpendicular forces of values m_1 m_2 ,$-m_0$, are applied at points x_1 , x_2 , x_0 , respectively. Mach (3) pointed out that Archimedes and many thinkers after him, among whom Galileo, Stevin, and Lagrange, believed to have found a proof of the equilibrium law of the lever as a consequence of two axioms :

i) Equal forces applied to equal lever arms are in equilibrium

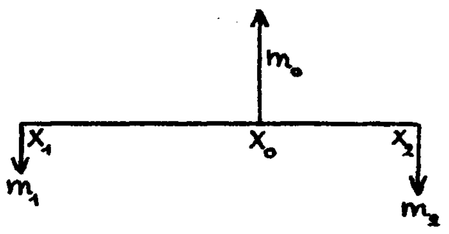
ii) if the forces are equal and the lever arms unequal, the balance inclinates to the side of greater length.

Mach showed that the implicit assumption of

$$(1) \qquad m_0 = m_1 + m_2$$

were made by all these people, from which the usual law follows, that may be written in the form of a **conservation law** for convenience:

$$(2) \qquad m_0 x_0 = m_1 x_1 + m_2 x_2$$



What would be the equilibrium law without that assumption ? Let us seek for more general equilibrium conditions in the form of

$$(1)' \qquad m_0\mathcal{E}(x_0) = m_1\mathcal{E}(x_1) + m_2\mathcal{E}(x_2)$$

$$(2)' \qquad m_0 p(x_0) = m_1 p(x_1) + m_2 p(x_2)$$

where the unknown force function $\mathcal{E}(x)$ and torque function $p(x)$ are even and odd respectively.

Let us analyse from a relativistic pont of view the simple lever problem. We postulate the existence of equilibrium laws of the precedent form, which are **invariant by translation** along the straight line of the lever : if we choose another arbitrary origin so that all coordinates x are replaced by x + y, then for any value of y the following conditions are also satisfied :

$$m_1 p(x_1+y) + m_2 p(x_2+y) = m_0 p(x_0+y)$$

$$m_1 \mathcal{E}(x_1+y) + m_2 \mathcal{E}(x_2+y) = m_0 \mathcal{E}(x_0+y)$$

This is a strong constraint on the functions $p(x)$ and $\mathcal{E}(x)$. Indeed, functional equations are readily obtained as necessary conditions, by considering a special case :

Equal arms ($x = x_0-x_1 = x_2-x_0$) and equal applied forces at the ends ($m = m_1 = m_2$). In the "reference frame" where y = 0 the torque is zero and the force law reads :

$$(3) \quad 2\, m\, \mathcal{E}(x) = m_0\mathcal{E}(0)$$

In the "reference frame" translated by length y the two equilibrium laws read :

$$(4) \quad m(\mathcal{E}(y+x) + \mathcal{E}(y-x)) = m_0 \mathcal{E}(y)$$

$$(5) \quad m(p(y+x) + p(y-x)) = m_0\, p(y)$$

The ratios of (4) by (3) and of (5) by (3) give the functional equations (we put $\mathcal{E}(0) = 1$ for convenience) :

$$(6) \quad \mathcal{E}(y+x) + \mathcal{E}(y-x) = 2\,\mathcal{E}(x)\mathcal{E}(y)$$

$$(7) \quad p(y+x) + p(y-x) = 2\,\mathcal{E}(x)p(y)$$

The following physical assumption allows to obtain the general form of the solutions :

Compensation principle :

For any distribution of finite forces, whether continuous or not, applied at finite distances along the lever, it is possible to find one choise of the compensating force (m_0 ,x_0) in order to obtain equilibrium. Mathematically, this amounts to assuming integrability of the functions $\varepsilon(x)$ and $p(x)$ on any finite interval. It turns out that the compensation principle implies that the force and torque functions are continuous and derivable to any order (the proof is given in reference (1) and will not be repeated here) . It follows that functional equation (6) can be solved by differentiation, and the following general solution is obtained:

$$(8) \quad \varepsilon(x) = \cosh(x/R)$$

where R is an universal constant, taking real or imaginary values only.

