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EQUATIONS AUX DERIVEES PARTIELLES

REGULARITY PROPERTIES OF THE GENERALIZED HAMILTONIAN FLOW

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1 Introduction

Let S be a symplectic manifold with boundary ∂S and let $p : S \longrightarrow \mathbf{R}$ be a smooth (C^∞) function with $dp|_{\partial S} \neq 0$. Following [MS] (see also sec. 24.3 in [H]), one defines the generalized Hamiltonian flow of p as follows.

Let $\varphi \in C^\infty(S)$ be a defining function of ∂S , i.e. $\varphi > 0$ in $S \setminus \partial S$ and $\varphi = 0$ on ∂S (φ might be only locally defined around ∂S). Assume that

$$\{\varphi, \{\varphi, p\}\} \neq 0.$$

We are going to define the flow of p on the zero level set

$$\Sigma = p^{-1}(0).$$

Consider the following subsets of Σ :

$$G = \{\sigma \in \Sigma : \varphi(\sigma) = H_p \varphi(\sigma) = 0\} \quad (\text{glancing set}),$$

$$G_d = \{\sigma \in G : H_p^2 \varphi(\sigma) > 0\} \quad (\text{diffractive set}),$$

$$G_g = \{\sigma \in G : H_p^2 \varphi(\sigma) < 0\} \quad (\text{gliding set}),$$

$$G^k = \{\sigma \in G : H_p^j \varphi(\sigma) = 0 \quad \forall \quad j = 0, 1, \dots, k-1\},$$

$$G^\infty = \bigcap_{k=2}^{\infty} G^k.$$

The *gliding vector field* H_p^G on G is defined by

$$H_p^G = H_p + \frac{H_p^2 \varphi}{H_\varphi^2 p} H_\varphi.$$

Definition ([MS]). Let $I \subset \mathbf{R}$ be an interval. A curve $\gamma : I \longrightarrow \Sigma$ is called a *generalized integral curve (bicharacteristic)* of p if there exists a discrete subset B of I such that:

(i) if $t \in I \setminus B$ and $\gamma(t) \in (S \setminus \partial S) \cup G_d$, then there exists

$$\gamma'(t) = H_p(\gamma(t));$$

(ii) if $t \in I \setminus B$ and $\gamma(t) \in G \setminus G_d$, then there exists

$$\gamma'(t) = H_p^G(\gamma(t));$$

(iii) for each $t \in B$, $\gamma(t+s) \in S \setminus \partial S$ for all small $s \neq 0$ and there exist the limits $\gamma(t-0) \neq \gamma(t+0)$ which are points of one and the same integral curve of φ on ∂S .

Clearly, such a curve γ has discontinuities at the points of B . To get a continuous curve we have to identify some pairs of points on ∂S . Consider the following equivalence relation on Σ : $x \sim y$ iff either $x = y$ or $x \in \Sigma \cap \partial S$, $y \in \Sigma \cap \partial S$ and x and y lie on one and the same integral curve of φ on ∂S . The quotient space $\tilde{\Sigma} = \Sigma / \sim$, which carries a natural structure of a manifold with boundary, is called *compressed characteristic set* and the projection $\tilde{\gamma}$ of a generalized integral curve γ on $\tilde{\Sigma}$ is a continuous curve called *compressed integral curve* of p .

In what follows we assume that

$$G^\infty = \emptyset.$$

In this case one can define a flow

$$F_t : \tilde{\Sigma} \longrightarrow \tilde{\Sigma} \quad , \quad t \in \mathbf{R},$$

such that $\{F_t : t \in \mathbf{R}\}$ is a compressed integral curve of p for each $\sigma \in \tilde{\Sigma}$ (cf. [MS]). It was shown in [MS] that the maps F_t are continuous.

Remark. It is clear from the definition that the maps F_t depend on φ . In general φ is only locally defined and so in such cases $\{F_t\}$ is a local flow defined for small $|t|$. However, the integral curves of p , disregarding their parametrization, are globally defined and do not depend on φ . To avoid the inconvenience caused by the change of the parameter along integral curves, one may consider maps between cross-sections of a given integral curve (the same definition as that of a Poincaré map). Since the problem we deal with below is of local nature, and locally the maps between cross-sections and F_t have equivalent behaviour, we consider the maps F_t as if they were globally defined.