Now, the necessary form of the function p(x) can be shown to be

$$(9) \quad p(x) = R \sinh(x/R)$$

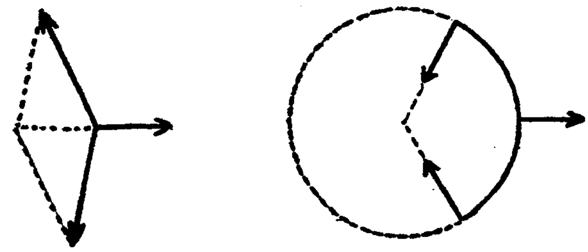
hint : show that p(x) is proportional to the derivative of the function $\varepsilon(x)$ by taking the difference of equations (3) and (4) in the case of an infinitesimal translation dy .

Remarks :

a) The normalisation factor R in (9) is chosen in order that one recovers Archimedes law in the special case $R \rightarrow \infty$ and in the general case when $x \ll R$.

b) The preceding deduction can be applied without any change to the equilibrium of forces passing through the same point : concurring forces or forces applied perpendicularly to an arc of circle.

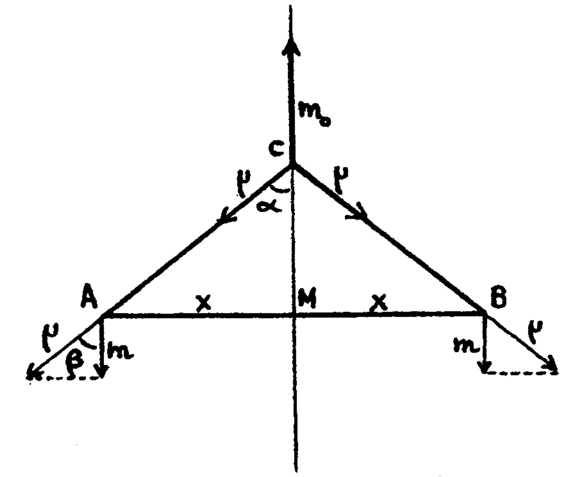
If one replaces translational by rotational invariance, the lengths x are replaced by the angles Θ , the periodicity constrains the constant R to imaginary values, and we obtain $\varepsilon = \cos\Theta$, $p = \sin\Theta$,which in Euclidean space are nothing but the projections of a unit force on orthogonal axes.



c) In one dimension, the force law and the torque law, its derivative, must give one sole condition : it follows that opposite forces applied to a solid line and parallel to it are in equilibrium.

2) Non-Euclidean geometry :

The actual value of the universal constant R can only be given by measurements. Its physical interpretation requires the consideration of a many-dimensional case. Let us analyze the two-dimensional experiment where the compensating force m_0 is applied to the vertex C of the solid triangle ABC, and equal forces m are applied to B and A: **(10)**,



The application of forces m is equivalent to that of forces

$$(10) \quad \rho = m/\cos\beta$$

along the straight lines AC and BC.

The equilibrium of the solid line AC requires application of opposite forces to its both ends, and the same holds true for BC.

The three forces applied to vertex C nullify each other, giving :

$$(11) \qquad m_o = 2\,\mu \cos\alpha$$

Hence, the equilibrium condition takes the form :

$$(12) \qquad m_o = 2\, m \cos\alpha / \cos\beta$$

As it amounts to the same to apply the compensating force to vertex C as in the middle M of AB (because opposite forces applied to both ends of a straight solid line parallel to this line nullify each other at equilibrium), the equilibrium condition reads as previously

$$(13) \qquad m_o = 2\, m \cosh x/R$$

By a comparison of both forms (12) and (13) of the equilibrium condition we obtain the **purely geometric relation :**

$$(14) \qquad \cos\alpha / \cos\beta = \cosh x/R$$

The sum of the angles of the triangle ACM is $\pi + \alpha - \beta$. As $\alpha > \beta$ if R is real, $\alpha < \beta$ if R is imaginary, and $\alpha = \beta$ in the limit $R \to \infty$, we obtain the following **classification of geometries**, according to the values of R, which therefore appears to be the **curvature of space (10) :**

R real : Lobachevskian geometry

R imaginary : spherical geometry

R : Euclidean geometry

Pushing the point C upwards or downwards to infinity (this is possible only when R is real) we obtain **two parallels to CM through the point A.**

When $\alpha \to 0$ the angle β tends to a limit which is given by

$$(15) \qquad \cos\beta = 1 / \cosh(x/R)$$

or equivalently by Lobacheski's famous formula for the angle of parallelism :

$$(16) \qquad \mathrm{tang}\,\frac{\frac{\pi}{2}-\beta}{2} = \exp -x/R$$

3) Dynamics of particle collisions.