Note that in general the maps F_t are not smooth. This is easily seen for

$$S = T^*(\Omega \times \mathbf{R}), \tag{1}$$

Ω being a domain in $\mathbf{R}^n$ with smooth boundary $\partial\Omega$, and p given by

$$p(x, \xi) = \sum_{i=1}^n \xi_i^2 - \xi_{n+1}^2. \tag{2}$$

An elementary argument shows that if Ω is the interior or the exterior of a ball in $\mathbf{R}^n$, then the maps F_t are Hölder continuous with Hölder exponent $\frac{1}{2}$, and $\frac{1}{2}$ is the maximal number with this property.

It is natural to ask if the maps F_t are Hölder continuous in the general case. In the present talk we consider some partial results in this direction.

Given S and p as in the beginning of this section, fix an arbitrary metric d on $\tilde{\Sigma}$ generating its topology.

Theorem 1. *Let $\rho_0 \in \tilde{\Sigma}$ and $T_0 > 0$ be fixed. There exist constants $C > 0$ and $\alpha > 0$ such that*

$$d(F_t \rho_0, F_t \rho) \leq C(d(\rho_0, \rho))^\alpha \quad (3)$$

for every $\rho \in \tilde{\Sigma}$ and every t with $|t| \leq T_0$.

For $k = 2, 3, \dots$ denote

$$G_+^k = \{\sigma \in G^k : H_p^k \varphi(\sigma) > 0\}.$$

Theorem 2. *Let K be a compact subset of Σ and $T_0 > 0$ be such that*

$$F_t(K) \subset G_g \cup \bigcup_{k=2}^{\infty} G_+^k \quad \forall t \in [0, T_0]. \quad (4)$$

Denote by $\tilde{K}$ the projection of K in $\tilde{\Sigma}$. Then there exist constants $C > 0$ and $\alpha > 0$ such that

$$d(F_t \sigma, F_t \rho) \leq C(d(\sigma, \rho))^\alpha$$

for all $\sigma, \rho \in \tilde{K}$ and $t \in [0, T_0]$.

It is natural to expect that the assertion of Theorem 2 remains true without assuming (4). Actually the proof of Theorem 2 is much easier than that of Theorem 1. That is why below we restrict our attention to Theorem 1. A scheme of its proof is given in section 3.

2 Motivation

In this section we briefly discuss a problem comming from the scattering theory, which indicates that regularity properties of the generalized Hamiltonian flow might be useful.

Let Ω be a domain in $\mathbf{R}^n, n \geq 3, n$ odd, with C^∞ boundary $\partial\Omega$ such that

$$K = \overline{\mathbf{R}^n \setminus \Omega}$$

is compact. Define S and p by (1) and (2), respectively.

The *scattering operator* related to the wave equation in $\mathbf{R} \times \Omega$ with Dirichlet boundary conditions on $\mathbf{R} \times \partial\Omega$ can be represented as an unitary operator

$$S : L^2(\mathbf{R} \times S^{n-1}) \longrightarrow L^2(\mathbf{R} \times S^{n-1})$$

(see [LP1]). The kernel of $S - \text{Id}$, which can be considered as a distribution

$$s_K(t, \theta, \omega) \in \mathcal{D}'(\mathbf{R} \times S^{n-1} \times S^{n-1}),$$

is called the *scattering kernel*.

The following **problem** arises: is it true that there exists subset R of full Lebesgue measure in $S^{n-1} \times S^{n-1}$ such that

$$\text{sing supp } s_K(t, \theta, \omega) = \{-T_\gamma : \gamma \in \mathcal{L}_{\omega, \theta}\} \quad (5)$$

for all $(\omega, \theta) \in R$? Here $\mathcal{L}_{\omega, \theta}$ is the set of all (ω, θ) -rays in Ω , i.e. infinite continuous curves in Ω with incoming direction ω and outgoing direction θ which are projections of generalized integral curves of p in S . By T_γ we denote the sojourn time of $\gamma \in \mathcal{L}_{\omega, \theta}$ (see [PS1] or ch. 1 in [PS2] for the precise definitions). There is no doubt that the right-hand side of (5) contains certain geometric information about the obstacle K , and so if (5) holds for sufficiently many pairs (ω, θ) , one could get this information knowing the singularities of the scattering kernel for the same pairs (ω, θ) . It is already known that this can be done for a special class of obstacles K . More precisely, the answer to the above question is affirmative, provided K is a finite union of disjoint convex bodies ([PS3]). Using this fact, it was shown in [S] that if K and L are two obstacles, each of them being a finite disjoint union of convex bodies, satisfying an additional condition (H) of M. Ikawa [I], and if

$$\text{sing supp } s_K(t, \theta, \omega) = \text{sing supp } s_L(t, \theta, \omega)$$

for almost all $(\omega, \theta) \in S^{n-1} \times S^{n-1}$, then $K = L$. For convex obstacles K and L such a result was established by Majda [Ma] (see also Lax and Phillips [LP2]).

Turning back to the question posed above, let us consider one possible way to deal with it. In fact, it follows from the results in [PS3] that to give an affirmative answer, it is sufficient to establish the existence of a set R of full Lebesgue measure in $S^{n-1} \times S^{n-1}$ such that for $(\omega, \theta) \in R$ there are no (ω, θ) -rays of mixed type in Ω , i.e. (ω, θ) -rays having non-trivial segments lying on $\partial\Omega$. To do so consider a fixed (ω, θ) -ray γ of mixed type in Ω and take a point (z, ζ) contained in a gliding segment of γ (lying entirely on $\partial\Omega$). Denote by G_t the generalized geodesic flow on $S^*(\Omega)$ generated by the flow F_t , $S^*(\Omega)$ being the cosphere bundle of Ω . Then taking a sufficiently large rational number $q > 0$, we have

$$\omega = \text{pr}_2 G_{-q}(z, \zeta) \quad , \quad \theta = \text{pr}_2 G_q(z, \zeta),$$

where $\text{pr}_2(y, \eta) = \eta$. This can be written as

$$(\omega, \theta) = W_q(z, \zeta),$$

$W_q : S_{\partial\Omega}^*(\Omega) \longrightarrow S^{n-1} \times S^{n-1}$ being defined by

$$W_q(y, \eta) = (\text{pr}_2 G_{-q}(y, \eta), \text{pr}_2 G_q(y, \eta)).$$

The choice of (z, ζ) now shows that

$$(\omega, \theta) \in W_q(S^*(\partial\Omega)). \quad (6)$$

More generally, it is clear that the existence of a (ω, θ) -ray of mixed type is equivalent to the existence of a rational $q > 0$ with (6). Consequently, the set of those pairs (ω, θ) for which there exist (ω, θ) -rays of mixed type is contained in

$$R_0 = \bigcup_{q \in \mathbf{Q}, q > 0} W_q(S^*(\partial\Omega)).$$

Since

$$\dim S_{\partial\Omega}^*(\Omega) = \dim S^{n-1} \times S^{n-1} = 2(n-1),$$

and $\dim S^*(\partial\Omega) = 2n-3$, it is natural to expect that $W_q(S^*(\partial\Omega))$ has Lebesgue measure zero in $S^{n-1} \times S^{n-1}$. This will be so provided W_q has some "good" regularity properties, which could be eventually derived from corresponding properties of the flows G_t and F_t . Unfortunately, our Theorems 1 and 2 do not provide such properties.

3 Sketch of the proof of Theorem 1

A standard compactness argument shows that the assertion of Theorem 1 is a consequence of the following (local) lemma.