The preceding deductive scheme of the laws of statics can be transposed just by changing words. Particle collisions are local processes which have been extensively studied in my previous work (1,2). The transposition is made possible because of the existence of the **rapidity, an additive velocity parameter,** along any geodetic line, which is associated to the relativistic invariance. The parameter m becomes the mass of particles, and formulas (1)' and (2)' are the conservation laws of energy and momentum respectively, the expressions of which being given by

$$\mathcal{E}(x) = \cosh(x)$$

and $$p(x) = \sinh(x)$$

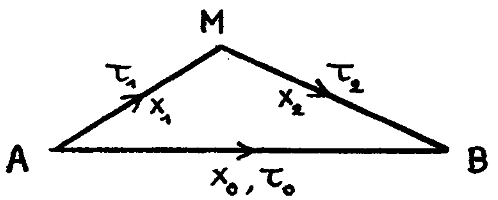
in terms of the rapidity x. The actual velocity is related to the rapidity by the formula established in reference (1) and which will be recovered in the next section :

$$(17) \qquad v = c\ \mathrm{tangh}\ x$$

It may be shown that the velocity space is a Lobachevskian space, the curvature being the constant c in the preceding formula. This constant appears to be **the maximal velocity of any energy propagation,** which like R can only be determined by measurements. We recover special relativistic dynamics, dispensing with the principle of invariance of the speed of light.

4) Kinematics.

Metrical properties of space-time intervals can be investigated in a fully analogous manner, by means of conservation rules. Consider for instance two travellers starting on a journey simultaneously at the point A, and meeting again simultaneously at the point B. The first travels at constant velocity from A to the intermediate point M, and then at another constant velocity from M to B. The second travels directly and at constant velocity from A to B. Since the coincidences at the end points are absolute, any observer, whether at rest or in motion, will agree with the "conservations" of elapsed time and covered distance AB for the two ways of travelling from A to B.



If we write the time and space functions in the form

$$t = \tau\, \varepsilon(x)$$

and $$r = \tau\, p(x)$$

the space-time "conservations" take in one dimension a form analogous to the lever equilibrium laws (1)' and (2)', where x and m are now replaced by the rapidity and the eigen time (the time indicated by the clock of the traveller) respectively. So we obtain immediately

$$\varepsilon \sim \cosh x$$

$$p \sim \sinh x$$

and $$v = r/t \sim \text{tangh } x$$

It is easy to derive the Lorentz transformation from these expressions.

References.

1. COMTE C. 1986, Eur.J.Phys., 7, 225-235

2. COMTE C. Proceedings of the Int. Conf. "Physical Inter--pretations of Relativity Theory", London, Imp.College, Sept.1988, and to be published in Eur.J.Phys.

3. EINSTEIN A. 1917, Ann.Phys.49

4. EINSTEIN A. 1918, Ann.phys. 55, 241

5. KRETSCHMANN E. 1917, Ann.Phys. 53, 575

6. LEVY-LEBLOND J.M. 1976, Am.J.Phys., 44, 271

7. MACH. E. : Die MECHANIK, historisch-kritisch dargestellt, 1883, and

   BORN. M. : EINSTEIN'S THEORY of RELATIVITY, Dover 1962

8. NEWTON I. PRINCIPIA MATHEMATICA PHILOSOPHIAE NATURALIS (translation of the original Latin by Andrew Motte, 1729 ; traduit du latin par Madame la Marquise du Chastelet, 1759)

9. COMTE C. and LOGIOTATOU M. to be published

10. BONOLA R. (1906) LA GEOMETRIA NON-EUCLIDEA (english translation : Dover 1955