Lemma 1. *Let $\rho_0 \in \tilde{\Sigma}$ be fixed. There exist a neighbourhood U_0 of ρ_0 in $\tilde{\Sigma}$ and constants $T > 0, C > 0, \alpha > 0$ such that (3) holds for all $\rho \in U_0$ and $t \in [0, T]$.*

Denote again by ρ_0 an element of Σ the projection of which in $\tilde{\Sigma}$ coincides with ρ_0 . It follows by [MS] (cf. also sec. 24.3 in [H]) that there exist local coordinates

$$(x, \xi) = (x_1, \dots, x_n; \xi_1, \dots, \xi_n)$$

around $\rho_0 = (0, 0)$ in S such that $\varphi = x_1$, i.e. locally

$$S = \{(x, \xi) : x_1 \geq 0\} \quad , \quad \partial S = \{(x, \xi) : x_1 = 0\},$$

and

$$p(x, \xi) = \xi_1^2 - r(x, \xi'),$$

r being a smooth function. Throughout we use the notation

$$x' = (x_2, \dots, x_n) \quad , \quad \xi' = (\xi_2, \dots, \xi_n).$$

Define the metric d by

$$d((x, \xi), (y, \eta)) = \max_{1 \leq i \leq n} \max\{|x_i - y_i|, |\xi_i - \eta_i|\},$$

and set

$$F_t(x, \xi) = (x(t), \xi(t)).$$

There are several cases for ρ_0 .

case 1. $\rho_0 \in S \setminus \partial S$. In this case locally around ρ_0 the generalized integral curves of p coincide with the integral curves of the Hamiltonian vector field H_p , so the assertion follows trivially with $\alpha = 1$.

case 2. $\rho_0 \in G_d$. This means that $\frac{\partial r}{\partial x_1}(\rho_0) > 0$. Then there exists a neighbourhood V_0 of ρ_0 in S and a constant $c > 0$ with $\frac{\partial r}{\partial x_1}(\rho) \geq c$ for all $\rho \in V_0$. Choose a neighbourhood U_0 of ρ_0 and $T > 0$ such that $F_t(U_0) \subset V_0$ for all $t \in [0, T]$. It then follows by Lemma 24.3.4 in [H] that for each $\rho \in U_0$ the generalized integral curve $\{F_t\rho : t \in [0, T]\}$ has at most one reflection. Using this one can easily derive that the assertion of the lemma holds with $\alpha = \frac{1}{2}$.

case 3. $\rho_0 \in G_g$. As in the previous case, we find neighbourhoods $U_0 \subset V_0$ of ρ_0 and $c > 0$ such that $\frac{\partial r}{\partial x_1}(\rho) \leq -c$ for each $\rho \in V_0$. Using Lemma 24.3.5 from [H] we find a constant $C' > 0$ such that if $\{F_t\rho : t \in [0, T]\}$ is a reflecting bicharacteristic (in this case it is equivalent to say that the bicharacteristic is not entirely contained in G_g), then we have

$$\eta_1^2(t) + y_1(t) \leq C'(\eta_1^2(0) + y_1(0))$$

for all $t \in [0, T]$, where

$$F_t(\rho) = (y(t); \eta(t)).$$

From this the assertion of the lemma follows easily with $\alpha = \frac{1}{2}$.

case 4. $\rho_0 \in G^k \setminus G^{k+1}, k \geq 3$. Let $(\tilde{x}'(t), \tilde{\xi}'(t))$ be the integral curve of the vector field H_p^G on G with initial conditions $\tilde{x}'(0) = x'(0)$, $\tilde{\xi}'(0) = \xi'(0)$. Set

$$e(t) = \frac{\partial r}{\partial x_1}(0, \tilde{x}'(t), \tilde{\xi}'(t)),$$

$$f(t) = |x'(t) - \tilde{x}'(t)| + |\xi'(t) - \tilde{\xi}'(t)|.$$

Given $\rho \in \Sigma$, define $e_\rho(t)$ and $f_\rho(t)$ as $e(t)$ and $f(t)$, respectively, replacing ρ_0 with ρ .

Choose neighbourhoods $U_0 \subset V_0$ of ρ_0 and $T > 0$ so small that H_p^k has a constant sign in V_0 and $F_t U_0 \subset V_0$ for all $t \in [0, T]$. Later we will have to eventually take smaller U_0 and T .

In the case under consideration we have

$$e(t) = at^{k-2} + \lambda(t)t^{k-1}$$

for some constant $a \neq 0$ and some smooth function $\lambda(t)$ (cf. [MS] or [H]). Fix $L > 0$ with

$$|\lambda(t)| \leq \frac{L}{2}, \quad |\lambda'(t)| \leq \frac{L}{2} \quad \forall t \in [0, T].$$

Using standard facts from the theory of differential equations, it follows that if U_0 is small enough, then there exists a constant $c > 0$ such that for every $\rho \in U_0$ we have the representation

$$e_\rho(t) = a_0 + a_1 t + \dots + a_{k-2} t^{k-2} + at^{k-2} + \mu(t)t^{k-1} \quad (7)$$

with

$$|a_i| \leq c\delta \quad \forall i = 0, 1, \dots, k-2; \quad |\mu(t)| \leq L, \quad |\mu'(t)| \leq L \quad \forall t \in [0, T], \quad (8)$$

where

$$\delta = d(\rho_0, \rho). \quad (9)$$

We may assume that $T \leq \frac{1}{2}$, then (7) and (8) imply

$$at^{k-2} - 2c\delta - Lt^{k-1} \leq e_\rho(t) \leq at^{k-2} + 2c\delta + Lt^{k-1} \quad , \quad t \in [0, T]. \quad (10)$$

Next, we distinguish two subcases.

Subcase 4.1. $a < 0$. Fix an arbitrary $\beta > 0$. The assertion of Lemma 1 follows immediately from the following

Lemma 2. U_0 and $T > 0$ can be chosen so small that there exists a constant $A > 0$ with

$$d(F_t \rho_0, F_t \rho) \leq A(d(\rho_0, \rho))^{\frac{1-\beta}{2}} \quad \forall \rho \in U_0, \quad \forall t \in [0, T].$$

Proof of Lemma 2. Set

$$\epsilon = \frac{\beta(k-2)}{1 + (1+\beta)(k-2)}$$

and choose $T > 0$ such that

$$T \leq \frac{\epsilon|a|}{2(k+1)L}.$$

Take $\rho \in U_0$ and set $\delta = d(\rho_0, \rho)$. Then $y_1(0) \leq \delta, |\eta_1(0)| \leq \delta$. The choice of T yields

$$(1 - \frac{\epsilon}{2})at^{k-2} - 2c\delta \leq e_\rho(t) \leq (1 + \frac{\epsilon}{2})at^{k-2} + 2c\delta$$

for all $t \in [0, T]$. Using the inequalities (24.3.7) in [H], it is not hard to see that there exists a constant $C_1 > 0$ such that

$$f_\rho(t) \leq C_1(\delta + |a|t^{k+1}) \quad , \quad y_1(t) \leq C_1(\delta + |a|t^k) \quad , t \in [0, T].$$

Set

$$h(t) = \frac{\partial r}{\partial x_1}(y(t), \eta'(t)),$$

and note that there exists a constant $C_2 > 0$, which does not depend on ρ , with

$$|h(t) - e_\rho(t)| = \left| \frac{\partial r}{\partial x_1}(y(t), \eta'(t)) - \frac{\partial r}{\partial x_1}(0, \tilde{y}'(t), \tilde{\eta}'(t)) \right| \leq C_2(f_\rho(t) + y_1(t))$$

for $t \in [0, T]$ (cf. p. 436 in [H]). Consequently, one finds a constant $C_0 > 0$ with

$$(1 - \frac{\epsilon}{2})at^{k-2} - C_0\delta \leq h(t) \leq (1 + \frac{\epsilon}{2})at^{k-2} + C_0\delta \quad (11)$$

for $t \in [0, T]$.

As in [H], we see that

$$|h'(t) - e'_\rho(t)| \leq \text{const}(f_\rho(t) + y_1(t) + |\eta_1(t)|)$$

for all $t \in [0, T]$ for which $h'(t)$ exists. Using an argument similar to that above, we find a constant $C_0 > 0$ (we may assume this is the same constant as in (11)) such that

$$(k - 2 - \frac{\epsilon}{2})at^{k-3} - C_0\delta \leq h'(t) \leq (k - 2 + \frac{\epsilon}{2})at^{k-3} + C_0\delta \quad (12)$$

for all $t \in [0, T]$.

Consider the function

$$g(t) = \eta_1^2(t) - y_1(t)h(t).$$

It is clearly continuous and $g'(t)$ exists almost everywhere in $[0, T]$. Set

$$t_\delta = \left(\frac{2C_0\delta}{\epsilon|a|} \right)^{\frac{1}{k-2}} = \text{const}\delta^{\frac{1}{k-2}}.$$

For those $t \in [t_\delta, T]$ for which $g'(t)$ exists, (11) and (12) imply

$$\frac{g'(t)}{g(t)} \leq \frac{k-2+\epsilon}{(1-\epsilon)t},$$

and integrating the latter inequality gives

$$g(t) \leq \text{const} \frac{g(t_\delta)}{\delta^{\frac{k-2+\epsilon}{(1-\epsilon)(k-2)}}} = \text{const} \frac{g(t_\delta)}{\delta^{1+\beta}}$$

for $t \in [t_\delta, T]$.

On the other hand, it follows easily by the definition of t_δ that $g(t) \leq \text{const} \delta^2$ for $t \in [0, t_\delta]$. Therefore $g(t) \leq \text{const} \delta^{1-\beta}$ for all $t \in [0, T]$. Consequently, $y_1(t) \leq \text{const} \delta^{1-\beta}$ and $|\eta_1(t)| \leq \text{const} \delta^{\frac{1-\beta}{2}}$ in $[0, T]$. Applying a standard argument from the theory of differential equations to the rest of coordinate functions, one gets

$$d(F_t \rho_0, F_t \rho) \leq \text{const} \delta^{\frac{1-\beta}{2}}$$

for all $t \in [0, T]$. This completes the proof of Lemma 2.

Subcase 4.2. $a > 0$. This case is easier than the previous one. One can define t_δ in a similar way and show that the integral curve $F_t \rho$ has no reflections for $t \in [t_\delta, T]$, provided U_0 is small enough and $\rho \in U_0$. In this way we find

$$d(F_t \rho_0, F_t \rho) \leq \text{const} \delta^{\frac{1}{k-2}}$$

for all $t \in [0, T]$.

References

- [I] M. Ikawa: *Decay of solutions of the wave equation in the exterior of several strictly convex bodies*. Ann. Inst. Fourier **38** (1988), 113-146.
- [H] L. Hörmander: *The Analysis of Linear Partial Differential Operators*, vol. III, Springer-Verlag, Berlin, 1985.
- [LP1] P.Lax, R.Phillips: *Scattering Theory*. Academic Press, New York, 1967.
- [LP2] P.Lax, R.Phillips: *The scattering of sound waves by an obstacle*. Comm. Pure Appl. Math. **30** (1977), 195-233.
- [Ma] A.Majda: *A representation formula for the scattering operator and the inverse problem for arbitrary bodies*. Comm. Pure Appl. Math. **30** (1977), 165-194.

- [MS] R. Melrose, J. Sjöstrand: *Singularities in boundary value problems*, I. Comm. Pure Appl. Math. **31** (1978), 593-617.
- [PS1] V. Petkov, L. Stoyanov: *Singularities of the scattering kernel and scattering invariants for several strictly convex obstacles*. Trans. Amer. Math. Soc. **312**, (1989), 203-235.
- [PS2] V. Petkov, L. Stoyanov: *Geometry of Reflecting Rays and Inverse Spectral Problems*. John Wiley & Sons, Chichester, 1992.
- [PS3] V. Petkov, L. Stoyanov: *Sojourn times of trapping rays and the behaviour of the modified resolvent of the Laplacian*. University of Bordeaux I – C.N.R.S., Preprint 9209, 1992.
- [S] L. Stoyanov: *An inverse scattering result for several convex bodies*. Preprint, 1992.

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